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PLASMA CYCLOTRON RADIATION AND FUSION REACTORS

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(*Received* 26 *April* 1960)

Abstract—The theory of cyclotron radiation from high-temperature plasma confined within a magnetic field is applied to the estimation of the radiation loss from plasma confined in fusion reactors. It is shown how a metallic reflector placed around the plasma should limit the seriousness of this loss. In the case of a mirror machine, comparison is made with the mirror loss and the electrical power dissipated in the coils producing the magnetic field. Ratios of power lost are found as functions of β/η (plasma to magnetic pressure ratio/absorption coefficient of the reflector) and βBR (B is the field strength, R the mean radius) which are also compared with the nuclear yield. For the stabilized pinch and stellarator, cyclotron loss is compared with nuclear yield.

With β not less than 0·1 it is shown that cyclotron loss can be unimportant in all three types of D-T reactor. For the D-D mirror machine the minimum necessary value of BR would be about 2×10^8 gauss cm, whilst for the pinch or stellarator the minimum necessary current or BR, based only on radiation loss, is 18×10^6 amperes or $3{\cdot}5 \times 10^6$ gauss cm respectively.

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1. INTRODUCTION

PROPOSALS for containing a plasma under the conditions necessary for a thermonuclear reactor require the plasma to be wholly or partly immersed within a strong magnetic field (ARTSIMOVICH, 1958; POST 1958; SPITZER, 1958; THONEMANN, 1958). It is well known, however, that charged particles radiate energy when moving in a circular orbit, a particular case being the spiral motion of electrons in a confining magnetic field (LIENARD, 1898; SCHOTT, 1912; LANDAU and LIFSHITZ, 1948; SCHWINGER, 1949). This radiation has come to be known variously, but equivalently, as cyclotron, synchrotron, or magnetic bremsstrahlung. Cyclotron radiation has been treated theoretically by the above authorities and was first observed, in good quantitative agreement with theory, in high-energy, circular, particle accelerators (ELDER, LANGMUIR and POLLOCK, 1948). In these machines it eventually represents an enormous loss of energy from high-energy electrons. Likewise, a hot plasma ($kT \geq 0{\cdot}1$ $\times$ electron rest mass energy) can lose energy through the cyclotron radiation of its electrons when placed in a magnetic field (ALFVÉN, 1950; TRUBNIKOV and KUDRYAVTSEV, 1958 (abbreviated to T & K)).

The purpose of this paper is to estimate this loss in the case of certain suggested thermonuclear reactors and to compare it with other losses which are additionally incurred by other known processes. Finally the sum of these is compared with the output of fusion energy in order to estimate some necessary parameters for a power-producing reactor.

The calculations are based on the results of TRUBNIKOV *et al.* (1958), whose work has led to an important re-appraisal of the effects of cyclotron radiation. It has there been pointed out that, in a very hot plasma, relativistic effects cause the radiation spectrum to extend effectively over many (10 or 20) multiples of the fundamental cyclotron frequency ω_0 and to become essentially a continuous rather than a line spectrum. At frequencies much higher than either ω_0, or the plasma frequency ω_p, the plasma is partially transparent to radiation and does not reflect it internally. It is therefore capable of radiating as a *quasi* black body up to relatively high frequencies. This implies that most of the cyclotron radiation escaping from a plasma of practical density would have a wavelength $\sim 0{\cdot}1 - 0{\cdot}01$ mm (infra-red). Consequently Budker (T & K, 1958) has suggested that this radiation may be returned to the plasma by a metal reflector placed around it to mitigate losses. This important concept is included in the calculations presented here.

The mirror machine is considered separately from the stabilized pinch and stellarator because of its radically different concepts. Since the operating temperature of a mirror machine (200–1000 keV) has

to be much higher than the others (10–50 keV) one expects that cyclotron radiation is likely to be far more crucial in its case, especially so if deuterium alone were used as a fuel.

2. THE THEORY OF THE CALCULATION OF CYCLOTRON RADIATION AND ABSORPTION FROM HOT PLASMA

An electron, moving at velocity v, with components $v_\perp$ and $v_\parallel$ perpendicular and parallel to a uniform static magnetic field of strength B_0, radiates total power q given by (LANDAU and LIFSHITZ, 1948):

$$q = \frac{2}{3}\frac{e^2}{c^3}\omega_0^2\gamma^2 v_\perp^2. \tag{2.1}$$

where $\omega_0 = |eB_0/m_0c|$, m_0 is the rest mass, e the charge (e.s.u.), c the velocity of light *in vacuo* and $\gamma \equiv (1 - (v/c)^2)^{-1/2}$.

The spectrum of the radiated power consists of an infinite series of lines at frequencies ω given by $r\omega_0/\gamma = \omega\left(1 - \frac{v_\parallel \cos\theta}{c}\right)$ where $r = 1, 2, \ldots, \infty$ and θ is the angle between the direction of propagation and the magnetic field. The term in brackets gives the Doppler shift of the relativistic harmonics seen by a stationary observer. The expression for the intensity of radiation per unit solid angle Ω at each line frequency is given by TRUBNIKOV (1958) and may be derived from standard formulas given, for example, in LANDAU and LIFSHITZ (1948) or SCHWINGER (1949). It is a slight modification to the expression due to SCHOTT (1912)

$$\frac{dI}{d\Omega} = \sum_{r=1}^{\infty}\frac{dI_r}{d\Omega}$$

$$= \frac{e^2\omega^2}{2\pi c}\sum_{r=1}^{\infty}\left[(\cot\theta - \frac{v_\parallel}{c}\sin\theta)^2 J_r^2 + \left(\frac{v_\perp}{c}\right)^2 J_r'^2\right] \tag{2.2}$$

The argument of the Bessel function J_r and its derivative J_r' is $\gamma\frac{\omega}{\omega_0}\frac{v_\perp}{c}\sin\theta$.

Assuming for estimation that $v_\parallel \sim v_\perp \sim v$ and $\cos\theta \sim 1$, then for v/c small the principal part of the shift about the rth harmonic is the Doppler shift $|\Delta\omega_r| \sim \frac{v}{c} r\omega_0$. For a Maxwellian distribution of electrons at temperature $T_e (kT_e \ll m_0c^2)$, the shifts become comparable with the distances between lines when $\Delta\omega_r \geq (r+1)\omega_0 - r\omega_0$ namely when $rv/c \geq 1$ or $r \geq \sqrt{m_0c^2/kT_e}$. The spectrum of the electron distribution at sufficiently high harmonics is *quasi* continuous as TRUBNIKOV points out.

It is assumed that the electrons are randomly distributed in phase so that there is no correlation between their radiation fields. This is equivalent to there being no co-operative interaction, or bunching, and implies that the energy radiated from n electrons in a particular volume is proportional to n and not n^2. This appears justified, at least in experiment with accelerated beams (ELDER, *et al.*, 1948). To calculate the net radiation escaping from a large but finite volume of plasma, one must calculate the transmission and absorption properties of plasma to its own cyclotron radiation, namely the real and imaginary parts, $n(\omega)$ and $\alpha(\omega)$ respectively, of the refractive index as functions of frequency. TRUBNIKOV *et al.* (1959) and T & K (1958) have calculated $\alpha(\omega)$ both directly from the Boltzmann equation and also by applying the principle of detailed balancing. Their calculations neglect collisions, viz. those processes which give rise to collision bremsstrahlung and its inverse absorption. The effect of re-absorption at low frequencies is to cut away the height of the emission spectrum $I(\omega)$ in excess of that given by the Rayleigh-Jeans law for each polarization,

$$\frac{dI(\omega)}{d\Omega} = \frac{\omega^2 kT}{8\pi^3c^2} \quad \begin{array}{l}\text{(power per unit emitting area}\\ \text{per unit angular frequency}\\ \text{per unit solid angle).}\end{array} \tag{2.3}$$

For a given volume of plasma there will be a limiting frequency ω^*, usually $\gg\omega_0$, beyond which the entire emission falls below the Rayleigh-Jeans black body intensity. For frequencies $<\omega^*$ the plasma is opaque ('black') while for frequencies $>\omega^*$ it is partially transparent.

The emission spectrum falls almost exponentially $\sim\exp -2/3r$ where $r = \frac{\omega}{\omega_0} \gg 1$ (LANDAU and LIFSHITZ, 1948) so that emission beyond ω^* is only approximately $\frac{9}{2}\frac{\omega_0}{\omega^*}$ of the total and may be neglected in making an estimate. Thus once ω^* is estimated, the Rayleigh-Jeans expression cut off beyond ω^* gives almost the entire spectrum of cyclotron radiation from a finite volume of plasma. T & K (1958) estimate ω^* for a plasma slab of thickness L from their value of $\alpha(\omega)$ by putting $L\alpha(\omega^*) \equiv 1$. For the sake of simplicity their numerical calculations were confined to polarized radiation propagated entirely perpendicular to the static magnetic field $B_0 (\theta = \pi/2)$. With this simplification, of the two possible polarizations, the wave with electric vector perpendicular to B_0

dominates. An approximate expression for $\alpha(\omega)$ for perpendicular propagation, due to TRUBNIKOV, is

$$\alpha(\omega) \cong \frac{\omega_p^2}{c\omega_0}\sqrt{\pi\mu}\,\frac{3}{2g} \times \exp -\mu\left[g^{1/3} - 1 + \frac{9}{20}g^{-1/3} + \ldots\right] \quad (2.4)$$

where $g \equiv \frac{9}{2\mu}\frac{\omega}{\omega_0}$ and $\mu \equiv m_0c^2/kT_e$

and $\omega_p = \sqrt{4\pi ne^2/m_0}$ is the plasma frequency with n = electron density. The expansion is only valid for $\mu \gg 1$ and $g \gg 1$. If one defines a 'magnetic length' $l \equiv B/4\pi ne$ then

$$1 \equiv \frac{L}{l}\frac{\sqrt{\pi}}{3}\frac{\omega_0}{\omega^*}\,\mu^{3/2}\exp -\mu\left[g^{*1/3} - 1 + \frac{9}{20}g^{*-1/3}\right] \quad (2.5)$$

is an expression determining $\frac{\omega^*}{\omega_0}$ in terms of μ and $\Lambda \equiv L/l$. The limits of validity for the expansion and its evaluation are $g^* = \frac{9}{2\mu}\frac{\omega^*}{\omega_0} \gg 1$, $\mu \gg 1$ which implies $\frac{9}{2}\frac{\omega^*}{\omega_0} \gg \mu \gg 1$. In addition there is the further requirement $\frac{\omega^*}{\omega_0} \geq \mu^{1/2}$, but this is contained in the foregoing. The derivation assumes that the real part of the refractive index of the plasma $n(\omega)$ is essentially unity. While TRUBNIKOV *et al.* (1959) have investigated $n(\omega)$ they find that its calculation leads to excessive algebraic complication. However, one may note that $n(\omega)$ differs from unity only by a small quantity of order $\omega_p^2/\omega_0\omega$. One is justified in taking n as effectively unity if $\frac{\omega^*}{\omega_0} \gg \frac{\omega_p^2}{\omega_0^2}$.

It will be seen that these limitations can nevertheless correspond to regions of great interest in fusion applications.

Because the validity and accuracy of these results has been questioned (DRUMMOND and ROSENBLUTH, 1960) one should observe that TRUBNIKOV *et al.* (1959) also estimate the effect of the angular dependency of the radiation which tends to be confined to a cone in the perpendicular direction. They have also omitted the smaller component of polarization $E//B_0$. These two modifications to the simple ($\theta = \pi/2$) analysis are incorporated in the final results displayed in the Geneva paper which are adopted here. Even so, in view of the assumptions and approximations made, the results can only be regarded generally as an estimate (with perhaps an error better than 50 per cent) when applied to specific cases. It must also be noted that the parameters derived subsequently here depend rather sensitively on these results.

The total cyclotron radiation from unit volume of plasma disregarding re-absorption and assuming $kT_e \ll m_0c^2$ is (T & K, 1958)

$$Q_c = \frac{4}{3}\frac{e^2}{c}\omega_0^2\, n\,\frac{kT_e}{m_0c^2}\left(1 + \frac{5}{2}\frac{kT_e}{m_0c^2} + \ldots\right) \quad (2.6)$$

Let S be the power radiated from unit surface of the plasma. In a plasma of limited extent only $K(<1)$ of the total power radiated within its volume eventually escapes from the surface. In a long cylinder of plasma of radius R

$$2\pi RS = K\pi R^2 Q_c. \quad (2.7)$$

In their calculations in plane geometry T & K (1958) define K as

$$2S = KLQ_c. \quad (2.8)$$

Assuming that there is little difference in the calculation of S as between the cylindrical and plane cases, one may take over the results from the slab to the cylinder by writing R for L. The flux of radiation from the plasma surface is found by integrating the Rayleigh-Jeans spectrum as far as ω^* which is evaluated numerically from (2.5). Strictly, ω^* is a function of θ, a correction which has been allowed for by TRUBNIKOV *et al.* (1959). S and Q_c are substituted in (2.8) to give K and T & K (1958) have thus shown that K may be expressed numerically in terms of T_e and the quantity Λ defined as

$$\frac{4\pi neL \text{ (or } R)}{B} = \Lambda(K,T_e). \quad (2.9)$$

If, in addition, the plasma cylinder is surrounded by an imperfectly reflecting shield which absorbs a small fraction $\eta(\ll 1)$ of radiation falling upon it, the net loss of radiated power per unit volume of plasma is

$$\frac{2\pi R\eta S}{\pi R^2} = \eta KQ_c. \quad (2.10)$$

3. LOSSES FROM A MAGNETIC MIRROR MACHINE

(a) *Particle loss*

In the present concept of the mirror machine, a plasma of very hot ions and electrons is confined between two mirror fields (POST, 1958; ARTSIMOVICH, 1958). For the half-life of containment of the particles POST gives

$$t_{1/2} = 2{\cdot}6 \times 10^{10}\,\frac{W^{3/2}}{n}\ \text{sec.} \quad (3.1)$$

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where W in keV is the mean ion energy and n is the ion density cm^{-3}. The containment time depends only weakly on the mirror field ratio and we shall not consider here the possibility of enhancing the confinement by rotating the plasma (ANDERSON *et al.*, 1958). Allowing for the fact that slow particles are scattered out more rapidly than faster ones, each ion on average carries away approximately $\frac{1}{3}W$. Neglecting the contribution from escaping electrons the power loss per cm^{-3} of plasma is then

$$Q_M = \tfrac{1}{3}Wn/t_{1/2}$$
$$= 2{\cdot}1 \times 10^{-20}\, n^2 W^{-1/2} \text{ ergs cm}^{-3} \text{ sec}^{-1}. \qquad (3.2)$$

(b) *Radiation loss*

The trapped ions lose energy through collisions to electrons which radiate energy from the plasma. In a steady state the electrons maintain an equilibrium temperature T_e with mean energy 3/2 kT_e(keV), usually considerably less than W. The rate of energy transfer from an ion of energy W to a Maxwellian distribution of electrons is given by SPITZER (1956), from which can be found the rate of energy transfer Q_{ie} from ions to electrons per unit volume.

$$Q_{ie} = 1{\cdot}4 \times 10^{-21}\, n^2 W (kT_e)^{-3/2} \text{ ergs cm}^{-3} \text{ sec}^{-1}. \quad (3.3)$$

The assumption of equilibrium between collision transfer of energy and its radiation gives

$$Q_{ie} = \eta K Q_e. \qquad (3.4)$$

Let us define β as the ratio of plasma pressure, due mainly to ions, to the magnetic pressure

$$\beta \equiv \frac{8\pi n}{B^2}(1{\cdot}6 \times 10^{-9} W \text{ keV}). \qquad (3.5)$$

It is believed that β should be less than 0·1 in mirror systems to ensure stability (BERKOWITZ *et al.*, 1958). Substituting β in (2.6) gives

$$Q_e = 2{\cdot}6 \times 10^{-25}\, n^2 W \frac{kT_e}{\beta} \text{ ergs cm}^{-3} \text{ sec}^{-1} \quad (3.6)$$

and hence from (3.3), (3.4) and (3.6),

$$\beta/\eta = 1{\cdot}8 \times 10^{-4} K (kT_e)^{5/2}. \qquad (3.7)$$

Defining Γ as the ratio of the nett loss from cyclotron radiation to the nett loss from the mirrors

$$\Gamma = \frac{\eta K Q_e}{Q_M}. \qquad (3.8)$$

Since from (3.2) and (3.3)

$$\frac{Q_{ie}}{Q_M} = 0{\cdot}071 \left(\frac{W}{kT_e}\right)^{3/2} \qquad (3.9)$$

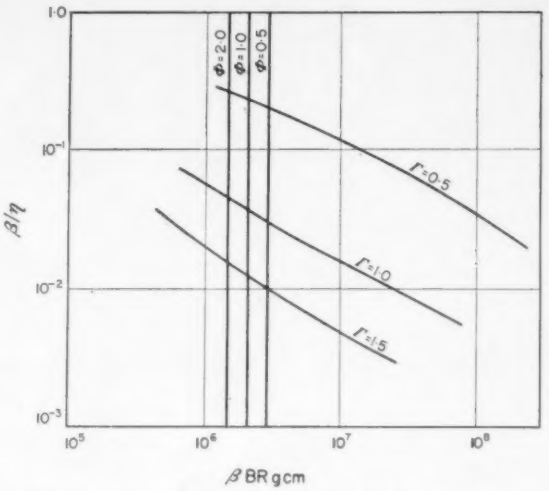

FIG. 1.—β/η against βBR gauss cm for ion energy $W =$ 200 keV.

when (3.4) holds

$$\Gamma = 0{\cdot}071 \left(\frac{W}{kT_e}\right)^{3/2}. \qquad (3.10)$$

Using (2.9) and (3.5) gives

$$\beta BR = 6{\cdot}8\, W\Lambda \text{ gauss cm}. \qquad (3.11)$$

Fixing W and Γ one may eliminate T_e by (3.10) and K by (3.7) to conveniently relate β/η and βBR to these two parameters as shown in the diagrams (Figs. 1, 2 and 3).

Additional quenching of electrons in order to lower T_e and reduce the cyclotron radiation does not help maintain the energy of the ions. Although radiation alone may be reduced, it can be seen from (3.9) that the overall transfer of energy from ions Q_{ie} is even further increased.

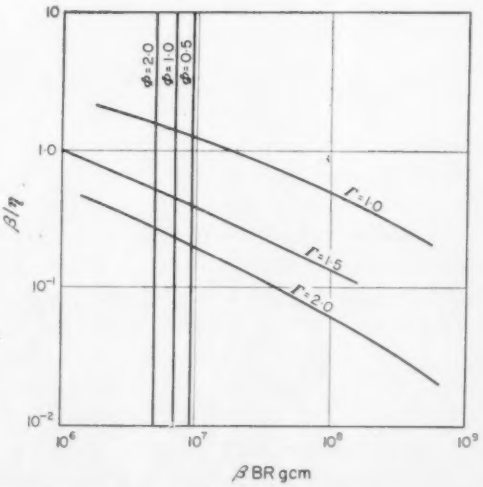

FIG. 2.—β/η against βBR gauss cm for ion energy $W =$ 500 keV.

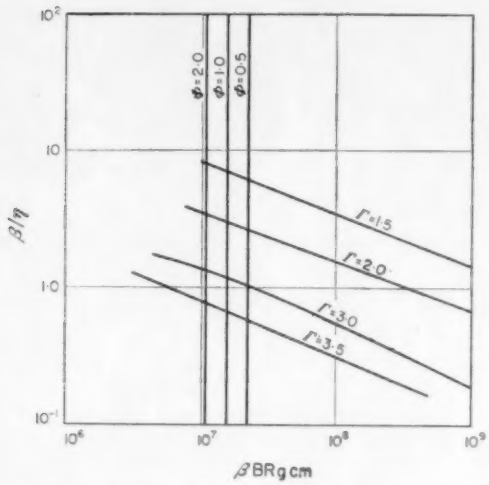

FIG. 3.—β/η against βBR gauss cm for ion energy $W =$ 1000 keV.

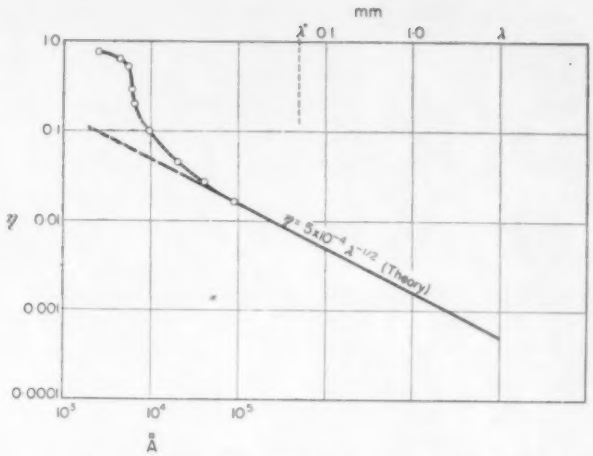

FIG. 4.—Absorption coefficient η as a function of wavelength λ.

The limits to the validity of TRUBNIKOV's theory are given approximately by,

$$kT_e \leqslant 150 \text{ keV}, \quad \Lambda \geqslant 300, \quad K \leqslant 0{\cdot}2.$$

With the aid of (3.10) the first condition gives $\Gamma \geqslant 0{\cdot}071 \left(\frac{W}{150}\right)^{3/2}$ with (3.11) the second gives $\beta BR \geqslant 2 \times 10^3 W$. From (3.4), with the use of (3.2) and (3.6), $K = \frac{\Gamma}{\eta} Q_M/Q_e = 5 \times 10^5 \frac{\beta}{\eta} \Gamma^{5/3} W^{-5/2}$. Hence the third condition is $\frac{\beta}{\eta} \leqslant 4 \times 10^{-7} W^{5/2} \Gamma^{-5/3}$.

These three conditions restrict the validity to regions in which the calculated curves are shown.

(c) *Reflector loss*

The value of η may be estimated using the well-known result for skin loss upon the reflection at the surface of a conductor giving (in the approximation $\eta \ll 1$)

$$\eta \cong 2 \times 10^{-6} \left(\frac{\omega}{2\pi\sigma}\right)^{1/2} \tag{3.12}$$

where ω is the angular frequency of the incident radiation and σ is the conductivity of the reflector in mhos cm^{-1}, ω_0 is given by $1{\cdot}8 \times 10^7 B$ (gauss) rad. sec^{-1} and we assume the relevant frequency $\omega = \omega^* = 10\,\omega_0$, which gives $\eta \cong 10^{-2} \left(\frac{B}{\sigma}\right)^{1/2}$. Assuming a copper reflector with $\sigma = 0{\cdot}5 \times 10^6$ mhos cm^{-1} and a field $B = 2 \times 10^5$ gauss gives $\eta = 10^{-2}$. The wavelength $\lambda = \lambda^*$, corresponding to ω^*, is approximately $10^3 B^{-1}$ cm, i.e. $\sim 0{\cdot}005$ cm. In Fig. 4 the theoretical value of η at all relevant wavelengths is compared with those available from experiments at $\lambda < 10^{-3}$ cm, $\eta > 0{\cdot}02$. Agreement with experiment is good for $\lambda > 4 \times 10^{-4}$ cm and one expects that η can be $\sim 0{\cdot}01$ for the wavelengths of radiation which will be encountered in practice, viz. 0·005 cm, $\lambda \leqslant 5 \times 10^{-3}$ cm.

(d) *Coil loss*

We assume the coils are cylindrical of radius R and thickness εR and of conductivity σ (e.m.u). The coil loss per unit volume of plasma Q_R is

$$Q_R = B^2/8\pi^2\varepsilon\sigma R^2 \text{ ergs cm}^{-3} \text{ sec}^{-1}. \tag{3.13}$$

Defining $\Phi \equiv Q_R/Q_M$ and using (3.2) with W in keV

$$\Phi = \frac{10^3 W}{(\beta BR)^2 \varepsilon\sigma}. \tag{3.14}$$

It is known that σ may be increased above its room temperature value for many materials by refrigeration (cryogenic techniques) but we shall not consider this possibility in detail here. For example we take $\sigma = 5 \times 10^{-4}$ e.m.u. (copper at room temperature) and $\varepsilon = 0{\cdot}25$ giving

$$\Phi = \frac{8 \times 10^6 W^{5/2}}{(\beta BR)^2} \tag{3.15}$$

bearing in mind that Φ may perhaps be increased effectively by a factor of 5 or so.

(e) *Collision bremsstrahlung loss*

At high electron temperature ($kT_e \sim 200$ keV) electron-electron radiation becomes of comparable importance to ion-electron radiation. The total Bremsstrahlung from unit volume of plasma is

suitably written as (KOFOED-HANSEN *et al.*, (1958):

$$Q_B = 5{\cdot}4 \times 10^{-24}\, n^2 (kT_e)^{1/2} f \text{ ergs cm}^{-3} \text{ sec}^{-1}. \quad (3.16)$$

where $f = f(kT_e)$ is unity for $kT_e \leqslant 10$ keV and thereafter monotonically increases with T_e (e.g. $f = 6$ when $kT_e = 500$ keV). Using (3.2) and (3.10)

$$\frac{Q_B}{Q_M} = 1{\cdot}1 \times 10^{-4} W \Gamma^{-1/3} f(0{\cdot}17\, \Gamma^{-2/3} W) \quad (3.17)$$

Q_B/Q_M is found to be small in our applications.

4. FUSION REACTORS

The mirror machine

Let Q_F be the fusion power liberated per unit volume of reacting plasma. Most of this power will appear outside the plasma as energetic neutrons and from subsequent exoergic nuclear reactions. We assume that fusion power and all losses may be converted to electrical energy with an overall efficiency of $\frac{1}{3}$. Neglecting the fusion energy released within the plasma on charged particles, ions must be injected into the machine at sufficient power Q_I per unit volume to sustain the plasma against losses, so that

$$Q_I = Q_{ie} + Q_M = (\Gamma + 1) Q_M. \quad (4.1)$$

Including the power required to make good the coil losses, a condition for a net power producing reactor is

$$\frac{1}{3}(\eta K Q_e + Q_B + Q_M + Q_F) > Q_I + \frac{2}{3} Q_R \quad (4.2)$$

or, using (3.8) and (4.1) and neglecting Q_B/Q_M

$$\frac{1}{2}\frac{Q_F}{Q_M} > \Gamma + \Phi + 1. \quad (4.3)$$

Now $$Q_F = 1{\cdot}6 \times 10^{-9}\, an^2 (\overline{\sigma v})_W\, U \text{ ergs cm}^{-3} \text{ sec}^{-1} \quad (4.4)$$

where U is the total energy released per reaction in keV and $(\overline{\sigma v})_W$ is the reaction cross-section velocity product averaged for a Maxwellian distribution of mean energy W keV (KOFOED-HANSEN *et al.*, 1958).

$$\text{Hence } \frac{1}{2}\frac{Q_F}{Q_M} = 3{\cdot}8 \times 10^{10}\, a (\overline{\sigma v})_W\, U W^{1/2}. \quad (4.5)$$

Using a 50 : 50 mixture of deuterium and tritium (D-T), $a = \frac{1}{4}$, while for pure deuterium (D-D), $a = \frac{1}{2}$. Assuming a suitable blanket of exoergic material U may be as high as 30×10^3 keV (Post, unpublished). For the D-T reaction we compute Γ and Φ for $W =$ 200 keV, at which energy the $(\overline{\sigma v})_W W^{1/2}$ versus W curve becomes relatively flat.

$$\text{For } W = 200 \text{ keV}: \frac{1}{2}\frac{Q_F}{Q_M} = 2{\cdot}7$$

$$\Gamma + \Phi < 1{\cdot}7. \quad (4.6)$$

For the D-D reaction we assume all the tritium formed is eventually consumed and take $U = 30 \times 10^3$ keV.

For D-D, the $(\overline{\sigma v})_W W^{1/2}$ curve increases up to $W = 1$ MeV, above which nuclear scattering becomes important.

$$\text{For } W = 1000 \text{ keV}: \frac{1}{2}\frac{Q_F}{Q_M} = 4{\cdot}7$$

$$\Gamma + \Phi < 3{\cdot}7. \quad (4.7)$$

A power reactor may therefore only operate on those regions of the β/η versus βBR diagrams where the inequalities in $\Gamma + \Phi$ are satisfied.

Evidently as one strives to satisfy the inequalities, by proceeding generally from bottom-left to top-right on the diagrams, the technical difficulties of producing high β and BR and low η increase.

The stellarator and stabilized pinch

While the above conditions apply specifically to the mirror machine, it is of interest to compare them with one which may be obtained for other proposed fusion reactors of the stellarator or stabilized pinch type (SPITZER *et al.* (1958) and THONEMANN *et al.* (1958)). We derive the condition that a sufficient plasma temperature be sustained in a steady state by the release of fusion energy in charged (non-escaping) particles,

$$\eta K Q_e + Q_B = Q_F. \quad (4.8)$$

That is we ignore the possibility of heating the plasma externally. We consider only a D-D plasma since one can show that the requirements for D-T are far less exacting.

Assuming that $kT_i = kT_e = 50$ keV and with $\beta = 8\pi nk(T_i + T_e)/B^2$ one finds

$$\beta/\eta \simeq 80K \quad \text{and} \quad \beta BR \simeq 13{\cdot}6\Lambda \text{ gauss cm}. \quad (4.9)$$

The relation between K, Λ at 50 keV (T & K, 1958) is used to give the approximate result

$$\log_{10}\beta/\eta \simeq 2{\cdot}9 - 0{\cdot}52 \log_{10}\beta BR. \quad (4.10)$$

In the stabilized pinch the current I is approximately $5\,BR$ amperes, while the line density $N = \pi R^2 n$ is another important parameter. Assuming $\beta = 0{\cdot}1 = \eta$ one finds $I = 18 \times 10^6$ amperes and $N = 10^{18}$ ions

TABLE 1

β/η	BR(gauss cm)	N(cm^{-1})
10	$4{\cdot}2 \times 10^{8}$	$1{\cdot}4 \times 10^{13}$
1	$3{\cdot}5 \times 10^{5}$	$9{\cdot}5 \times 10^{16}$
10^{-1}	$2{\cdot}8 \times 10^{7}$	$6{\cdot}1 \times 10^{20}$

cm^{-1}. Probably I and N will need to be at least as great as this for other reasons, so that cyclotron radiation loss is not decisive.

5. CONCLUSIONS

The minimum necessary value of BR in gauss cm required to reduce cyclotron radiation and coil losses sufficiently to achieve net power from a fusion mirror reactor depends rather critically on the values of β, η, Γ and Φ which can be achieved, as is illustrated in the diagrams. The practical limitations on these quantities are not known very clearly at present. One can hardly expect to achieve $\beta > 0{\cdot}1$ or increase the maximum values of $\Gamma + \Phi$ given in relations (4.6) and (4.7). However, Φ might be reduced by cryogenic technology by a factor of five or so below the values shown, and it should be possible to make η considerably less than 0·1.

Assuming $\beta = 0{\cdot}1$, $\eta < 0{\cdot}1$, $\beta/\eta > 1$, the combined known losses do not preclude the possibility of a D-T 200 keV mirror reactor with $BR \geqslant 2 \times 10^{7}$ gauss cm, or a D-D 1 MeV mirror reactor with $BR \geqslant 2 \times 10^{8}$ gauss cm. Coil losses predominate for the former and cyclotron losses for the latter reaction. Since attainable regions of β/η and βBR lie only just within the valid limits of the theoretical calculations, the numerical results can only be taken as estimates.

In the stellarator or stabilized pinch using the D-D reaction, the radiation requirements on BR or current may be no more severe than those demanded by other considerations, e.g. if $\beta = 0{\cdot}1$, $\eta \leqslant 0{\cdot}1$, then $BR \geqslant 3{\cdot}5 \times 10^{6}$ gauss cm or $I \geqslant 18 \times 10^{6}$ amperes. Cyclotron radiation should have little consequence for the D-T reaction.

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MEASUREMENTS OF THE PROTON ENERGIES FROM SCEPTRE III

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Abstract—Measurements of the energies of the protons emitted from a deuterium discharge in Sceptre III using nuclear plate cameras, give information on the motions of the fusing deuterons in the discharge. Experimental results on proton energies from four series of exposures at different stabilizing magnetic fields using cameras observing the discharge at nominal angles of 45°, 90° and 135° are given, together with the results of exposures of the cameras to mono-energetic protons from the A.E.I. electrostatic generator to find their instrumental linewidths. These results show that the peaks of the proton spectra observed at 45° and 135° to the discharge axis are due to fusions of deuterons whose centre of mass velocity along the axis is 5×10^7 cm/sec. The broadening of the proton spectra observed at 90° is due to a centre-of-mass motion of the fusing deuterons in the radial plane of about 2×10^7 cm/sec. It is also indicated that azimuthal motions occur in this plane. All these motions are largely independent of the stabilizing magnetic field. It is also apparent from the plates that the fusion region of the discharge is displaced from the centre of the torus tube at the observation section.

1. INTRODUCTION

A SERIES of energy measurements on the protons emitted in Sceptre III and IIIA (ALLIBONE *et al.*, 1958; WARE, 1959b) has been carried out for a number of discharge conditions, in order to obtain information about the mode of fusion of the deuterons. These measurements, which were all based on the determination of proton energy from range in nuclear emulsion, may be divided into three classes:

(1) The determination of the mean energy of protons emitted at different angles with respect to the discharge axis in order to obtain information about the velocity component of the centre-of-mass of the fusing deuterons along the discharge axis.

(2) The measurement of the shape of the proton spectra observed at each of these angles to obtain information about the distribution of the deuteron velocities along this axis as described by HUNT (1960) and about components of the deuteron motion in the plane perpendicular to this axis, as described in Section 5.2.1. of the present paper. To do this, the shape of the proton spectra at each of these angles was compared with the shape for mono-energetic protons found from a separate measurement using the proton beam from the Van de Graaff accelerator.

(3) The comparison of the energies and densities of protons arriving at different parts of the detector in order to obtain information on possible azimuthal deuteron motions round the discharge axis, as discussed in Section 5.2.2.

The present paper collects the results of recent experimental work and those reported earlier.

2. METHOD OF MEASUREMENT

The proton energies have been measured at nominal angles 45°, 90° and 135° to the direction of positive current flow in the Sceptre discharge, by measuring their ranges in Ilford C2 nuclear emulsions mounted in cameras of the type already described (JONES *et al.*, 1959). These were of two kinds: the first, which observed at 90° to the discharge axis, was called type *A* and the second, which observed at 135° and 45°, type *B*. The arrangement is shown schematically in Fig. 1. Each exposure corresponded to several thousand Sceptre discharges, so the results obtained represent the integrated effect of these.

In the type *A* cameras, the geometric construction was such that only protons emitted within an angular range of about 6° approximately normal to the discharge axis could reach the photographic plates directly. Only tracks whose angles lay in the range set by the relative positions of the defining aperture and emulsion were accepted. The range of angles to the discharge axis accepted in the *B* type cameras was approximately 30°.

The proton acceleration due to changing magnetic fields was calculated to be negligible and the slight bending of the proton trajectories by the estimated magnetic fields does not affect the conclusions drawn here. In the subsequent discussion therefore, linear proton trajectories are assumed throughout.

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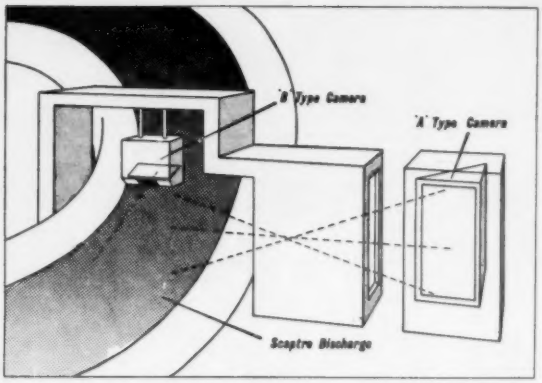

FIG. 1.—Schematic diagram of cameras in position on Sceptre. (Not to scale).

3. RESULTS

Four series of exposures have been carried out in either Sceptre III or Sceptre IIIA under the conditions given in Table 1.

The first three sets of exposures were performed with both *A* and *B* type cameras, while series *Z* was performed with only an *A* camera. In addition, all the cameras were exposed to effectively mono-energetic 3·0 MeV protons, obtained by scattering the Van de Graaff beam at 90° off a 0·33 mg/cm² gold foil. The energy spread introduced in this way was calculated to be about 1·5 per cent of the proton energy. This affected the observed widths at half-maximum by only a few keV and has been allowed for.

The results of the first three series of exposures are given in Table 2. The errors are discussed in Section 4. A histogram of the results at $B_{\phi_0} = 1400$ gauss is given in Fig. 2.

3.1 *Widths of proton spectra*

In Table 3, the widths of the proton spectra from Sceptre exposures are shown compared with the exposures of the cameras to 3·0 MeV protons from the Van de Graaff for the particular camera and foil thickness used.

The errors estimated for the half-widths of the Van de Graaff spectra are composed in all cases of about ±5 keV, due to the uncertainty of fitting a smooth curve to the results. Because of the small *B* camera slit, each plate was exposed three times to the scattered beam in order that tracks should be recorded over the whole plate area. Additional small errors are allowed for possible beam energy changes and for variations of the moisture content of the emulsion between these exposures.

For all the Sceptre exposures there is an increased error due to the fact that the much smaller track

TABLE 1

Exposure series	B_{ϕ_0} field (g)	Peak gas current (kA)	D_2 pressure (μ)	Turns ratio	Condenser voltage (kV)	Assembly specification
W	500	100	1·8	16	25	Sceptre III
X	1000	100	1·4	16	25	Sceptre IIIA
Y	1400	120	1·4	16	25	Sceptre IIIA
Z	500	75	1·8	16	25	Sceptre IIIA

TABLE 2

Series	Initial B_ϕ field (g)	Copper foil used (in.)	Nominal angle of observation	Relative mean proton energy (MeV)	Energy shift between 45° and 135° (MeV)
W	500	0·0005 0·0001 0·0005	45° 90° 135°	3·06 ± ·02 3·00 ± ·04 2·88 ± ·01	0·18 ± ·025
X	1000	0·0005 0·0005 0·0005	45° 90° 135°	3·05 ± ·01 3·01 ± ·03 2·82 ± ·02	0·22 ± ·02
Y	1400	0·0005 0·0005 0·0005	45° 90° 135°	3·11 ± ·02 3·02 ± ·03 2·91 ± ·01	0·20 ± ·02

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TABLE 3

Camera type	Exposure series	Width at ½ max. (MeV)	Comparable width at ½ max. measured on the Van de Graaff (MeV)
A	*W*	0·175 ± ·020	0·125 ± ·005
A	*X*	0·215 ± ·015	0·150 ± ·010
A	*Y*	0·170 ± ·015	0·145 ± ·010
A	*Z*	0·150 ± ·011	0·125 ± ·005
B	*X*	0·215 ± ·015 (45°)	0·155 ± ·015
B	*X*	0·190 ± ·015 (135°)	0·155 ± ·015
B	*Y*	0·200 ± ·015 (45°)	0·160 ± ·015
B	*Y*	0·200 ± ·015 (135°)	0·145 ± ·015

density made it necessary to scan large areas of plate with the consequent increase in possible foil non-uniformity effects. In addition, there was greater difficulty in fitting a smooth curve to the Sceptre results. Some of the results for both the Van de Graaff and Sceptre exposures include a small error because of difficulty in scanning due to slight damage to the emulsion surface.

In the *B*-type cameras the widths include a small effect due to the range of angles with respect to the discharge axis that they record (HUNT, 1960). In the *A* cameras this effect was negligible because of the much narrower angular range.

The histograms for the exposures of the *A* camera, with a 2 in. × 1/8 in. slit, to the mono-energetic protons and to the discharge, as well as a *B* camera exposure to the mono-energetic protons, have already been published (JONES *et al.*, 1959).

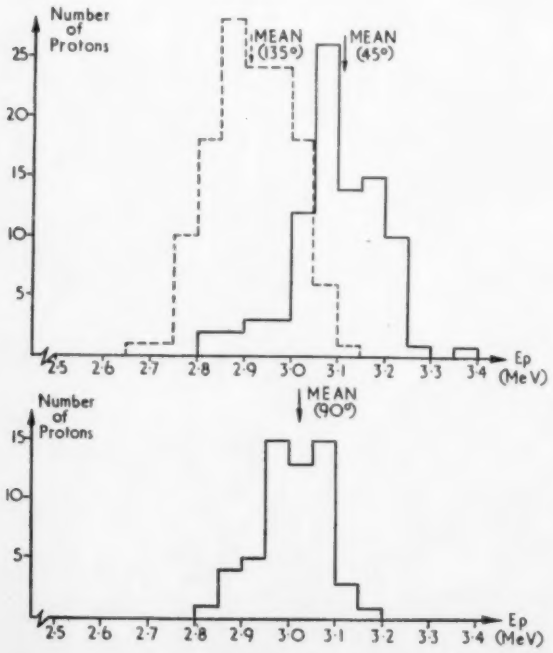

FIG. 2.—Sample histograms for exposure series *Y*.

3.2. *Further information from plate exposures*

3.2.1. *Energy changes along the plate.* Comparisons have been made between the energies of protons recorded at each end of the plate. For the *A* camera mounted on the outer wall, end 1 was the higher end in the vertical plane.

Table 4 shows the differences in mean energies between ends.

TABLE 4

Series	Angle of observation	Difference in energy between ends (+ve for end 1 greater) (MeV)
Z	90°	+ ·034 ± ·027
Z	90°	+ ·035 ± ·017
X	90°	+ ·019 ± ·025

The errors quoted are statistical with an uncertainty of about ±5 keV added to allow for the difference in area of foil traversed by protons hitting either end of the plate, all these foils being found uniform within this limit using a β-gauge.

In series *Y*, it was only possible to use an earlier version of the *A* type camera which did not permit the measurement of such energy differences.

3.2.2. *Density and angle variation along the plate.* The distribution of track density on the *B* camera plates for the *W*, *X* and *Y* exposure series indicated that the protons came from a region whose centre was displaced about ½ to 1 in. outwards from the axis of the discharge tube. From the range and mean value of the angles of the tracks in different regions of the large *A* camera plates, it was deduced that the centre of the fusion system was 1 ± 0·3 in. above the torus axis for series *Z* and 0·4 ± 0·2 in. above for series *X*. In series *W* two large *A* camera plates were exposed at the top and side of the torus. From these it appears

that 85 per cent of the protons emerged from a cylindrical region less than 6 in. in diameter, whose centre was slightly displaced from the torus axis.

4. DISCUSSION OF ERRORS IN MEAN ENERGY

The errors quoted in Table 2 apply to the values of proton energy observed at each of the three angles, expressed on a separate relative scale for each set of observations. If the results for proton energies at different conditions are to be compared, the associated errors must be increased. The main factors considered in determining the quoted errors are (a) statistical deviations of the results, (b) thickness variations in shielding foil, and (c) differences arising from different emulsion batches used.

If absolute values for the proton energies are required, allowance must be made for further uncertainties (JONES *et al.*, 1959). The errors on line widths are discussed in Section 3.

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5. DISCUSSION OF RESULTS

5.1. *Motion along the discharge axis*

The results in Table 2 indicate that there was no significant change in the energy shift between the proton spectra observed at 45° and 135° to the discharge axis, for the three values of B_{ϕ_0} field used, 500, 1000 and 1400 gauss. The protons in the region of the peaks of these spectra arise from deuteron fusions for which the centre-of-mass velocity component along the discharge axis in the direction of positive gas current, $(v_{cm})_\phi$, is $5 \pm 1 \times 10^7$ cm/sec. Ion drift velocities of $(1–2) \times 10^6$ cm/sec along the discharge axis in the direction of positive current have been observed by measuring the Doppler shift of the *OV* and *CV* lines from impurity ions in the discharge (HUGHES and KAUFMAN, 1959). On the basis of these measurements, it has been suggested (WARE, 1959a) that the mean deuteron speed along the discharge axis should be about 5×10^6 cm/sec. The relation of this figure to the value of 5×10^7 cm/sec obtained from the proton measurements has been discussed elsewhere (HUNT, 1960; HERDAN and HUGHES, 1960).

5.2. *Motion in the plane perpendicular to the discharge axis*

5.2.1. *Line widths.* If the fusing deuterons are moving at random in the radial plane of the discharge, of if they have a directed motion either radially or azimuthally, then the proton spectra observed by the cameras will be broader than for mono-energetic protons. The broadening will be greatest in the 90° camera. Starting from the shape of the proton spectra observed by the camera for effectively mono-energetic protons and applying the theory given in the Appendix, one can find the width of spectrum expected at 90° to the discharge axis produced for a particular value of the v_{cm} component in the radial plane.

By comparing the calculated widths with those experimentally observed, values for the velocity components were found for the conditions used. These are given in Table 5.

TABLE 5

Series	v_{cm} component $(v_{cm})_r$ in radial plane (cm/sec)
W	$2{\cdot}4 \pm 0{\cdot}6 \times 10^7$
X	$2{\cdot}3 \pm 0{\cdot}5 \times 10^7$
Y	$1{\cdot}9 \pm 0{\cdot}5 \times 10^7$
Z	$1{\cdot}8 \pm 0{\cdot}5 \times 10^7$

These results indicate that there is a definite broadening effect in all cases, and that the v_{cm} component causing it is approximately 2×10^7 cm/sec and is largely independent of B_{ϕ_0}.

The widths of the distributions in exposures at 45° and 135° are discussed by HUNT (1960).

5.2.2. *Energy changes along the plate.* Line width considerations cannot distinguish between radial motions and azimuthal motions of the fusing deuterons, whereas a difference in energy observed at opposite ends of the plate would indicate an azimuthal motion. For both the large *A* camera plates exposed in series *Z*, a difference of the same sign was observed between the mean energies at the two ends of the plates. Together these plates indicated a difference which was non-zero with a significance of greater than 99 per cent and is consistent with an azimuthal v_{cm} component of about 2×10^7 cm/sec. A similar component was observed in series *X*. The sense of the motion is anti-clockwise when viewed along the B_{ϕ_0} direction, B_{ϕ_0} and *E*, the electric field, being anti-parallel.

Although energy differences are observed between the ends of the *B* camera plates their interpretation is much more complicated as there are also energy changes along the plate due to the range of angles with the axis that the camera observes. The calculation of these effects is very uncertain, being dependent on the position of the discharge.

5.3. *Summary*

As a result of the above observations it is concluded that:

(a) There is a component of motion of the centre-of-mass of the fusing deuterons along the discharge

axis in the direction of positive current flow. The value of this component, for the fusions corresponding to the means of the proton spectra, is $5 \pm 1 \times 10^7$ cm/sec for the three values of B_{ϕ_0} investigated.

(b) The centre-of-mass velocity of the fusing deuterons has a component in the plane perpendicular to the discharge axis. For an axially symmetric motion this component is found, from the broadening of the proton spectrum observed at 90°, to be about 2×10^7 cm/sec and largely independent of B_{ϕ_0} field. From measurements of the spectral broadening it is not possible to distinguish between azimuthal, radial, or randomly orientated motions in this plane. The measurements of energy changes along the plates did, however, establish the existence of azimuthal motions. Their magnitude could be sufficient to account fully for the observed broadening at 90°, but the accuracy of the measurements is not sufficient to exclude contributions from radial or randomly orientated motions.

(c) The centre of the fusion region at the observation section is displaced upwards in Sceptre IIIA by about 1 in. for $B_{\phi_0} = 1000$ gauss. In Sceptre III for $B_{\phi_0} = 500$ gauss, it appears to be displaced about 1 in. outwards, but for Sceptre IIIA, this outward displacement appears less for higher fields.

(d) The minor diameter of the region of fusion appears to be about 6 in.

Acknowledgements—The idea of measuring the proton energy differences to investigate the mechanism of the fusion reaction was the result of discussions between DR. D. R. CHICK, DR. S. E. HUNT and the authors. The contributions made by DR. S. E. HUNT to the subsequent work are gratefully acknowledged.

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Thanks are also due to DR. T. E. ALLIBONE, F.R.S., Director of this Laboratory, for permission to publish this paper. The work has been supported in part by an Atomic Energy Research Establishment Contract for thermonuclear research.

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APPENDIX

The broadening of the proton spectrum observed by the *A* type cameras must arise from motion of the fusion system in the plane perpendicular to the discharge (ϕ) axis. The possibility that the broadening could arise from the known motion along the ϕ axis is excluded in the present case, because both the small range of angular acceptance of the cameras and the range of $(v_{cm})_\phi$ values predicted by HUNT (1960) are insufficient to cause any appreciable broadening of the observed spectrum.

The simplest axially symmetric form that the motion perpendicular to the ϕ-axis could take corresponds to a single value

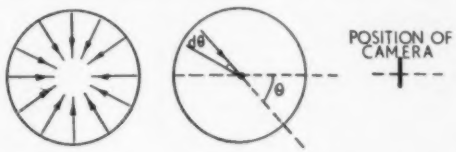

FIG. 3.—Diagram showing equivalent radial form of randomly directed v_{cm} components in the radial plane of the discharge.

of $(v_{cm})_p$, the component of the centre-of-mass velocity of the fusing deuterons in the plane perpendicular to the ϕ-axis, randomly orientated in this plane. The spectrum broadening for this simplest case will now be derived under the following assumptions:

(a) The reacting system is symmetrical about the ϕ-axis.

(b) The yield from the fusion reaction is independent of the angle of emission of the proton with respect to the direction of the centre-of-mass velocity of the fusion.

(c) The fusion region subtends a small angle at the detector.

(d) $v_{cm}/v_0 \ll 1$ where v_0 is the proton velocity in the centre-of-mass system, and also the relative kinetic energy of the fusing deuterons is small.

The relation between the proton energy, E, and its laboratory angle of emission, θ, with respect to v_{cm_p}, for the case of one deuteron striking another at rest in the laboratory system is:

$$E^{\frac{1}{2}} = \tfrac{1}{2}(2M_2 v^2_{cm})^{\frac{1}{2}} \cos\theta + \frac{1}{2}\left\{2M_2 v^2_{cm}\cos^2\theta + \frac{4M_3}{(M_2+M_3)}\left[Q - \frac{M_1(M_2-M_3)}{M_3}v^2_{cm}\right]\right\}^{\frac{1}{2}} \quad (1)$$

where M_1, M_2 and M_3 are the masses of the deuteron, proton and triton respectively and Q is the Q-value of the D(d,p)T reaction.

Since any contribution of $(v_{cm})_\phi$ to the energy observed for protons in the plane perpendicular to the ϕ-axis may be neglected for the cases at present of interest, v_{cm} in equation (1) may be replaced by $(v_{cm})_p$. The relative kinetic energy of collision which affects the fusion yield may, however, include a contribution from the motion along the ϕ-axis.

The problem will first be discussed for the case of fusions occurring between deuterons striking deuterons at rest. In view of assumption (b) any randomly directed system of $(v_{cm})_p$ components may be rearranged in the equivalent radial form shown in Fig. 3. Considering the protons emitted along the directions

of observation from the fusions whose $(v_{cm})_p$ components lie in the elemental angle θ to $\theta + d\theta$, their number is proportional to $d\theta$ and they will have an energy spread E to $E + dE$, corresponding to the change in angle of emission with respect to $(v_{cm})_p$ from θ to $\theta + d\theta$. If $F(E)dE$ is the number of protons in this energy range, we then have the relation

$$F(E)\,dE = k\,d\theta \quad \text{or} \quad F(E) = k\frac{d\theta}{dE}. \tag{2}$$

The quantity $\frac{d\theta}{dE}$ has been calculated for the case $(v_{cm})_p = 2{\cdot}7 \times 10^7$ cm/sec, and used to obtain the graphs of $F(E)$ against θ and E shown in Fig. 4, to illustrate the form of the function.

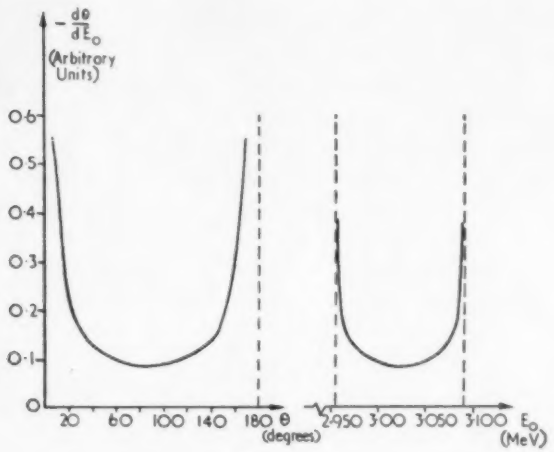

FIG. 4.—Graphs of $-\frac{d\theta}{dE_0}$ plotted against θ and E_0 for $(v_{cm})_p = 2{\cdot}7 \times 10^7$ cm/sec and $\frac{M_3}{M_2 + M_3}\,Q = 3{\cdot}020$ MeV.

In order to predict the form of the observed spectrum the instrumental line shape of the detection system must be combined with the intrinsic shape of the proton spectrum. Let the instrumental effect be such that a mono-energetic proton group of energy E_0 gives a distribution whose intensity recorded in the range E to $E + dE$ is proportional to $\Phi(E,E_0)\,dE$. The combination of this shape with the function $F(E)\,dE$ for the proton spectrum gives the relation

$$\psi(E)\,dE = \int_{E_1}^{E_2} \Phi(E,E_0)\,dE\,F(E_0)\,dE_0 \tag{3}$$

where $\psi(E)\,dE$ is proportional to the intensity actually observed in the range E to $E + dE$, the integration being carried out with respect to dE_0 between the limits E_1 and E_2 of the intrinsic proton spectrum. Since $F(E_0) = k\frac{d\theta}{dE_0}$, we may rewrite equation (3) as

$$\psi(E) = k\int_0^{\pi} \Phi(E,E_0)\,d\theta \tag{4}$$

since the limiting values E_1 and E_2 of the intrinsic spectrum correspond to the values 0 and π for θ.

The values of E_0 for particular values of θ are found from equation (1). Then, using $\Phi(E,E_0)$, the relative intensity to be expected at an energy E, as determined from the experimental curves obtained from the Van de Graaff exposures, the relative magnitude of $\psi(E)$ is evaluated by graphical integration of equation (4). By repetition of this procedure for different values of E, the shape of $\psi(E)$ is obtained. The curve for $\psi(E)$ based on the instrumental line shape of the $A2$ camera with a 0·0001 in. copper foil is shown in Fig. 5 for the case of $(v_{cm})_p = 2{\cdot}7 \times 10^7$ cm/sec.

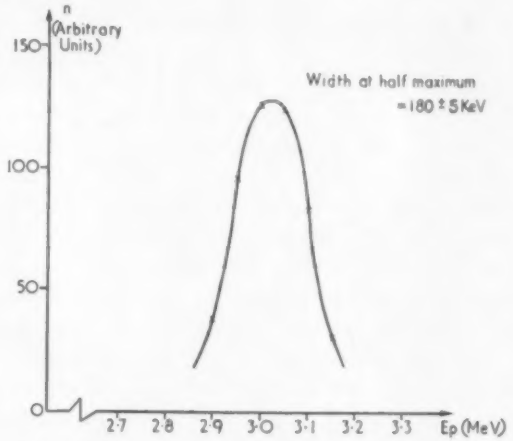

FIG. 5.—Calculated curve deduced for camera for radial component of $(v_{cm})_p = 2{\cdot}7 \times 10^7$ cm/sec.

The discussion has, so far, been limited to the case of deuterons fusing with others at rest in the laboratory. It may be shown that for the most general type of collision, the values of E and $\frac{d\theta}{dE}$ differ only slightly from those calculated for the above particular case, provided that $\frac{v_{cm}}{v_0} \ll 1$, and also the relative kinetic energy of the pair of deuterons is small. In particular, if this energy is less than 60 keV, the effect of differing modes of collision on E and on $\frac{d\theta}{dE}$ is less than 3/4 per cent. These effects may thus be neglected and the restriction on the type of collision removed.

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AN ACHROMATIC SYSTEM OF EXTRACTION FOR PROTON SYNCHROTRONS

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Abstract—An achromatic modification of the PICCIONI extraction system for weak focusing synchrotrons is described. A quadrupole magnet placed between the target and the extraction magnet is used to cancel the momentum resolution of the extracted beam produced by the synchrotron magnet. The extraction magnet is made radially focusing to give a beam of small dimensions in the exit region.

1. INTRODUCTION

IN the PICCIONI extraction system as used on the Cosmotron (PICCIONI, 1955) the beam at the extraction magnet has an increased width since the magnetic path from the target causes a momentum resolution of the beam. The energy spread of the extracted beam is produced by the Landau effect in the target and the betatron oscillations which have not been completely removed by the lip on the target, and although it is very small (a few MeV) the momentum resolution of weak focusing synchrotrons is very high. After being deflected outwards by the extraction magnet, this broadened beam must travel through the fringe field of the synchrotron magnet. Here it is defocused radially in a distorting manner and the resulting beam may be so wide that it is difficult to handle with a quadrupole channel of convenient size. In addition, since with the resolution each momentum is occupying a different part of the radial phase space, the phase space of the whole beam is increased several times.

It can be shown that, if a quadrupole of a certain strength is placed halfway between the target and the extraction magnet, the system so produced is achromatic, the protons at the extraction magnet having positions and directions which do not depend on their energy. This is not however a focus in the usual sense and an image of the source is not produced, the beam appearing to come from a virtual source some distance away.

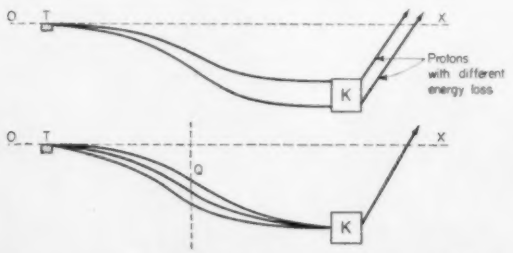

FIG. 1.—Schematic diagrams comparing the normal PICCIONI scheme and the achromatic modification.

It is usual to put the extraction magnet at or near to a vertical focus of the beam from the target. In the present system the quadrupole introduced is defocusing vertically and increases the distance between the target and the focus. It may however still be possible to place the extraction magnet near this focus.

2. REQUIREMENTS FOR AN ACHROMATIC SYSTEM

We consider the motion of a proton leaving the target T with a radial displacement α cm, measured outwards from a reference radius (OX in Fig. 1) and a divergence β cm rad^{-1}. The proton oscillates about its closed orbit which is a distance η from the reference line and passes through the quadrupole Q and then on to the extraction magnet K. η is given by

$$\eta = \Delta E \frac{dE}{dr}$$

where ΔE is the difference in energy from the reference energy and dE/dr is the 'energy compaction' ratio.

Let ρ be the displacement of the particle about its closed orbit. Then at T

$$\begin{bmatrix} \rho \\ \rho' \end{bmatrix}_T = \begin{bmatrix} \alpha - \eta \\ \beta \end{bmatrix}$$

where $'$ denotes differentiation with respect to θ, the azimuthal co-ordinate,

and at Q

$$\begin{bmatrix} \rho \\ \rho' \end{bmatrix}_Q = \begin{bmatrix} E & F \\ G & H \end{bmatrix} \begin{bmatrix} \alpha - \eta \\ \beta \end{bmatrix}$$

where

$$\begin{bmatrix} E & F \\ G & H \end{bmatrix}$$

is the transfer matrix from target to quadrupole.

At the quadrupole the proton suffers a deflexion

$$\Delta\rho' = K(y - \zeta)$$

where y is the radial position of the proton measured from the reference line, ζ is the position of the centre of the quadrupole and K is its strength.

Then $$y = \eta + \rho = \eta + E(\alpha - \eta) + F\beta.$$

Thus we find at the extractor magnet

$$\begin{bmatrix} \rho \\ \rho' \end{bmatrix}_K = \begin{bmatrix} D & B \\ C & A \end{bmatrix} \times \begin{bmatrix} E(\alpha - \eta) + F\beta \\ G(\alpha - \eta) + H\beta + K\{\eta + E(\alpha - \eta) + F\beta - \zeta\} \end{bmatrix}$$

where

$$\begin{bmatrix} D & B \\ C & A \end{bmatrix}$$

is the transfer matrix from the quadrupole to the extractor magnet.

We have $$y_K = \rho_K + \eta$$
$$y_K{}' = \rho_K{}'$$

and collecting terms we find

$$\begin{aligned} y_K &= \eta\{1 - DE - BG + KB - KBE\} \\ &\quad + \alpha\{DE + BG + KBE\} \\ &\quad + \beta\{DF + BH + KBF\} + \zeta\{-KB\} \\ y_K{}' &= \eta\{-CE - AG + KA - KAE\} \\ &\quad + \alpha\{CE + AG + KAE\} \\ &\quad + \beta\{CF + AH + KAF\} + \zeta\{-KA\}. \end{aligned}$$

To make the system achromatic we must make the coefficients of η in the above expressions zero, that is, we must make the motion independent of the closed orbit.

The quadrupole strength required to make the coefficient of η in the expression for y_K equal to zero is $K_1 = (1 - DE - BG)/\{B(1 - E)\}$ and in the expression for $y_K{}'$

$$K_2 = (-CE - AG)/\{A(1 - E)\}.$$

Equating K_1 and K_2 we have $(E \neq 1)$

$$A(1 - DE - BG) = -B(CE + AG)$$

whence $A = E$ since $AD - BC = 1$.

If $A = E$ a quadrupole strength can be found which will make the transfer from target to extractor magnet achromatic. This condition is satisfied if the quadrupole is placed symmetrically between the target and the extraction magnet. In this case the matrices are related by

$$\begin{bmatrix} E & F \\ G & H \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

and the strength of the quadrupole is

$$K = \frac{2C}{1 - A}.$$

The expressions for the displacement and divergence of the proton at the extraction magnet reduce to

$$\begin{aligned} y_K &= \frac{2AD - 1 - A}{1 - A}\alpha + \frac{2DB - B}{1 - A}\beta - \frac{2BC}{1 - A}\zeta \\ y_K{}' &= \frac{2AC - 2A}{1 - A}\alpha + \frac{2AD - 1 - A}{1 - A}\beta - \frac{2AC}{1 - A}\zeta \end{aligned} \tag{1}$$

The position of the centre of the beam at the extraction magnet is no longer determined by the energy loss in the target, but by the position of the centre of the quadrupole ζ. Rewriting equations (1) we have

$$\begin{bmatrix} y_K - \left\{-\frac{2BC}{1 - A}\zeta\right\} \\ y_K{}' - \left\{-\frac{2AC}{1 - A}\zeta\right\} \end{bmatrix} = \frac{1}{1 - A} \times \begin{bmatrix} 2AD - 1 - A & 2DB - 2B \\ 2AC - 2A & 2AD - 1 - A \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \tag{2}$$

If the distance from target to extraction magnet is approximately one half a betatron oscillation (i.e. one-quarter of an oscillation from target to quadrupole) A and D will be small. The matrix in equation (2) will reduce to the form

$$\begin{bmatrix} -1 & -L \\ 0 & -1 \end{bmatrix} \quad \text{where} \quad L \simeq +2B.$$

Thus the beam at the extraction magnet willl appear to have come from a source through a drift length L. By giving the extraction magnet a radial field gradient the beam can be brought to a focus as it passes through the fringe field of the magnet; in this non-linear field the beam has a small radial dimension and all the particles see almost the same field and in consequence the distortion of the phase space is minimized.

Although the beam travels only a relatively small azimuthal distance round the accelerator as it goes from the extraction magnet to the outside of the

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machine, some further momentum resolution will take place. This can, however, be allowed for by arranging to leave a small dispersion in the beam at the magnet, which is then eliminated by the remaining magnetic path.

Suppose Δy_K, $\Delta y_K'$ are the differences in position and divergence at K of a proton of energy loss ΔE necessary to make its path coincide with that of a proton of no energy loss at some suitable point outside the accelerator. From the 'energy compaction' ratio we can obtain the value of $\Delta\eta$ equivalent to this ΔE.

Let $\quad \Delta y_K/\Delta\eta = L \quad$ and $\quad \Delta y_K'/\Delta\eta = L'$.

These quantities can be evaluated by trajectory integration. In general L and L' will depend on ΔE, but for the range of ΔE of interest (10 MeV say) this variation can be neglected. L and L' must include the effect of the slope of the field of the extraction magnet.

The η dependent parts of y_K and y_K' are

$$\eta\{1 - DE - BG + KB - KBE\} = \Delta y_K$$
$$\eta\{-CE - AG + KA - KAE\} = \Delta y_K'.$$

Thus we must ensure that

$$L = 1 - DE - BG + KB(1 - E)$$
$$L' = -CE - AG + KA(1 - E),$$

giving a value for the quadrupole strength

$$K = \{L - 1 + DE + BG\}/\{B(1 - E)\}$$

and the relation between L, L' and the matrix elements

$$L' = -CE - AG + A\{L - 1 + DE + BG\}/B$$

from which we may obtain (since $AD - BC = 1$)

$$E = BL' - AL + A.$$

Since the quadrupole and extraction magnet must in practice be in straight sections of the accelerator, the matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

will be fixed in value, and a required value of E can be obtained. By varying the position of the target the matrix containing this value of E can be found.

Because of the change of position of the target, the distance of the apparent source from the extractor magnet will be changed and a new value of the gradient of the field of the extraction magnet will be needed to focus the beam in the same position. This change of gradient in turn changes the values of L and L' and thus the process above must be iterated until acceptable values of both the target position and field gradient of the extraction magnet are obtained.

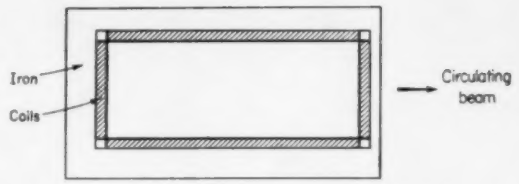

FIG. 2.—The quadrupole magnet (schematic).

3. THE QUADRUPOLE MAGNET

The quadrupole magnet referred to above must, like the PICCIONI extraction magnet, be close to the circulating beam in the machine. The distance between the usable aperture and the edge of the frame must be small to enable reasonably small energy loss in the target to move the protons from the circulating beam into the quadrupole aperture, and also to minimize the region of constant field gradient (n-value) required in the machine. This can be achieved by using a quadrupole of the type described by HAND and PANOFSKY (HAND, 1959) in which the iron return path and the current carrying conductors are made as thin as possible (see Fig. 2). Also by adding a second winding on three sides of the quadrupole aperture, it is possible to make the magnetic centre of the quadrupole adjustable, and thus the position of the beam in the extraction magnet may be controlled.

The strength of the quadrupole is moderate. For NIMROD (PICKAVANCE, 1959) (in which $r_0 = 1878$ cm, $n = 0{\cdot}6$ and $Q_R = 0{\cdot}71$) it would be located in a straight section 135° before the extraction magnet, and would have a strength of approximately 250 gauss/cm for a length of 60 cm.

CONCLUSION

Extensive numerical calculations have been made for the application of the achromatic scheme to the NIMROD accelerator. These calculations indicate that it should be possible to transport the beam from the target to the outside of the accelerator with only a very small increase in 'effective' phase area, and also that no shimming of the fringe field of the magnet will be required.

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SYNCHROTRON RADIATION FROM RUNAWAY ELECTRONS IN THE STELLARATOR*

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(*Received* 2 *June* 1960)

Abstract—The microwave radiation from low-density plasmas following ohmic heating discharges in the B-3 stellarator has been studied. The observed spectrum was found to be consistent with a calculated spectrum of synchrotron radiation from runaway electrons accelerated by the applied electric field. The deviation of the radiation frequency from the electron cyclotron frequency is attributed to Doppler shift caused by motion of the emitting electrons along the magnetic field. The maximum observed frequency deviation corresponded, within 10 per cent, to the maximum electron velocity which was kinematically possible, over an electron energy range extending from 25 kilovolts to almost 1 MeV.

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INTRODUCTION

IT has been realized that the stability of magnetically confined plasmas may be seriously impaired by the presence of a group of particles which are not a part of the normal Boltzmann distribution (BOHM and GROSS, 1949; AKHIEZER and FAYNBERG, 1951). In the stellarator an instability has been found (SPITZER, 1958; ELLIS *et al.*, 1960), the existence of which may be related to the phenomenon of electron runaway (DREICER, 1960). Information on runaway electrons has been obtained up to now by measurements of the X-rays emitted following loss of electron confinement (BERNSTEIN *et al.*, 1958). This paper reports experiments at microwave frequencies on the synchrotron radiation from runaway electrons in the stellarator.

Runaway electrons are created during the process of ohmic heating (COOR *et al.*, 1958). These electrons are accelerated by the induced electric field to energies well beyond the thermal energy and many of them survive the disappearance of the main body of plasma electrons. Initially, the velocity of the fast electrons is directed approximately along the magnetic field lines. However, plasma and atomic scattering convert a portion of the longitudinal motion into transverse motion, which then gives rise to synchrotron radiation. When the plasma density has risen such that the electron plasma frequency is in the vicinity of the observation frequency, the microwave radiation pattern is quite complex (HEALD, 1956). However, after instabilities have removed most of the plasma electrons, it is possible to observe synchrotron radiation from the runaway electron beam without the additional complication of strong plasma interaction.

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1. SYNCHROTRON RADIATION

Several recent analyses of the synchrotron radiation from hot plasmas in thermal equilibrium have been given (TRUBNIKOV and KUDRYAVTSEV, 1958; DRUMMOND and ROSENBLUTH, 1960). However, these papers are not relevant to our experimental situation, since we wish to describe radiation from a group of electrons which are not in thermal equilibrium. These electrons constitute a beam passing through a background of weakly-ionized helium atoms. In our experiments the electron beam is not mono-energetic, but may possess an energy spectrum extending to hundreds of kilovolts. The high energy electrons in the beam scatter little during the observation time of about 10 milliseconds. The low energy electrons ($T < 50$ kilovolts) scatter badly but thermalize quickly as a result of ionization and excitation of the neutral atoms. In either case, because the transverse energy is a small fraction of the electron rest energy, the electrons will emit synchrotron radiation predominantly at the fundamental frequency.

The synchrotron radiation from electrons with a total velocity $\beta = v/c$ and a component velocity $\beta_\parallel$ along the magnetic field will be Doppler shifted to an extent

$$\omega/\omega_0 = (1 - \beta_\parallel \cos\theta)^{-1}(1 - \beta^2)^{1/2} \qquad (1)$$

in a vacuum, where ω is the radiation frequency, ω_0 is the electron cyclotron frequency eB/m_0c, and θ is the angle between the magnetic field and the direction of the radiation. The B-3 stellarator (GORMAN *et al.*, 1958), with an effective aperture diameter of 4 centimetres and a bending radius of 25 centimetres, can confine electrons to a maximum energy of about 1 MeV at a confining field of 10 kilogauss. Electrons

with this energy will radiate at the fundamental frequency over a range extending from 5 kilomegacycles (kMc) to 160 kMc.

The intensity of the synchrotron radiation from one electron per unit solid angle $d\Omega$ in a frequency interval $d\omega$ at a frequency ω is given by TRUBNIKOV (1958):

$$\frac{dI(\theta)}{d\Omega\, d\omega} = \frac{e^2\omega^2}{2\pi c} \sum_{n=1}^{\infty} \left[\left(\frac{\cos\theta - \beta_\parallel}{\sin\theta} \right)^2 J_n^2(\kappa) + \beta_\perp^2 J_n'^2(\kappa) \right] \times \delta[n\omega_0(1-\beta^2)^{1/2} - \omega(1-\beta_\parallel\cos\theta)] \quad (2)$$

where $\beta_\perp$ is the electron velocity perpendicular to the magnetic field, $J_n(\kappa)$ are Bessel functions of order n, $\kappa = n\beta_\perp \sin\theta(1-\beta_\parallel\cos\theta)^{-1}$ and n is the harmonic number. For the special case where $\beta_\perp \ll 1$ most of the radiation is emitted in the first harmonic, the Bessel functions can be written in an approximate form, and one may simplify equation (2) to

$$\frac{dI(\theta)}{d\Omega\, d\omega} = \frac{e^2\omega^2\beta_\perp^2}{8\pi c} \left[\frac{(\cos\theta - \beta_\parallel)^2 + (1-\beta_\parallel\cos\theta)^2}{(1-\beta_\parallel\cos\theta)^2} \right] \times \delta[\omega_0(1-\beta^2)^{1/2} - \omega(1-\beta_\parallel\cos\theta)] \quad (3)$$

2. THE EXPERIMENT

Observations of microwave radiation at fixed frequencies of 70, 35, 24, and 10 kMc were made following ohmic heating discharges in the stellarator, as the magnetic field fell toward zero. Microwave horn antennas inside the vacuum system (35 and 70 kMc) and outside the glass windows (10 and 24 kMc) viewed the plasma column perpendicularly. Signals passed from the antennas along waveguide to superheterodyne receivers with bandwidths of 16 megacycles and sensitivities of 10^{-12} watts. Simultaneous measurements were made of microwave radiation, plasma current and voltage, the magnetic confining field, and the electron density. The plasma current was measured by means of a toroidally wound pick-up loop encircling the discharge tube. The electric field accelerating the electrons was inferred from the voltage across the ceramic insulator of the vacuum tube. The magnetic field was measured by integrating the induced voltage in a pick-up loop encircling the confining field current. The electron density was deduced from the phase shift of an 8·6 mm microwave beam propagated across the plasma. Data were presented and photographed on oscilloscopes.

We note the following results which indicate a synchrotron origin for the microwave noise emitted from the helium discharge afterglow.

(i) Radiation was detected by each of the receivers when the cyclotron frequency of the plasma electrons was close to the receiver frequency. Because of the Doppler shift the magnetic resonance spectrum was quite broad.

(ii) When the receiver frequency was much smaller than the electron cyclotron frequency at maximum magnetic field strength, it was possible to measure the full magnetic resonance spectrum. The magnetic field varied almost linearly from 15 kilogauss ($\omega_0/2\pi = 42$ kMc) to zero in 6 milliseconds following the discharge. During this time the power detected at 10 kMc was measured as a function of time.

In order to minimize the interaction of the runaway beam with the plasma and the neutral gas, discharges

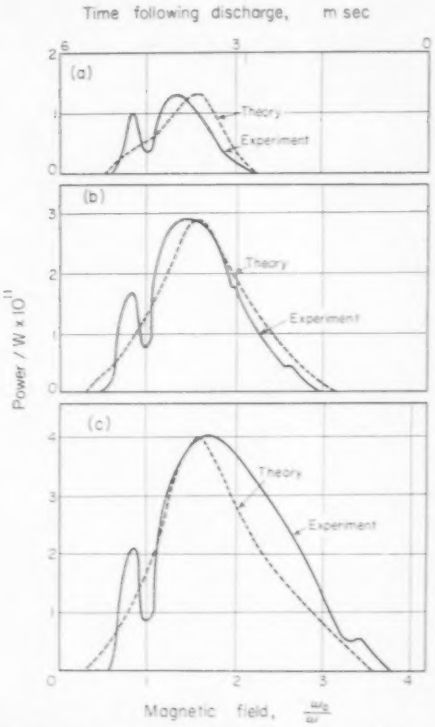

FIG. 1.—Intensity of synchrotron radiation in a 16 megacycle frequency interval at 10 kMc as a function of magnetic field (time) following three ohmic heating discharges in helium. The magnetic field is expressed as the ratio of the electron cyclotron frequency ω_0, to the observation frequency ω. Time increases to the left. The cyclotron resonance field ($\omega_0/\omega = 1$) is 3·6 kilogauss. The theoretical curves have been normalized at the resonance peaks.

were created in the stellarator at low pressure. Helium gas at a pressure of 0·14 microns of mercury was partially ionized ($n_e \sim 5 \times 10^{10}$ cm^{-3}) by a 150 volt, 250 kilocycle breakdown voltage applied for 100 microseconds across the ceramic insulator of the discharge tube. Then a unidirectional electric field of 0·17 volts/cm, induced around the stellarator vacuum tube, accelerated some of the electrons to high energies. At higher pressures an electric field of this strength will cause the gas to break down completely.

TABLE 1.—CHARACTERISTICS OF THREE OHMIC HEATING DISCHARGES IN HELIUM

Discharge	*A*	*B*	*C*
Neutral density	5×10^{12} cm^{-3}	5×10^{12} cm^{-3}	5×10^{12} cm^{-3}
Magnetic field (kilogauss)	$15(1 - \alpha t)$ $\alpha = 1{\cdot}67 \times 10^{3}$ sec^{-1}	$15(1 - \alpha t)$ $\alpha = 1{\cdot}67 \times 10^{3}$ sec^{-1}	$15(1 - \alpha t)$ $\alpha = 1{\cdot}67 \times 10^{3}$ sec^{-1}
$\int E\,dt$ (volt cm^{-1} sec)	$2{\cdot}0 \times 10^{-5}$	$3{\cdot}2 \times 10^{-5}$	$3{\cdot}9 \times 10^{-5}$
Initial plasma current (amperes)	<10	35	60
Maximum electron density	$0{\cdot}4 \times 10^{11}$ cm^{-3}	$1{\cdot}0 \times 10^{11}$ cm^{-3}	$1{\cdot}5 \times 10^{11}$ cm^{-3}
β_{max}	0·66	0·83	0·87
β_{min}	0·4	0·4	0·4
Measured energy input to plasma (joules)	0·05	0·2	0·4
	Calculated from synchrotron analysis		
Initial runaway current (amperes)	5	20	30
Energy input to runaway beam (joules)	0·02	0·1	0·3

However, at low pressure, plasma losses are not greatly overcompensated by the electron production. Hence the electron density did not increase substantially while the electric field was applied to the plasma.

The results obtained from three discharges in helium are shown in Fig. 1, where the solid lines show the observed radiation intensity as a function of magnetic field (time). Discharge conditions (listed in Table 1) were held constant, except that the ohmic heating voltage was removed from the plasma at different times. Dips in the observed power, due to absorption by residual plasma electrons, are observed when the cyclotron resonance condition $\omega = \omega_0$ is fulfilled. The cyclotron resonance absorption provided an absolute calibration of the magnetic field; the width of the absorption is caused by field inhomogeneity, which amounts to ± 8 per cent within the effective aperture of the discharge tube. An analysis of these data is given in Section 3.

(iii) Over an electron energy range extending from 25 kilovolts to almost 1 MeV the maximum Doppler shift of the radiation was consistent with that calculated from the applied (electric field) × (time). This experiment and the results are discussed in Section 4. Experimental conditions were the same as those described above, except that the gas pressure was 1 micron.

(iv) Radiation was detected by means of an antenna placed at a port which allowed a view along the magnetic field lines. The spectrum observed was the same as that obtained from the antenna at the 90° observation port. We infer that the original directional properties of the radiation have been largely lost by scattering and diffraction inside the vacuum tube.

3. ANALYSIS OF THE SYNCHROTRON SPECTRUM

In principle an analysis of the spectrum of the synchrotron radiation should enable one to determine the parallel velocity distribution of the runaway electrons. Several practical difficulties confront such a programme. The most formidable of these is the fact that scattering and diffraction will tend to make the radiation isotropic. Therefore, although the receiving antenna may be highly directional, radiation emitted at different angles θ to the magnetic field will be detected with an unknown efficiency.

We invert the problem and inquire whether it is possible to predict the gross features of the observed synchrotron radiation line under several simplifying assumptions. The synchrotron power emitted by an electron beam in a small frequency interval $d\omega$ at a frequency ω is a function of the magnetic field, the parallel velocity spectrum of the beam and the transverse velocity. The power observed depends also on the efficiency with which the antenna receives radiation at different angles. In our analysis we have introduced the following approximations:

(i) The number of runaway electrons diminishes in time in proportion to the magnetic flux contained inside the vacuum tube. This assumption is consistent with the observed decay of runaway current, in the absence of plasma current instabilities. We further assume that the relative number of electrons in a given velocity interval is independent of time. This simplification must obviously be incorrect for electrons which undergo large angle scattering, but we take care of this crudely by introducing a low velocity cut-off discussed under (3).

(ii) The electrons gain transverse energy by means

of atomic scattering, but lose some of this energy to the diminishing magnetic field. The effect of these competing processes for $\beta_\perp \ll \beta_\parallel$ is given by

$$d(\beta_\perp^2) = K\,dt + \beta_\perp^2 \frac{dB}{B} \qquad (4)$$

where K is a scattering parameter. From multiple scattering theory $K \cong 1{\cdot}8 \times 10^{-12} N(1-\beta^2)\beta^{-1}$ for the scattering of relativistic electrons by neutral helium atoms of density N. If the scattering is weak, β and hence K may be taken as constant. We neglect scattering which occurs during the acceleration period of the electrons and compute the transverse electron energy attained following the termination of the acceleration field. When the magnetic field varies linearly as $B = B_0(1-\alpha t)$, the solution of equation (4) gives the following time dependence for $\beta_\perp^2$:

$$\beta_\perp^2 = K(t - 1/\alpha)\ln(1-\alpha t). \qquad (5)$$

Under the experimental conditions listed in Table 1 the maximum value of $\beta_\perp^2$ predicted from equation (5) is $\beta_\perp^2 = 0{\cdot}04$. The transverse energy is thus small enough to allow the use of equation (3) with a maximum error of 5 per cent.

(iii) In order to determine the radiation at a frequency ω it is necessary to integrate equation (3) over the range of parallel electron velocities in the beam ($\beta_{\min}$ to $\beta_{\max}$). In our calculation we have taken the quantity $\beta_\perp^2 n_\beta$ which appears in the integrand to be independent of β. This assumption is reasonable if the rate of production of runaway electrons is constant while the accelerating field is applied to the plasma. In this case $n_\beta \propto (1-\beta^2)^{-3/2}$, while $\beta_\perp^2 \propto (1-\beta^2)\beta^{-1}$ from considerations of scattering. Then $\beta_\perp^2 n_\beta \propto (1-\beta^2)^{-1/2}\beta^{-1}$, a function which is equal to $2{\cdot}4 \pm 0{\cdot}4$ in the velocity interval $0{\cdot}4 < \beta < 0{\cdot}9$. This analysis breaks down for electrons of lower velocity. However, radiation from such electrons is not so significant, since the electrons will tend to thermalize in a time short compared with the observation time. The actual situation is complicated, but as an approximation we introduce a low velocity cut-off, $\beta_{\min}$, which is the minimum electron velocity which can survive ionization and scattering losses until the magnetic field falls through resonance; we have set $\beta_{\min}$ equal to 0·4. The value of $\beta_{\max}$ was computed from the magnitude and duration of the applied electric field.

(iv) The efficiency with which the receiving antenna detected the radiation is assumed independent of the emission angle θ. This assumption received support from the experiments described in Section 2.

The results of the analysis are indicated by the dashed lines of Fig. 1, where the theoretical curves have been normalized at their maxima. (The cyclotron resonance absorption by the residual plasma electrons has not been included in the calculation.) Note that, since these curves apply to the case in which observations are made at a fixed frequency while both the magnetic field and electron density are varying, the shape of the curves is quite different from the frequency spectrum of radiation with fixed magnetic field and electron density. This is why the theoretical curves in Fig. 1 peak for ω less than ω_0 rather than for ω greater than ω_0 as would be the case for fixed magnetic field and electron density.

It can be seen that the general shape and breadth of the observed resonance spectrum are fairly well predicted. There is, however, a large underestimate of the radiation intensity at magnetic fields slightly below the cyclotron resonance ($\omega = \omega_0$). (This corresponds to radiation at a frequency slightly above the cyclotron frequency.) It does not appear to be possible to account for this discrepancy on the basis of radiation at the second harmonic. The most likely explanation is that the radiation originates from the low-velocity electrons ($\beta_\parallel < 0{\cdot}4$) which have not been included in the analysis. Another obvious failure of the simple theory is that it does not predict the observed absence of radiation below a magnetic field of 0·6 ω or 2 kilogauss. One can trace this discrepancy to the assumption of a time independent parallel velocity spectrum. Actually, as the magnetic field decays, electron drifts in the stellarator bends cause a loss of the high-energy electrons. This effect is not an important factor in our analysis when the magnetic field is greater than 3 kilogauss.

From geometrical considerations it is possible to estimate the efficiency with which the antenna receives radiation from the stellarator. This estimate yields an efficiency of 2 per cent with an uncertainty of about a factor of three. We have then calculated the runaway beam currents required to produce radiation of the measured intensity. These currents, listed in Table 1, constitute only about one-half of the measured plasma currents, which are almost entirely due to runaway electrons. The estimated energy in the runaway beam is also about one-half of the measured energy input to the plasma. The discrepancy between the measured and calculated quantities is a result of the large uncertainty in the computation of the antenna efficiency.

4. ELECTRON VELOCITIES FROM THE MAXIMUM DOPPLER SHIFT

Although the exact shape of the synchrotron resonance spectrum depends on a number of parameters

which cannot easily be determined, the maximum Doppler shift should depend only on the maximum electron velocity. To test whether the maximum Doppler shift could be predicted over a wide range of electron energies, discharges in helium were terminated at various times so that the maximum electron energy varied from 25 kilovolts to 2 MeV. The direction of electron flow was opposite the direction of the magnetic field lines.

The magnetic field at which the radiation at 70 kMc cut-off was measured. β_{max} was then computed from equation (1) with $\cos\theta = 1$. Fig. 2 shows results

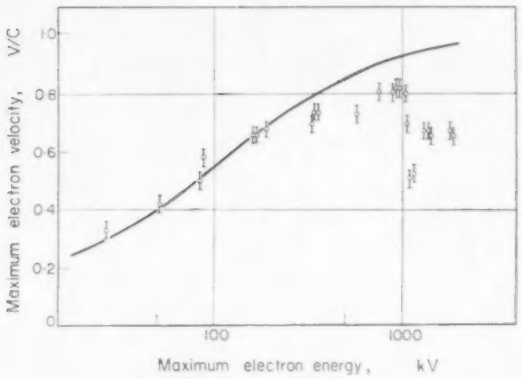

FIG. 2.—Results of a series of discharges in helium in which the ohmic heating voltage was removed at different times. The maximum electron velocity determined from the maximum Doppler shift of the 70 kMc radiation is plotted vs. the maximum electron energy computed from the magnitude and duration of the electric field. The curve represents the relativistic velocity-energy relation.

obtained with an initial magnetic field of 20 kilogauss, where the maximum electron velocity as determined from the Doppler shift calculation is plotted against the maximum energy computed from kinematics. The agreement between the synchrotron prediction and the kinematical calculation is good until the maximum possible electron energy was increased to almost 1 MeV. The interpretation of this data is that some electrons are accelerated in the stellarator up to the maximum velocity kinematically permissible. At the 1 MeV point the plasma current, at the time the electric field was removed from the plasma approached the Kruskal limit (KRUSKAL *et al.*, 1958). When the discharge duration was prolonged until the plasma current reached the KRUSKAL limit, we observed that the intensity of the synchrotron radiation fell by 90 per cent, and the maximum electron energy contained in the stellarator was sharply reduced. We infer that high-energy electrons are poorly confined when the plasma current is at the KRUSKAL limit. This information has previously been obtained from X-ray measurements (BERNSTEIN *et al.*, 1958). Our data are not inconsistent with a complete loss of runaway electrons at the KRUSKAL limit, because the residual synchrotron radiation may originate from electrons accelerated by residual electric fields persisting after the driving voltage has been removed from the plasma.

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MOUVEMENT D'UNE PARTICULE ET EQUILIBRE D'UN PLASMA EN GEOMETRIE CYLINDRIQUE

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Sommaire—L'intégration exacte de l'équation de Boltzmann en l'absence de collisions peut être réalisée dans le cas où il existe un ordre de symétrie assez élevé: état stationnaire ayant la symétrie de révolution et la symétrie cylindrique. En faisant une hypothèse supplémentaire d'isotropie des vitesses, on oblige le mouvement d'ensemble du plasma à être celui d'un solide, le système étant self-consistent on obtient alors une relation entre les champs magnétiques, relation assez bien vérifiée experimentalement.

Abstract—The exact integration of the collisionless Boltzmann equation can be performed by assuming stationary conditions and cylindrical symmetry. With an additional assumption of isotropy on the velocity distribution, it turns out that the macroscopic motion is that of a solid body. We obtain a relation between the components of the magnetic field in agreement with experimental results.

I. INTRODUCTION

L'EQUILIBRE magnetohydrodynamique classique suppose les dimensions du plasma grandes vis-à-vis du rayon de giration des particules; cette hypothèse permet de considérer les tubes de force comme étant relativement indépendants.

Mais cette condition ne semble pas réalisée pour les ions dans la plupart des expériences.

De grands rayons de giration, une diffusion même faible dûe à des collisions binaires ou à des collisions lointaines (ondes d'instabilité, turbulence) établissent des liaisons entre les différentes régions du plasma.

Cette étude en partant de l'intégration exacte de l'équation de Boltzmann sans second membre dans le cas d'une géométrie cylindrique et en faisant une hypothese supplémentaire d'isotropie permet d'obtenir le mouvement d'ensemble des ions et des électrons. Les distributions des vitesses macroscopiques obtenues de cette façon sont celles de deux solides ayant des mouvements helicoidaux autour du même axe.

Ce mouvement se traduit par une relation entre les composantes axiales et tangentielles du champ magnétique, relation qui a été vérifiée dans un certain nombre de cas expérimentaux.

II. EQUATIONS GENERALES

L'équation de Boltzmann pour un groupe de particules en l'absence de collision peut s'écrire

$$\frac{df}{dt} = 0 \tag{1}$$

où f est la fonction de distribution dans l'espace des phases. Cette équation signifie que f est une constante du mouvement individuel. On est donc amené à rechercher les intégrales premières du système différentiel lié aux trajectoires de ces particules. Les équations de base seront les équations Hamiltoniennes du mouvement d'une particule chargée non relativiste.

Soit H cet Hamiltonien.

$$H = \frac{1}{2m}\left[(P_r - qA_r)^2 + \left(\frac{P_\theta}{r} - qA_\theta\right)^2 + (P_z - qA_z)^2\right] + q\Phi \tag{2}$$

m étant la masse de la particule, q sa charge, P_i ($i = r$, θ ou z) les moments généralisés liés aux coordonnées cylindriques r, θ et z; $\mathbf{w}$ sa vitesse, t le temps. $\mathbf{A}$ un potentiel vecteur lie au champ magnétique $\mathbf{B}$ Φ le potentiel scalaire correspondant lié au champ électrique $\mathbf{E}$

Le potentiel vecteur $\mathbf{A}$ sera défini par les équations suivantes

$$\begin{aligned} \nabla \times \mathbf{A} &= \mathbf{B} \\ A_r &= 0 \end{aligned} \tag{3}$$

ce qui donne après intégration

$$\begin{aligned} A_r &= 0 \\ A_\theta &= \frac{1}{r}\int_0^r r' B_z \, dr' \\ A_z &= -\int_0^r B_\theta \, dr' \end{aligned} \tag{4}$$

B_θ et B_z étant des fonctions de r', θ, z et t. Le potentiel Φ est lié au potentiel $\mathbf{A}$ et au champ électrique $\mathbf{E}$ par la relation

$$\nabla\Phi = -\left(\frac{\partial \mathbf{A}}{\partial t} + \mathbf{E}\right) \tag{5}$$

ce qui donne après intégration

$$\Phi = -\int_0^z E_z(r=0, z', t)\,dz' - \int_0^r E_r(r', \theta, z, t)\,dr' \tag{6}$$

Moyennant ces définitions l'Hamiltonien et les moments généralisés s'écrivent:

$$\begin{aligned} H &= \tfrac{1}{2}m\mathbf{w}^2 + q\Phi \\ P_r &= mw_r \\ P_\theta &= mrw_\theta + qrA_\theta \\ P_z &= mw_z + qA_z. \end{aligned} \tag{7}$$

La dérivée totale par rapport au temps d'une grandeur F s'écrit

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + [F, H] \tag{8}$$

ou [,] est le crochet de Poisson; en particulier l'équation de Boltzmann sans second membre se met sous la forme

$$\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + [f, H] = 0.$$

La résolution de cette équation aux dérivées partielles revient à chercher les formes indépendantes F vérifiant $dF/dt = 0$. Toute fonction arbitraire de ces formes la vérifiera aussi.

L'équation (8) appliquée à l'Hamiltonien H et aux moments P_θ et P_z s'écrit:

$$\begin{aligned} \frac{dH}{dt} &= \frac{\partial H}{\partial t} = q\frac{\partial \Phi}{\partial t} - q\mathbf{w}\cdot\frac{\partial \mathbf{A}}{\partial t} \\ \frac{dP_\theta}{dt} &= -\frac{\partial H}{\partial \theta} = -q\frac{\partial \Phi}{\partial \theta} + q\mathbf{w}\cdot\frac{\partial A}{\partial \theta} \\ \frac{dP_z}{dt} &= -\frac{\partial H}{\partial z} = -q\frac{\partial \Phi}{\partial z} + q\mathbf{w}\cdot\frac{\partial A}{\partial z} \end{aligned} \tag{9}$$

Lorsqu'il existe deux relations de la forme suivante pouvant provenir d'une symétrie des champs électriques et magnétiques

$$\begin{aligned} T\frac{\partial \mathbf{A}}{\partial t} + \Psi\frac{\partial \mathbf{A}}{\partial \theta} + K\frac{\partial \mathbf{A}}{\partial z} = 0 \\ T\frac{\partial \Phi}{\partial t} + \Psi\frac{\partial \Phi}{\partial \theta} + K\frac{\partial \Phi}{\partial z} = 0. \end{aligned} \tag{10}$$

T, Ψ et K étant des constantes.

On obtient l'intégrale première suivante

$$-TH + \Psi P_\theta + KP_z = \text{cste} \tag{11}$$

Deux intégrales indépendantes dans le cas de la symétrie cylindrique seront utilisées dans la suite; cette intégrale permet de plus l'étude de certaines déformations hélicoïdales.

III. MOUVEMENT INDIVIDUEL DANS LE CAS D'UNE GEOMETRIE CYLINDRIQUE STATIQUE

Les hypothèses seront alors les suivantes:

(a) la géométrie est une géométrie cylindrique de révolution: le champ magnétique ne comporte que des composantes $B_z(r)$ et $B_\theta(r)$ uniquement fonction de r.

(b) le champ électrique est radial et ne dépend que du rayon.

Dans ce cas on obtient les intégrales premières suivantes

$$\begin{aligned} H &= \text{cste} \\ P_\theta &= \text{cste} \\ P_z &= \text{cste}. \end{aligned} \tag{12}$$

On peut alors poursuivre l'intégration du système différentiel donnant les trajectoires des particules en posant

$$\begin{aligned} W_r^2 &= \frac{2}{m}[H - q\Phi(r)] - \left[\frac{P_\theta - qrA_\theta}{mr}\right]^2 - \left[\frac{P_z - qA_z}{m}\right] \\ &\equiv \varphi(r, H, P_\theta, P_z). \end{aligned} \tag{13}$$

Confinement d'une particule dans un cylindre de rayon **R**

Les équations (12) permettent par la connaissance de H, P_θ et P_z de déterminer les composantes de la vitesse d'une particule pour un rayon quelconque. Pour qu'une particule puisse atteindre ce rayon il faut que les composantes de cette vitesse soient réelles.

La seule qui puisse devenir imaginaire est la composante radiale w_r.

La condition de confinement s'écrit donc

$$\varphi(R, H, P_\theta, P_z) < 0 \tag{14}$$

On peut expliciter cette condition pour une particule se trouvant sur l'axe en utilisant les équations (7) soit

$$\mathbf{w}^2 - \frac{2q}{m}\Phi(R) - \frac{q^2}{m^2}A_\theta^2(R) - \left[w_z - \frac{q}{m}A_z(R)\right]^2 < 0 \tag{15}$$

On obtient ainsi dans un plan énergie radiale-vitesse axiale une région à l'intérieur de laquelle une particule située sur l'axe ne pourra atteindre les parois cylindriques de rayon R, en posant $\varepsilon = \dfrac{\mathbf{w}^2 - w_z^2}{\mathbf{w}^2}$ soit le rapport entre l'énergie cinétique radiale et l'énergie

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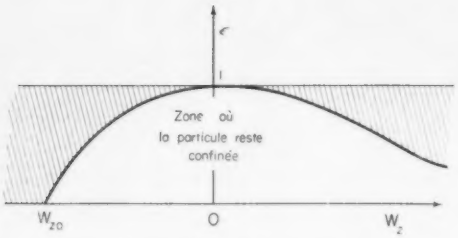

FIG. 1.

cinétique totale on obtient la région suivante (Fig. 1) ainsi qu'une vitesse limite w_{z0}

$$w_{z0} = \frac{\phi(R)}{A_z(R)} + \frac{q\bar{A}^2(R)}{2mA_z(R)}$$

IV. ETATS STATIONNAIRES D'UN PLASMA SANS COLLISIONS DANS UNE GEOMÉTRIE CYLINDRIQUE DE RÉVOLUTION

1. *Cas général*

L'équation de Boltzmann sans collision signifie que la fonction de distribution f est une constante du mouvement individuel.

En ne cherchant que les états statiques valables dans une géométrie cylindrique de révolution on impose de plus à la fonction de distribution les conditions suivantes:

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial z} = \frac{\partial f}{\partial \theta} = 0. \tag{16}$$

Dans ce cas la fonction f est une fonction arbitraire des 3 constantes calculées précédemment H, P_θ et P_z, ces 3 constantes étant exprimées en fonction de $\mathbf{w}$ et r par les équations (7).
soit

$$f(\mathbf{w}\, r) \equiv g(H, P_\theta, P_z). \tag{17}$$

Il est à remarquer que g est une fonction arbitraire mais que jusqu'à présent il n'a pas été tenu compte de la seconde partie des équations de Maxwell

$$\begin{aligned} \nabla \times \mathbf{E} &= 4\pi c^2 \sigma \\ \nabla \times \mathbf{B} &= 4\pi \mathbf{I} \end{aligned} \tag{18}$$

où c est la vitesse de la lumière, σ la densité de charge, $\mathbf{I}$ la densité de courant électrique. Pour que le système soit cohérent il sera nécessaire de faire le calcul de grandeurs macroscopiques et de prendre en considération le fait qu'il existe plusieurs types de particules.

Soit n_j la densité numérique et $\mathbf{v}_j$ la vitesse macroscopique des particules de masse m_j et de charge q_j on aura

$$\begin{aligned} n_j(r) &= \iiint g_i(H_j, P_{\theta j}, P_{zj})\, d^3w \\ v_{zj}(r) &= \frac{1}{n_j(r)} \iiint g_j(H_j, P_{\theta j}, P_{zj})\, d^3w\, w_z \\ v_{\theta j}(r) &= \frac{1}{n_j(r)} \iiint g_j(H_j, P_{\theta j}, P_{zj})\, d^3w\, w_\theta \end{aligned} \tag{19}$$

Il est facile de vérifier que $v_{rj} = 0$.
Les équations de Maxwell (18) s'écrivent alors

$$\begin{aligned} \Sigma_j n_j q_j &= \frac{-1}{4\pi r c^2} \frac{d}{dr}\left(r \frac{d\Phi}{dr}\right) \\ \Sigma_j v_{zj} n_j q_j &= \frac{-1}{4\pi r} \frac{d}{dr}\left(r \frac{dA_z}{dr}\right) \\ \Sigma_j v_{\theta j} n_j q_j &= \frac{-1}{4\pi} \frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rA_\theta)\right) \end{aligned} \tag{20}$$

ce qui en connaissant les fonctions de distributions relatives aux divers groupes de particules, ou plus exactement en sachant quelles sont les fonctions arbitraires g_j permet de calculer les potentiels vecteurs et scalaires par la résolution de système différentiel (20).

2. *Etude de certaines fonctions de distributions*

On se placera dans le cas où la fonction de distribution est isotrope autour d'une vitesse macroscopique moyenne au voisinage d'un point donné, ceci étant valable pour les divers types de particules; f_j sera une fonction de

$$[\mathbf{w} - \mathbf{v}_j(r_1)]^2 \text{ pour } r = r_1.$$

$\mathbf{v}_j(r_1)$ étant la vitesse moyenne des particules j au point considéré. En utilisant le fait que $f_j[(\mathbf{w} - \mathbf{v}_j(r_1))^2, r_1]$ est une fonction des Intégrales premières H_j, $P_{\theta j}$ et P_{zj} on obtient par une identification dans l'espace des vitesses que la fonction de distribution est une fonction arbitraire γ_j d'une combinaison linéaire des intégrales premières précédentes soit

$$f(\mathbf{w}, r) \equiv \gamma_j(H_j - \omega_j P_{\theta j} - u_j P_{zj}) \tag{21}$$

ω_j et u_j étant des constantes.

En considérant l'équation (21) on obtient directement les vitesses macroscopiques

$$v_{jr} = 0, \quad v_{j\theta} = \omega_j r, \quad v_{jz} = u_j. \tag{22}$$

Cette distribution des vitesses macroscopiques est celle d'un cylindre solide ayant un mouvement helicoïdal autour de son axe: la vitesse longitudinale étant égale à u_j et la vitesse angulaire de rotation à ω_i soit

$$U_j(\mathbf{w}, r) \equiv H_j - \omega_j P_{\theta j} - u_j P_{zj}.$$

$U_j(\mathbf{w}, r)$ représente une énergie qui peut être décomposée en une énergie thermique $\frac{m_j}{2}(\mathbf{w} - \mathbf{v}_j)^2$ et une énergie potentielle $\prod_j(r)$

$$\prod_j(r) = +q_j\omega_j r A_0 + q_j u_j A_z + q_j\Phi - \frac{m_j\omega_j^2 r^2}{2} \tag{23}$$

$\prod(r)$ se compose de 2 potentiels magnétiques, d'un potentiel électrostatique et d'un potentiel centrifuge.

3. *Cas où il y a deux types de particules de charges de signe opposé*

L'indice i sera réservé aux ions et l'indice e aux électrons; on supposera de plus que la configuration est basée sur un confinement magnétique pour une des deux assemblées, par exemple celle des électrons. Dans le cas d'un confinement du type pinch, en se plaçant dans un repère tel que $|u_i| \ll |u_e|$ la condition précédente s'écrit en posant $q = -q_e$ et $q_i = Zq$

$$|u_e A_z| \gtrsim |\phi|$$

ou en tenant compte des équations (20)

$$Zn_i(r) - n_e(r) \leqslant \frac{u_e^2}{c^2} n_e(r) \tag{24}$$

relation exprimant la quasi neutralité d'un plasma non relativiste. On obtient une relation analogue dans le cas d'un confinement par champ axial.

Les équations (20) seront donc remplacées en negligeant le terme $\frac{u_e^2}{c^2}$ devant l'unité par le système suivant

$$Zn_i(r) = n_e(r) = n(r) \tag{25}$$

$$\begin{aligned} q(u_i - u_e)n(r) &= \frac{1}{4\pi r}\frac{d}{dr}(rB_0) \\ q(\omega_i - \omega_e)n(r) &= \frac{-1}{4\pi r}\frac{d}{dr}(B_z). \end{aligned} \tag{26}$$

On peut résoudre partiellement ce système par une combinaison linéaire des équations (26):

$$B_{z0} - B_z = \frac{\omega_i - \omega_e}{u_i - u_e}(rB_0 - 2i) \tag{27}$$

On obtient donc une relation linéaire entre le champ magnétique longitudinal et le produit du champ magnétique tangentiel B_0 par le rayon. Dans la formule précédente B_{z0} est la valeur du champ magnétique sur l'axe et i le courant circulant dans un conducteur placé sur cet axe (Pinch inverse, triaxial pinch).

Un cas particulier intéressant est le cas où les deux fonctions f ont la même structure soit:

$$\begin{aligned} f_i(\mathbf{w}, \mathbf{r}) &\equiv \frac{1}{Z}\alpha\left[\frac{U_i(\mathbf{w}, r)}{kT_i} - a_i\right] \\ f_e(\mathbf{w}, \mathbf{r}) &\equiv \alpha\left[\frac{U_e(\mathbf{w}, r)}{kT_e} - a_e\right]. \end{aligned} \tag{28}$$

L'équation (25) peut alors s'écrire

$$\frac{\Pi_i(r)}{kT_i} = \frac{\Pi_e(r)}{kT_e} \tag{29}$$

où k est la constante de Boltzmann, T_i et T_e deux constantes ayant les dimensions d'une température, mais ne devenant les températures électroniques et ioniques que dans le cas où α est une exponentielle décroissante. L'étude numérique sera faite dans ce cas. On retrouvera comme cas particulier le potentiel de THONEMANN (1951) et la répartition radiale de BENNETT (1934).

La résolution de l'équation (29) donne:

$$\begin{aligned} \Phi(r) = &-\frac{\omega_e T_i + Z\omega_i T_e}{T_i + ZT_e} rA_0 - \frac{u_e T_i + Zu_i T_e}{T_i + ZT_e} A_z \\ &- r^2 \frac{m_e\omega_e^2 T_i + m_i\omega_i^2 ZT_e}{2q(T_i + ZT_e)} \end{aligned} \tag{30}$$

et pour la densité numérique:

$$\begin{aligned} n(r) = n\Bigg[&\left(\frac{1}{k\left(T_e + \frac{T_i}{Z}\right)}\right)\Bigg(q(\omega_e - \omega_i)rA_0 \\ &+ q(u_e - u_i)A_z + \left(m_e\omega_e^2 + \frac{m_i\omega_i^2}{Z}\right)\frac{r^2}{2}\Bigg)\Bigg] \end{aligned} \tag{31}$$

En rappelant les équations déjà obtenues, on forme avec l'équation (31) le système différentiel permettant de calculer les champs magnétiques moyennant la connaissance de la fonction α et d'un certain nombre de paramètres

$$\begin{aligned} q(\omega_i - \omega_e)n(r) &= \frac{1}{4\pi r}\frac{d}{dr}(rB_0) \\ B_{z0} - B_z &= \frac{\omega_i - \omega_e}{u_i - u_e}(rB_0 - 2i) \\ B_z &= \frac{1}{r}\frac{d}{dr}(rA_0) \\ B_0 &= -\frac{d}{dr}(A_z). \end{aligned} \tag{32}$$

On peut aussi adjoindre au système (32) les équations suivantes de manière à résumer les principaux résultats:

$$v_{jr} = 0, \quad v_{j0} = \omega_j r, \quad v_{jz} = u_j.$$

4. *Résumé des hypothèses et validité du modèle*

Les hypothèses qui ont été faites sont les suivantes

(1) l'absence de collisions.

(2) l'état stationnaire: les champs magnétiques et électriques sont indépendants du temps et par suite il n'y a pas, compte tenu de l'absence de collisions, de champ électrique longitudinal.

(3) l'isotropie des fonctions de distribution autour d'une vitesse macroscopique.

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On peut considérer que l'hypothèse (1) est vérifiée si l'on s'intéresse à la configuration instantanée d'un plasma et non à son évolution dans le cas où les deux conditions suivantes sont remplies:

(a) le temps que dure une collision est beaucoup plus petit que celui qui s'écoule entre deux collisions (définies par une déviation 90° par exemple).

(b) les longueurs caractéristiques L liées aux gradients qui existent dans un plasma ne sont pas d'un ordre très supérieur au libre parcours moyen.

L'hypothèse (2) peut être vérifiée si la variation des champs magnétiques est négligeable pendant le temps LV où V est la vitesse moyenne des particules, et si de plus les champs électriques d'induction sont faibles; on peut aussi remarquer qu'une combinaison linéaire des intégrales premières H, P_θ et P_z est une intégrale première valable dans des conditions beaucoup plus générales en particulier dans le cas de déformations hélicoïdales conservant le pas des courants.

L'hypothèse (3) est une première approximation pour une fonction de distribution. De plus lorsque la fonction α est une exponentielle la fonction de distribution choisie représente l'équilibre thermodynamique de chacune des 2 espèces considérées comme indépendantes (avec l'hypothèse de conservation de l'énergie et des moments pour chacune de celles-ci).

La solution envisagée dans ce cas reste donc valable en présence de collisions électrons-électrons ou ions-ions, seules les collisions électrons-ions n'ont pas été prises en compte, ces dernières se traduisent par une résistivité du plasma et une certaine viscosité des deux fluides l'un par rapport à l'autre. Dans les cas d'expériences usuelles l'énergie dépensée par effet Joule est faible vis-à-vis de l'énergie magnétique fournie; il s'ensuit que ces effets restent secondaires. La paroi par contre semble avoir une importance beaucoup plus grande sur le comportement du plasma.

RESUME DES RESULTATS

Moyennant les hypothèses précédentes, dans une géométrie cylindrique de révolution on peut déduire:
—qu'il existe une relation linéaire entre B_z et rB_θ,
—que le courant longitudinal est proportionnel à la densité $n(r)$ et le courant azimutal à $r\,n(r)$.

LISTE DES SYMBOLES UTILISES

$\mathbf{A}$ = potentiel vecteur (uem)
a = constante sans dimension
$\mathbf{B}$ = champ magnétique (uem)
c = vitesse de lumière
d = symbole différentiel total
$\mathbf{E}$ = champ électrique (uem)
e = charge de l'électron (ues) ou en indice, grandeurs se rapportant aux électrons
f = fonction de distribution dans l'espace des phases
g = fonction de distribution sur les intégrales premières
H = Hamiltonien
h_0 = distance de Debye caractéristique d'un plasma
I = densité de courant (uem)
i = courant dans un conducteur axial ou en indice grandeurs se rapportant aux ions
k = constante de Boltzmann
m = masse d'une particule
n = densité numérique
n_0 = densité numérique caractéristique (constante)
$\mathbf{P}$ = moment généralisé
p_0 = pression caracteristique
q = charge d'une particule (uem)
R = rayon de la paroi
r = rayon
r_0 = rayon caractéristique d'un pinch
T = température
t = temps
U = énergie macroscopique d'une particule
u = vitesse d'entrainement longitudinal d'un type de particule
v = vitesse macroscopique d'un type de particule
$\mathbf{w}$ = vitesse d'une particule
$X = rA_z$ en variable réduite
$Y = A_\theta - 2i \log \rho$ en variable réduite
z = coordonnée axiale
Z = nombre de charges d'un ion
α = fonction de distribution portant sur $\dfrac{U}{kT}$
γ = fonction de distribution portant sur U
∇ = vecteur symbolique gradient
∂ = symbole de la dérivée partielle
θ = coordonnée angulaire
λ = champ magnétique sur l'axe, variable réduite
μ = pas des courants, variable réduite
ν = courant i, variable réduite
Π = énergie potentielle
ρ = rayon réduit $\dfrac{r}{r_0}$
σ = densité de charges (uem)
Φ = potentiel scalaire (uem)
ω = vitesse de rotation macroscopique d'un type de particules.

RESULTATS NUMERIQUES ET CONFRONTATION EXPERIMENTALE

1. *Intégration des équations* (32)

La résolution du système différentiel a été faite dans le cas où le potentiel centrifuge $\frac{1}{2}\left(m_e\omega_e^2 + \dfrac{m_i}{Z}\,\omega_i^2\right)r^2$ est négligeable vis-à-vis des 'potentiels magnétiques' qui sont de l'ordre de grandeur de l'énergie thermique du plasma.

En choisissant comme fonction α la fonction e^{-x} le système (32) s'écrit:

$$n(r) = n_0 \exp\left[\frac{q}{k\left(T_e + \dfrac{T_i}{Z}\right)}\left((\omega_i - \omega_e)rA_\theta(r) + (u_i - u_e)A_z(r)\right)\right] \quad (33)$$

n_0 étant une constante de la dimension d'une densité (densité au centre d'un plasma lorsqu'il n'y a pas de conducteur axial).

Equations en variables réduites: En remplaçant q en uem par sa valeur $\frac{e}{c}$ e étant la charge de l'électron en unités électrostatiques et en posant

$$X = q\frac{\omega_e - \omega_i}{k\left(T_e + \frac{T_i}{Z}\right)} rA_z$$

$$Y = q\frac{u_e - u_i}{k\left(T_e + \frac{T_i}{Z}\right)} \int_0^r \left(B_\theta - \frac{\varepsilon i}{r}\right) dr \qquad (34)$$

$$\rho = \frac{r}{r_0} \text{ avec } r_0 = \frac{ch_0}{u_e - u_i}, \; h_0 = \left[\frac{k\left(T_e + \frac{T_i}{Z}\right)}{4\pi n_0 e^2}\right]^{\frac{1}{2}}$$

on obtient

$$\frac{\lambda}{2} + \mu^2 \frac{1}{\rho}\frac{dX}{d\rho} = \rho \frac{dY}{d\rho}$$

$$\frac{d}{d\rho}\left(\rho \frac{dY}{d\rho}\right) = \rho^{\nu+1} e^{-(X+Y)} \qquad (35)$$

où les paramètres λ, μ et ν sont

$$\lambda = \frac{B_{z0}c}{e(\omega_i - \omega_e)2\pi r_0^2 n_0}$$

$$\mu = \frac{u_e - u_i}{(\omega_e - \omega_i)r_0} \qquad (36)$$

$$\nu = \frac{2ie(u_e - u_i)}{k\left(T_e + \frac{T_i}{Z}\right)c}$$

avec comme conditions initiales pour $\rho = 0$

$$X = 0, \quad Y = 0, \quad \frac{dY}{d\rho} = 0$$

les valeurs calculées seront alors

$$\frac{n}{n_0}, \qquad \frac{B_z\lambda}{2B_{z0}\mu^2}, \qquad \frac{B_\theta\lambda}{2B_{z0}\mu^2}.$$

Conséquences des équations précédentes: on obtient les équations suivantes qui ne sont que des équations aux dimensions:

en posant $\quad n_0 k\left(T_e + \frac{T_i}{Z}\right) = p_0$

$$p_0 = 4\pi r_0^2 I_{z0}, \quad I_{z0} = n_0(u_e - u_i)\frac{e}{c} = \frac{B_{z0}\mu}{2\pi r_0 \lambda}$$

Dans le cas d'un pinch classique ($\nu = 0$) on peut chercher la condition pour que la densité de courant électrique I_z soit constante au voisinage de l'axe. Cette condition est obtenue en écrivant que le développement limité de la densité $n(r)$ au voisinage de $r = 0$ est de la forme $n_0 + d_4 r^4$.

Cette condition s'écrit au voisinage de l'axe

$$\frac{I_\theta}{B_\theta} = \frac{I_z}{B_z} \qquad (37)$$

ce qui se traduit par $\lambda = \mu^2$.

Cette relation est la relation 'force free field' classique. Dans la suite, les configurations de ce type vers lesquelles ont l'air d'évoluer les configurations expérimentales seront dénommées configurations C.F.F.F. (central force free field).

2. *Résultats expérimentaux obtenus dans des géométries cylindriques sans conducteur axial*

La comparaison des résultats expérimentaux et calculés porte sur les résultats obtenus par J. L. TUCK (1958) dans une chambre à striction linéaire, colombus S_4 et sur ceux obtenus par COLGATE (1958) sur un montage analogue (Figs. 2, 3 et 4).

La première vérification qui n'implique pas la distribution maxwellienne pour les fonctions γ_j porte sur l'existence d'une relation linéaire entre B_z et rB_θ (courbes suivantes). On peut considérer que l'on obtient avec une assez bonne approximation des droites si l'on élimine un certain nombre de cas:

(a) Le pinch est fortement décentré:
On n'a pas en général une connaissance exacte des lignes de champs mais la relation linéaire peut rester valable vis-à-vis de certaines déformations hélicoidales conservant le mouvement solide.

(b) Le plasma touche la paroi:
La courbe B_z, rB_θ est bien rectiligne dans sa partie centrale mais s'incurve lorsque l'on approche des parois.

Dans le cas d'une géométrie toroïdale à courbure faible on obtient aussi une région centrale linéaire si l'on peut déterminer le centre de la décharge avec précision et faire les corrections dues à la courbure des champs magnétiques.

3. *Courbes obtenus par l'intégration numérique du système différentiel (équation* 34)

(a) *Sans conducteur axial.* L'intégration numérique a été réalisée à Fontenay-aux-Roses par M. DE PALMA sur une machine à calculer digitale.

La Fig. 5 représente le plan des paramètres λ et μ^2 et les courbes des différents types:

La Fig. 6 permet à partir d'une courbe expérimentale de retrouver les paramètres λ et μ d'une courbe calculée qui pourrait lui correspondre et d'obtenir

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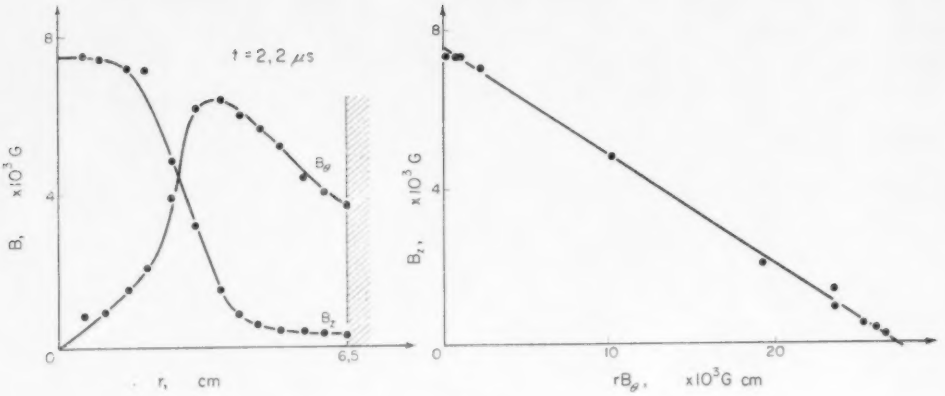


FIG. 2.

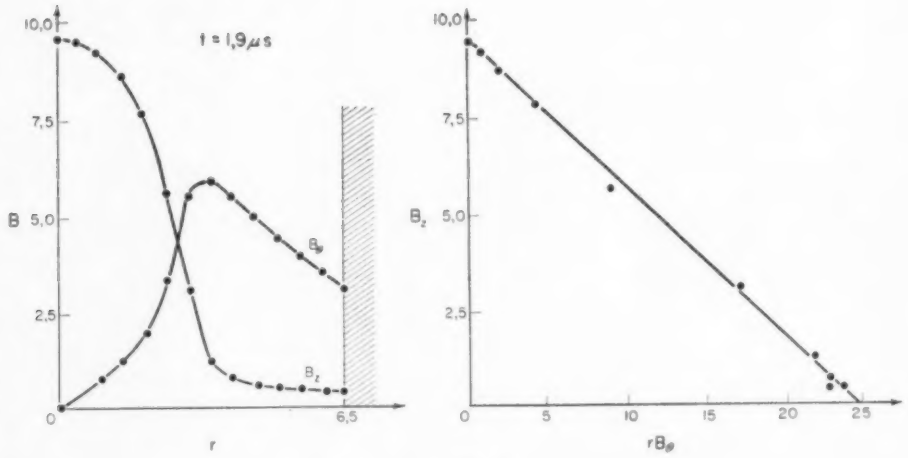


FIG. 3.

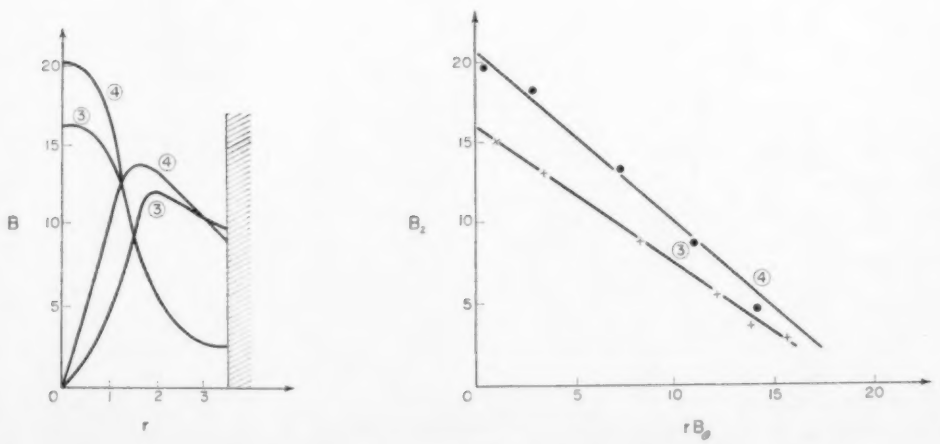


FIG. 4.

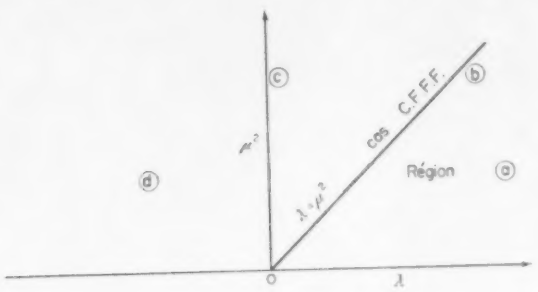
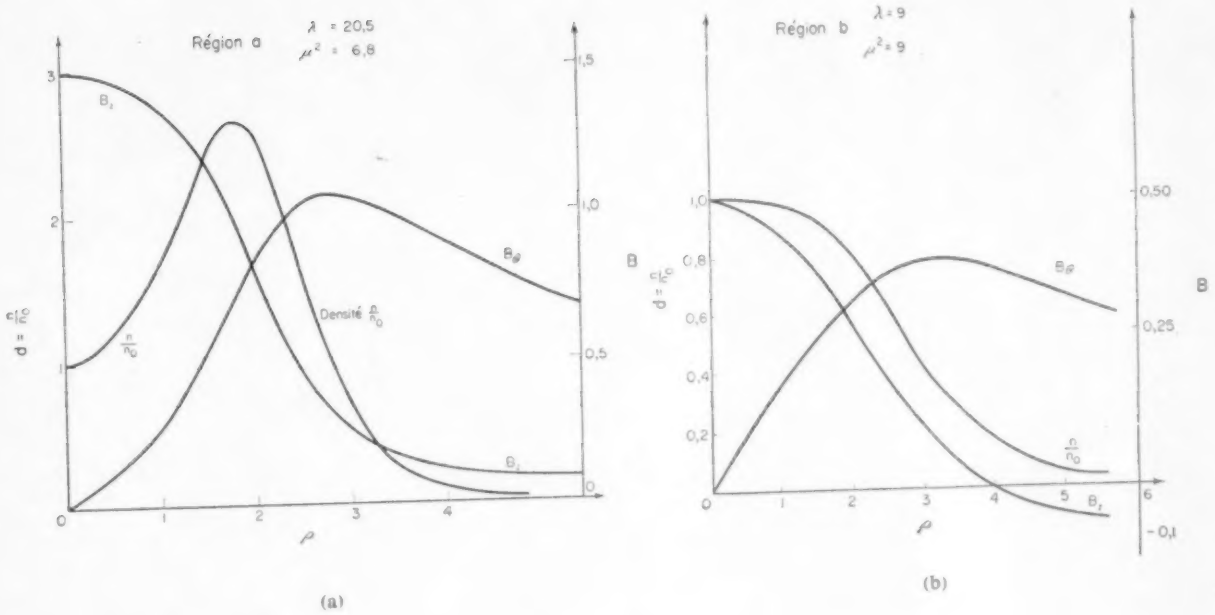
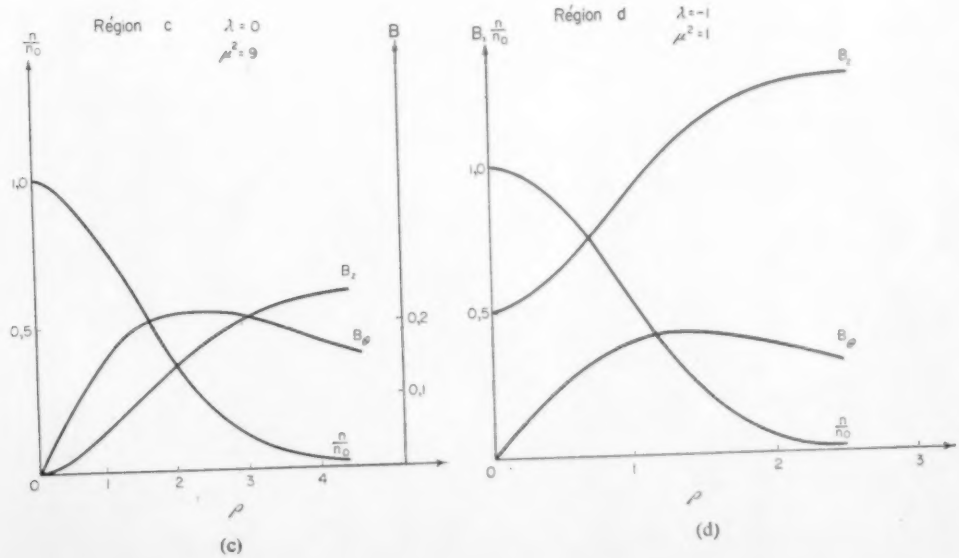


FIG. 5.





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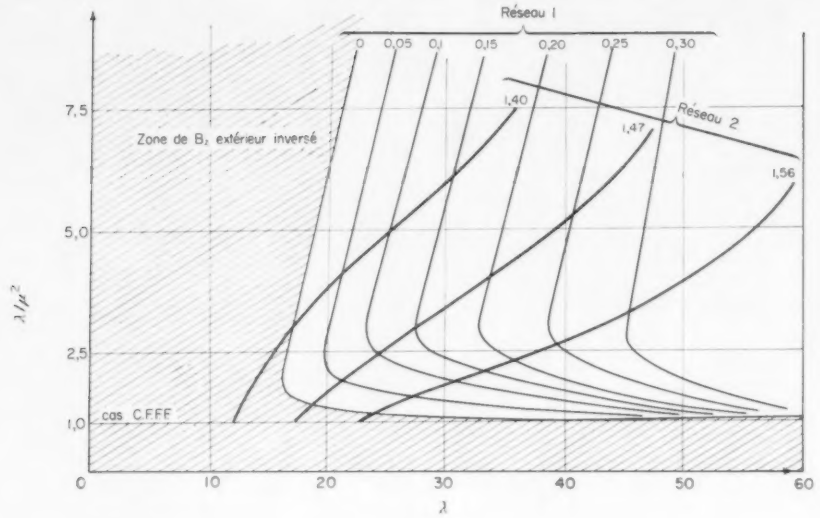

FIG. 6.

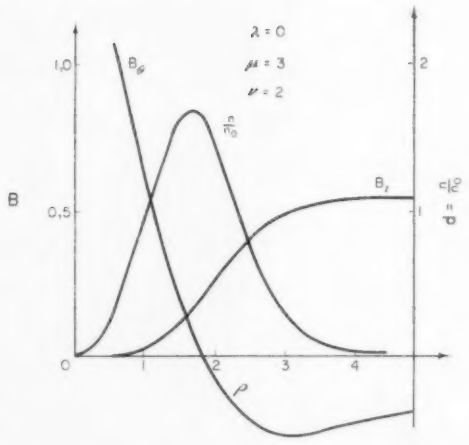

FIG. 7.

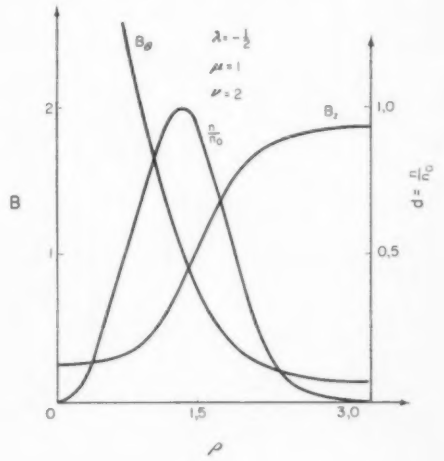

FIG. 8.

ainsi une répartition possible des pressions qui sont alors proportionnelles à la densité. Le plan λ, $\frac{\lambda}{\mu^2}$ comporte 2 réseaux de courbes: le réseau 1 correspond au rapport $\frac{B_z \text{ minimum}}{B_{z0}}$, le réseau 2 au rapport $\frac{B_{z0}}{B_0 \text{ maximum}}$.

(b) *Avec conducteur axial.* On sera dans le cas où $\nu > 0$. On peut dans ce cas s'attendre à des types de courbes correspondant à des 'triaxal pinch' ou à des 'hard core'. (Figs. 7 et 8.)

4. *Conclusion*

L'existence d'une relation linéaire entre le champ magnétique axial B_z et le produit rB_0 semble relativement bien vérifiée dans les cas considérés. Dans ces conditions la variation de la densité en fonction du rayon est relativement facile à déduire puisque celle-ci est proportionnelle au courant axial et au quotient $\frac{I_0}{r}$. De plus la forme des courbes calculées pour les champs magnétiques dans le cas d'une répartition Maxwellienne semble bien s'accorder avec les résultats expérimentaux des sondages magnétiques.

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RESEARCH NOTES

Spectroscopic observations of 'fluctuations' in the Sceptre IV discharge

(*Received* 7 *May* 1960)

1. INTRODUCTION

OBSERVATIONS of the variation of intensity with time of many spectral lines from Sceptre IV, CHICK (1959), show the common characteristic of rapid fluctuations superimposed on a slower, more regular, intensity-time variation. It has been suggested by WARE (1959) that these fluctuations are due to a hydromagnetic instability. WARE (1960) has shown that the instability perturbation will propagate as a wave. Attempts have been made on Sceptre IV to detect a wave velocity in the fluctuations of spectral line intensity.

2. EXPERIMENTAL PROCEDURE

Spectroscopic measurements can be made on the Sceptre IV discharge through two vertical quartz windows. The lines of sight passing normally through the centres of the windows cross the axis of the torus with a separation of 25 cm (see Fig. 1).

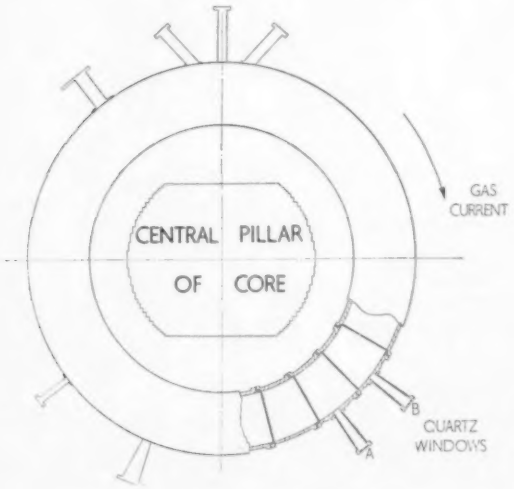

FIG. 1.—Line diagram of Sceptre IV.

Observations of the variation of intensities of spectral lines with time have been made using two monochromators with photomultiplier attachments, each monochromator viewing one of the two windows.

The observations were made on the OV line at λ2781 Å, the CuI resonance line at λ3247 Å and $D\beta$, λ4860 Å. The discharge parameters were: deuterium pressure $1{\cdot}6 \times 10^{-3}$ mm Hg; peak current about 2×10^5 amp; turns ratio 8:1, condenser bank 910 μF, charged to 20 kV. The parameter θ had a value of 2 when the initial toroidal stabilizing field was 1,250 gauss, where

$$\theta = \frac{I}{5b\,B_{\phi_0}}$$

and I = peak discharge current, in amp,
B_{ϕ_0} = initial toroidal magnetic field, in gauss,
b = tube radius, 15 cm in this case.

3. RESULTS

With a B_{ϕ_0} value of 1,250 gauss (i.e. $\theta \simeq 2$), most of the pairs of oscillograms of the OV intensity observed through the two windows showed marked similarities. A typical pair of oscillograms is shown in Fig. 2. A phase displacement will be

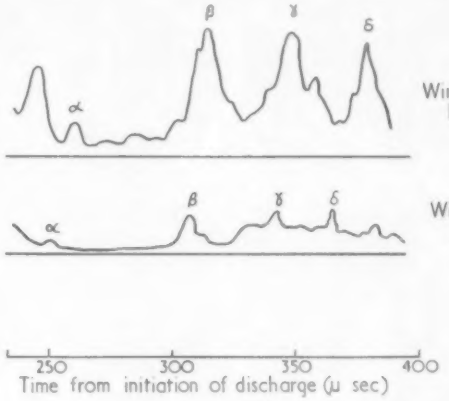

FIG. 2.—Oscillograms of OV intensity.

observed between corresponding features, and this is typical of the results when such features can be recognized (that is for about 80 per cent of the exposures). It will be seen that the phase displacements for each of the four features are approximately the same from one trace to the next. It is extremely unlikely that the effect could be due to purely random fluctuations and the observations therefore indicate a plasma disturbance which moves round the torus with a velocity of the order of 5×10^6 cm sec^{-1}, in the opposite direction to the discharge current. The velocities deduced in this way from features which occurred at varying times, t, after initiation of the discharge have values lying between $6{\cdot}9 \times 10^6$ cm sec^{-1} and $2{\cdot}2 \times 10^6$ cm sec^{-1}. Further work is needed to show whether this spread of velocities is a systematic variation during the duration of the discharge, or if it is due to variations in velocity from discharge to discharge. The mean values of the velocities measured from pairs of oscillograms is $5{\cdot}8 \times 10^6$ cm sec^{-1}. Some of the other oscillograms taken with values of B_{ϕ_0} greater than 1,250 gauss showed that a similar phenomenon also occurred at lower values of θ. There was less marked correlation between features on the oscillograms of the intensity of the CuI line, but a plasma disturbance travelling in the direction of the discharge current was indicated. A similar phenomenon has been observed in a study of the OV and CuI lines emitted from Sceptre III. None of the $D\beta$ oscillograms showed any correlation between features.

4. DISCUSSION

These observations can be interpreted as being due to a helical instability wave of the type discussed by WARE (1959),

or to a helical perturbation in plasma density which rotates without moving round the torus, or a combination of these motions. If the first interpretation is taken, WARE (1960) predicts a wave velocity;

$$V_w^z = \frac{1}{n_e e B_\theta} \cdot \frac{dp}{dr}$$

where n_e = electron density,
B_θ = the θ-component of the magnetic field in the plasma,
p = the pressure at a point r from the axis of the discharge.

V_w^z is taken to be positive in the direction of current flow. The OV light is emitted from the hot, central region of the Sceptre III discharge, KAUFMAN, HUGHES and WILLIAMS (1960), and it is likely that this is also the case in Sceptre IV. Consequently, the above results only apply to the central core of the discharge. Probe measurements on the discharge in Sceptre III under similar conditions, ALLEN and LILEY (1959), give average values of dp/dr and B_θ for the centre of the discharge to be $-1{\cdot}96 \times 10^4$ dyn cm^{-3} and 1,250 gauss. n_e can be calculated approximately by assuming that the deuterium filling the torus initially is fully dissociated and then compressed by the pinch effect to one-quarter of the volume of the torus. Thus the predicted value of $V_w^z = 2{\cdot}4 \times 10^6$ cm sec^{-1} (in Sceptre III). Preliminary probe measurements in Sceptre IV (ALLEN, 1960) indicate that the magnetic field configurations are similar to those in Sceptre III. So the theory also predicts a velocity of the order of a few $\times 10^6$ cm sec^{-1} in Sceptre IV. There is thus agreement between theory and experiment in sign and order of magnitude.

The specific intensity I of the OV line depends on the concentration of OV ions, N_1, the electron density, n_e and on the electron temperature, T_e. It has been observed that for any given discharge the ratio of the amplitude of fluctuations to the mean intensity is nearly the same for the OV line and the CV line at λ2271 Å. A study of the excitation mechanisms of these lines suggests that the fluctuations in the OV intensity are mainly due to variations in plasma density rather than electron temperature. However, a more detailed investigation is required before a firm conclusion can be reached on this point.

Acknowledgements—The author wishes to thank MR. D. L. WARREN for assistance with the experiments and also MR. R. F. HEMMINGS, MR. L. A. GREEN and MR. V. C. CLOKE, who were responsible for the engineering and running of Sceptre IV. Thanks are also due to DR. A. A. WARE for advice and encouragement and to DR. T. E. ALLIBONE, F.R.S., Director of the Laboratory, for permission to publish this work. This work is supported in part by a contract from the Atomic Energy Research Establishment, Harwell.

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Rectification in a 50-cycle discharge

(*Received* 12 *May* 1960)

INTRODUCTION

FIFTY cycle mains excitation has been used on the Russian thermonuclear apparatus Alpha for pre-ionization (KONSTANTINOV, 1959) and on Zeta for outgassing the stainless steel liner (MITCHELL, WHITTLE, JACKSON and CLARKE, 1959) and in both cases low pressure discharges were obtained. We have investigated similar discharges, scaled down by a factor of three in linear dimensions for experimental convenience and observed rectification, the gas current showing a marked tendency to flow in only one direction with respect to the axial magnetic field.

APPARATUS

The discharge tube was a two-gap aluminium torus of 400 cm mean path length, 35 cm bore and 6·3 mm wall thickness. The inner surface was coated with glass enamel, and a water-cooled stainless steel aperture 30 cm in diameter was fitted to intercept runaway electrons, thus protecting the enamel surface. A steady axial field of up to 1000 gauss was available, and the gas could be pre-ionized by connecting the two halves of the torus to a r.f. oscillator.

The torus linked a transformer core permitting a flux swing of 0·6 Wb, and having primary windings of 8, 10, 20 and 40 turns. The primary was connected to the 240 volt 50 c/s mains, suitable protective resistors being included in the circuit. The peak electric field in the torus could thus be varied from 1/10 to 1/50 V/cm. The primary current was measured with a current transformer and the discharge current with a Rogowski coil and a passive integrator.

OBSERVATIONS OF RECTIFICATION

Steady discharges were obtained under the following range of conditions:

Electric field	E_z	1/10 to 1/25 V/cm
Axial magnetic field	B_z	200 to 1000 gauss
Gas		H_2 D_2 He N_2 A
Gas pressure		$0{\cdot}2$ to $1{\cdot}0 \times 10^{-3}$ mm Hg
Line density		$0{\cdot}7$ to 7×10^{16} atoms/cm
Gas current	I_g	up to 4000 A.

The gas current was always below the Kruskal limit, that is $I_g < \frac{\pi r^2 B_z}{l}$, where r and l are the minor radius and axial length of the torus.

The discharge tended to behave as a rectifier, and over a small pressure range centered at $0{\cdot}5 \times 10^{-3}$ mm Hg complete rectification occurred, gas current flowing only when the electric and magnetic fields were in the same direction. This effect was independent of the direction of the magnetic field, the phase of the

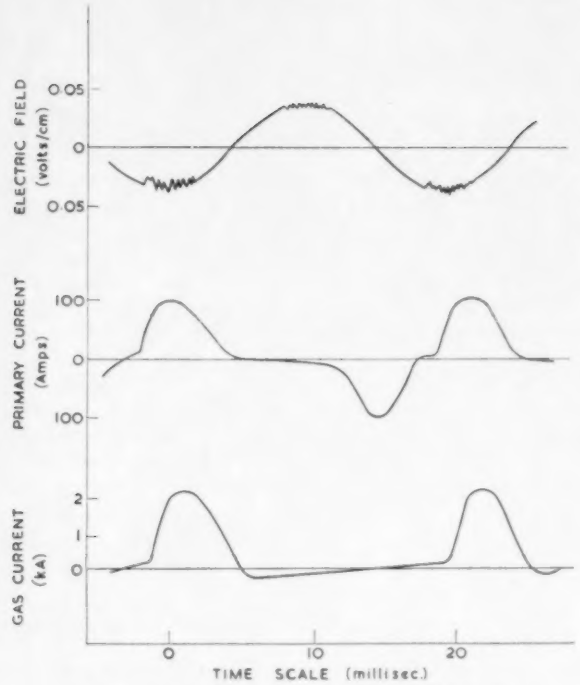

FIG. 1.—Typical waveforms of rectifying discharge.
Gas: Hydrogen Pressure: 8×10^{-4} mm Hg
B_z: 700 gauss.

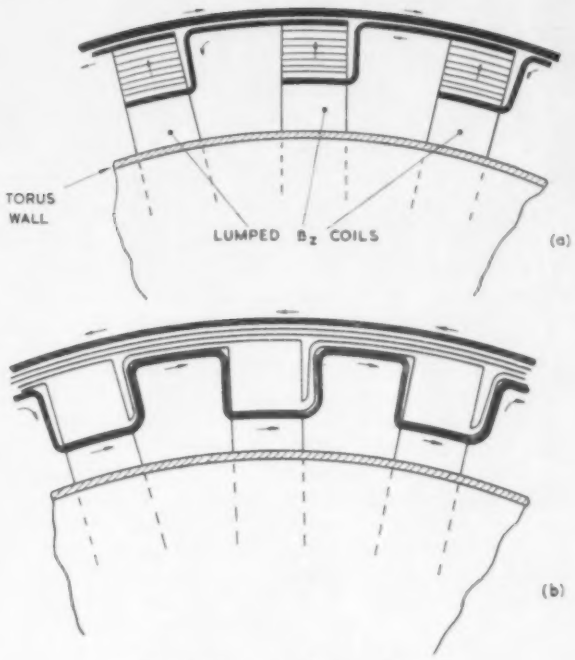

FIG. 2(a, b).—B_z coil connexions.
The solid lines show (a) the portions of conductor responsible for the transverse magnetic field, (b) the compensating windings. The arrows indicate the direction of current flow.

a.c. mains used and the magnitude and polarity of the r.f. pre-ionizing current. Typical waveforms are shown in Fig. 1.

A fully rectifying discharge could be maintained in hydrogen and helium without the need of r.f. pre-ionization. During the non-conducting half-cycle no light from the gas could be detected with a shuttered photomultiplier, and no detectable electron current was drawn by a positive probe.

CAUSE OF RECTIFICATION

On a suggestion by LYMAN SPITZER JR. that a transverse component of magnetic field might be the cause of rectification, a close examination of the axial field coil system was made. It was found that the method of connecting the coils did indeed produce a small transverse field component as shown in Fig. 2(a). The magnitude of the transverse field was calculated to be 0·8 per cent of the axial field at the outer torus wall, falling off inwardly as the square of the distance from the coil connexions. The direction of the transverse field B_T was such that $E_z \times B_T$ was directed towards the outer torus wall in the non-conducting half-cycle.

A set of compensating windings was fitted to the coils as shown in Fig. 2(b); these enabled the transverse field to be cancelled and reversed. As the current in the compensating coils was increased the rectifying action first ceased and then reappeared in the opposite sense.

A similar rectifying effect has been observed by BEZBATCHENKO *et al.* (1957) in a porcelain torus. They found that rectification ceased when the torus was lined with a stainless steel helix and deduced that it was caused by a transverse electric field arising from charge accumulation on the insulating torus walls.

In order to test this alternative explanation our torus was lined with sectors of copper foil which covered 90 per cent of the wall area. The rectifying properties were found to be completely unchanged.

BEZBATCHENKO *et al.* do not describe their axial field coil system in detail, but an examination of the diagram of their apparatus reveals two significant points. Firstly, the coil connexions as drawn would produce a transverse component of magnetic field. Secondly, the current flowing in the stainless steel helix due to the axial electric field would also produce a transverse magnetic field. It may be that the first component was sufficient to cause rectification and the second was sufficient to cancel the first.

CONCLUSIONS

Under conditions of low pressure and small electric fields, toroidal discharges in axial magnetic fields are profoundly affected by small transverse magnetic fields of the order of 1 per cent of the axial field. Great care is therefore necessary in the design of lumped axial field coils and in the layout of their electrical connexions if such effects are to be avoided. This is especially true of coil systems fed from high-voltage capacitor banks, as they tend to employ high ratios of current to axial field.

Acknowledgements—The author would like to thank DR. R. J. BICKERTON for helpful discussions and MR. J. W. HILL for experimental assistance.

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J. Nucl. Energy, Part C: Plasma Physics, 1961, Vol. 3, pp. 34 to 36. Pergamon Press Ltd. Printed in Northern Ireland

BOOK REVIEWS

Plasma Physics—*A course given by* S. CHANDRASEKHAR *at the University of Chicago*. Notes compiled by S. K. TEHRAN. University of Chicago Press, 1960, 217 pp., $1.75.

THIS monograph is a valuable addition to the rapidly expanding literature on theoretical plasma physics. The book has been prepared by Dr. Tehran from notes he compiled during a lecture series which Prof. Chandrasekhar delivered during the academic year 1957–58. By this means the fortunate reader is able to find a coherent presentation of a substantial portion of modern plasma theory and also to share the insight of one of the pioneer researchers in the field.

The lecture course as described in this book is devoted to detailed treatment of a number of select topics rather than a broad description of the entire field. Thus, for example, radiation from a plasma is not mentioned; and except for a brief section on the electrical conductivity of a partially ionized gas, the rest of the material treats only the fully ionized plasma.

There are seven chapters in all, the first of which is devoted to a brief discussion of Maxwell's equations along with the non-relativistic equation of motion for charged particles. A connexion between particle orbit theory and phase space distribution functions is sketched. Particle orbits in external force fields varying slowly in space or time are developed in the second chapter. The treatment is restricted to first order perturbation theory; and the concept of adiabatic invariants is introduced. The third chapter examines the concept of adiabatic invariants in considerable detail with the primary emphasis laid on the magnetic moment of gyrating particles.

A number of special applications of first order orbit theory are given in the fourth chapter. Included in this discussion are stationary equilibria, gravity and magnetic curvature drifts, and the general two dimensional problem with stationary magnetic fields. Stability problems are illustrated by discussion of Rayleigh-Taylor and flute instabilities in a plasma. The full power of first-order particle-orbit theory and the resultant hydromagnetic equations are exhibited in the fifth chapter which is devoted entirely to pinch stability calculations.

The sixth chapter is devoted to certain aspects of plasma oscillations in both zero temperature and finite temperature plasmas. The phenomenon of Landau damping is treated in some detail in connexion with longitudinal electrostatic oscillations of a finite temperature plasma. Unfortunately the electrostatic plasma instabilities related to two stream motion and certain non-Maxwellian distributions only started coming into vogue during the summer of 1958; consequently, they are not included in this book. The final chapter discusses transport phenomena under the assumption departures from thermal equilibrium are small and describable in terms of gradients of macroscopic quantities. Expressions are derived for diffusion, thermal conduction, and electrical conduction; in each instance a different technique is employed. The treatment of this chapter could have been unified by recourse to the Fokker-Planck equation as described in W. P. Allis's *Handbuch* article; however this approach is not mentioned in spite of the entry under "Fokker-Planck" in the subject index.

One or two minor objections may be voiced. There are a few instances of obvious inconsistencies; e.g. page 8 starts out assuming a uniform magnetic field only to end up by a discussion of its gradients. Certain misconceptions current in the literature are continued; i.e. the Larmor frequency and gyration frequency are used synonymously in spite of the fact these quantities differ by a factor of two. Yet these are but minor detractions from an otherwise excellent treatment marked by considerable clarity of mathematical presentation. A further strong recommendation is the economic price of this soft bound edition. By far the major adverse criticism is the size of the print which lies on the verge of readability. Finally, this reviewer would enter a plea that the self-consistent field formulation in plasma physics which is referred to with increasing frequency as the Vlasov equation continue to be so referred by general consent. The present book refers to this as the collisionless Boltzmann equation, although it has been pointed out that with equal logic it may be called the Boltzmannless collision equation.

P. L. AUER

Proceedings of the Fourth International Conference on Ionization Phenomena in Gases, Vol. I. North Holland Publishing Co., Uppsala.

THE first volume of these Proceedings includes the first two parts; (I) Fundamental Processes, (II) Different Types of Discharge and Applications. The sections into which these parts are divided make it easy for the reader to turn at once to the subjects which interest him; the concise contributions and clear titles aid greatly in this. The editor and publishers have done their work well, making the printed record of the conference a timely and valuable production.

Very useful summaries are given in the general papers of the present state of knowledge. Collisions between atoms and charged particles are discussed by Wade Fite, and Raether compares the various modes of charge multiplication in electric fields in an illuminating fashion. A condensed survey of radiation effects in gases is presented by Weissler, Lochte-Holtgreven describes investigations on the arc plasma and Nottingham reports on the caesium plasma diode transducer. Space forbids the mention of more than the leading topics dealt with in the individual papers.

Several measurements of charge exchange and electron capture cross-sections for multiply charged ions are reported in the first section of Part I. Ionization of inert gases by protons and in section C excitation of various gases by nuclear radiation have also been studied. Electron stripping from fast alkali atoms in a gas is a form of 'ionization' investigated in one paper. Theoretical calculations of charge transfer cross-sections in H and H_2 are given, and an interesting result predicting an extremely large detachment cross-section for H^-. A temporary negative ion N_2^- is strongly suggested. The attachment process by which O_2^- is formed concerns several authors, and one microwave study indicates a two-body attachment process at electron energies above 1·5 eV but a three-body interaction at low energy and sufficiently high pressure.

The effect of space charge on avalanche growth is demonstrated in many papers of the second section, with statistical treatments much emphasized. It is possible to derive Townsend's α from the

distribution in avalanche size within 2 per cent. The total absence of secondary ionization processes in hydrocarbons is established. In H_2 and air however, secondary coefficients have been measured at very high pd under precisely controlled conditions, and the striking effects of cathode surface conditions clearly shown. Non-uniform geometry does not exclude the use of α in calculations according to a revised estimate, and a fresh look at the Penning effect leads to the conclusion that this is not a true secondary ionization process at all. Stabilizing the dark discharge is found to extend this regime by an order of magnitude in spite of space charges.

In the section on radiation processes photo effects in the volume and at the cathode are further elucidated; particular studies derive the photodetachment cross-section for O^- and the excitation cross-sections in mercury and helium. The spectroscopy of auto-ionization is discussed.

The first section of Part II brings contributions to the theory of the positive column, including its oscillations, striations and noise. The maintenance of the d.c. glow discharge, its effective impedance and relaxation times are also treated. Other authors report on experimental surveys of high frequency low pressure discharges, on the theory of the electrodeless discharge, and on corona. This last includes corona in explosive gas mixtures, a stimulating study indeed.

The cathode phenomena in arcs still occupies many research workers, and new complexities in the modes of operation are suggested by experiments in which temperatures are carefully controlled. Spectroscopic studies of carbon, hydrogen and nitrogen arcs appear, and the temperature equilibrium prevailing is discussed. The behaviour of decaying plasmas and re-ignition of an arc receive attention in several papers.

A few contributions on sparks appear, discussing a wide range of topics. Propagation in long gaps, light emission (from which Stark constants for some spectral lines and recombination coefficients are derived) and the relation between 'whistlers' and lightning discharges are neighbours to a report of conditioning an enclosed air gap, due to nuclei in the volume. Very accurately reproducible breakdown potentials are measured here.

Applied science claims a large section, where investigation of Geiger counters, trigatrons, thyratrons, spark gaps etc. appear. Better understanding of these devices and their improvement are both attempted. Again radiation from the discharge proves a valuable tool (in investigating TR cells) and provides a valuable spectrum (Lyman continuum flash source.) Caesium diodes are discussed in two papers.

New methods of measurement form the short concluding section. Crossed beam measurements in H_2 (cf. Fite's lecture) and a magnetic probe responding up to 30 Mc/s are described; spectroscopy and mass-spectrometry find application in understanding discharges, and the corona discharge is suggested as a tool in understanding turbulence.

P. F. LITTLE

Proceedings of the Fourth International Conference on Ionization Phenomena in Gases. Edited by N. ROBERT NILSSON, 2 Vols. North Holland Pub. Co., Amsterdam. £12 6*s*.

THIS volume is the second of two containing the *Proceedings of the Fourth International Conference on Ionization Phenomena in Gases* held at Uppsala during August 1959. The majority of its papers are from laboratories engaged in controlled thermonuclear research and it records the progress and changes of emphasis since the 1958 Geneva Conference.

In common with most conference proceedings its contents are of uneven quality. There are over one hundred papers; some are fully written-up accounts of completed work whereas others are brief accounts of unfinished work. Such proceedings do have the advantage of enabling the present state of the whole subject to be surveyed provided that they are published speedily. In the present case, it is doubtful if such an attractive volume could have been produced more rapidly.

The volume consists of nine sections. The first four are concerned with general aspects of plasma physics and the remaining five with experiments on the production, confinement and heating of plasmas in specific geometries. The large machines are less in evidence than they were a few years ago and there is much more emphasis on small experiments to test specific questions.

The majority of the theoretical papers are concerned with what may be called plasma physics rather than magnetohydrodynamics. Thus there are discussions about the derivation of the time equations determining the behaviour of a plasma and whether it can be described satisfactorily by fluid equations. Many of the papers are concerned with the lack of isotropy of particle distribution functions caused by the infrequency of collisions at high temperatures. In contrast, several experiments have been designed to test the validity of the simpler theories. Such experiments are those on the propagation of Alfvén waves, the hard-core pinch and the structure of hydromagnetic shock waves.

At the Geneva Conference most of the groups studying high current gas discharges reported anomalously large energy losses. Much work has since been done in attempts to understand these; most of the groups report progress but their findings are not unanimous. An interesting feature of the work on mirror machines was a proposed new method of trapping particles by using spatially modulated magnetic fields. Perhaps the most controversial subject of the conference was the azimuthal pinch, which has been studied by many groups. There was dispute about whether a true thermonuclear reaction has yet been observed and this was only one of the interesting questions posed by the experiments.

R. J. TAYLER

Plasma Physics J. G. LINHART, North Holland Publishing Co., 1960. 278 pp., 50*s*.

THIS book is a useful addition to the meagre list of books devoted to the physics of plasmas. The first part deals with fundamentals, comprising chapters on the motion of particles, the fluid description of a plasma, equilibrium configurations, waves and instabilities, shock waves, and collision and relaxation processes. The second part, on applications, comprises two chapters, the first on controlled fusion, and the second on a variety of topics including the generation of electromagnetic waves, the conversion of chemical to electrical energy, particle accelerators, rocket propulsion, and energy storage. A great range of material is presented, much of it probably for the first time in the form of a text-book. The account of collision and relaxation processes is unusually clear, and much of the remainder of the book is admirable. In a few places the treatment seems rather cursory, as in the case of hydromagnetic waves, and in a few cases actually misleading, as in the description of the stabilized pinch. Occasionally, physical clarity is lost in the quest of mathematical rigour which, however, is not always achieved.

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It is a great pity that no account of experimental work is given, although a lengthy and valuable list of references contains a large amount of experimental material.

Some parts of the book seem to be addressed to the accelerator rather than to the plasma physicist—for example the treatment of equilibrium configurations neglecting collision, leading to neutralized beams rather than plasmas, and in the use of relativistic equations of motion for these beams.

A formidable list of errata is provided; this however, represents less than half of the mistakes visible on a casual first reading. A list of symbols is provided at the end of each chapter; this feature is valuable only if compiled with care, which is far from the case here. The references in the text are hardly adequate.

To summarize, this book is a useful work of reference on the theory of a plasma, but in the reviewer's opinion its value is sharply reduced by the neglect of experimental work.

M. G. RUSBRIDGE

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THE ENERGY DISTRIBUTION OF THE PROTON BEAM FROM THE SIX-METRE SYNCHROCYCLOTRON*

I. M. VASILEVSKII and IU. D. PROKOSHKIN

(*Received* 7 *December* 1958)

Abstract—The energy distribution and mean energy are given for the proton beam from the six-metre synchrocyclotron at the Joint Institute for Nuclear Studies, for various modes of acceleration. The energy spectrum is Gaussian, with a dispersion of 2·8 ± 0·3 MeV. The mean energy is accurate to 0·1 per cent.

1. INTRODUCTION

As a result of an increase in the intensity of the proton beam[1] extracted from the vacuum chamber of the synchrocyclotron at the Joint Institute of Nuclear Studies, it is now possible to measure interaction cross-sections to an accuracy of a few per cent. However, such an accuracy can only be realized if, at the time of measuring the cross-sections, one also makes a careful measurement of the energy of the proton beam. The reason for this is that the cross-sections increase rapidly with energy, particularly near the threshold of the reaction being studied. Thus, the cross-section for meson production in nuclear collisions at 650 MeV proton energy changes by 0·7 per cent per MeV, while at 350 MeV the rate of change is 3 per cent per MeV. In view of this rapid rise in cross-section at low proton energies one must know not only the mean energy but also the energy spectrum of the beam. A knowledge of the energy spectrum is particularly desirable when absorbers are used to reduce the energy of the protons, since this procedure considerably broadens the energy spread of the beam.

In the work reported here, we studied the energy spectrum of the proton beam at several proton energies between 150 and 670 MeV and in addition measured the mean proton energy while the accelerator was operating in various acceleration modes.

2. PROTON ENERGY SPECTRA

In order to measure the proton spectra we used a magnet having 100 cm diameter pole pieces providing a magnet field intensity of 16,000 oersted in the gap. The magnet was set up in the external proton beam (Fig. 1) and was capable of being turned through angles of up to 20°. Brass collimators containing vertical slits were placed in front of the magnet and from these emerges a flat beam of protons 0·1–0·7 cm

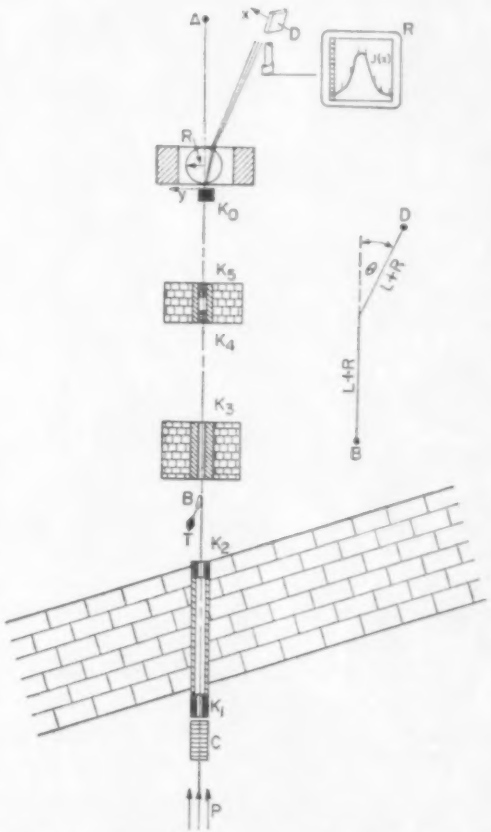

FIG. 1.—Experimental arrangement. *P*–proton beam; *C*–polyethylene absorber; K_0–K_5-collimators; *D*–proton detector; *R*–pen recorder; *B*–pulley for tightening wire.

high and 2 cm wide. After passing through the spectrometer, the protons were detected by a thin (0·1–0·2 cm) plastic scintillator viewed by a type FEU–19M photomultiplier. The pulses from the photomultiplier were integrated by an RC-network and the current (10^{-9}–10^{-7} amp) was measured by an EPPV–51 pen-recording potentiometer. The detector could be moved in a direction perpendicular to the proton beam by a synchronous motor and both the

* Translated from *Atomnaya Energiya* **7**, 225 (1959).

3A—(4 pp.)

x-co-ordinate shift and variations in the beam current density, J, were registered on the strip chart of the pen recorder (Fig. 1). Simultaneously, co-ordinate registering fiducial marks were inscribed on the chart, from which the x-co-ordinate could be determined to an accuracy of 0·02 cm. A typical curve of $J(x)$ is shown in Fig. 2. A second detector, similar to the one described above, was used to record the time variation of the proton beam intensity. The complete cycle of measurements for $J(x)$ took 5 minutes and during this time the magnetic field of the deflecting magnet was maintained constant to within less than 0·05 per cent.

As can be seen from Fig. 2, the function $J(x)$ is well described by a Gaussian curve. The dispersion σ of

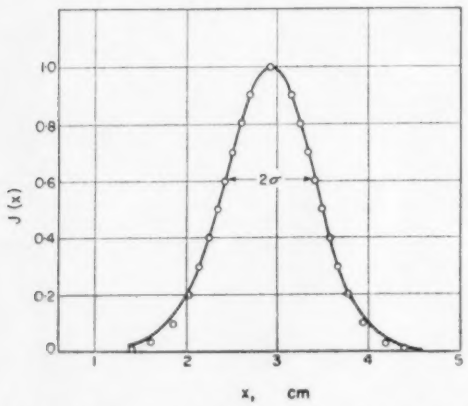

FIG. 2.—Experimental curve for $J(x)$. The circles denote values of the Gaussian function for standard deviation σ.

this curve is determined by a number of factors each of which is characterized by a partial dispersion σ_i. These include the energy spread of the beam (σ_E), the air-scattering of the protons in the beam (σ_s) and the collimator and scintillator widths (σ_x). These dispersions vary in different ways as the distance l (see Fig. 1) is increased and, as a result, the ratio σ_E/σ_s passes through a slanting maximum in the region $3 < l/R < 10$. We chose $l/R = 8$ and this value provided a spectrometer resolution $dE/E\,dx \approx 1$ per cent cm^{-1}, corresponding to a ratio σ_E/σ_s equal to 0·5. To reduce σ_s, a thin-walled helium filled polyethylene tube was placed in the path of the beam, which reduced the magnitude of the scattering several times.

The proton energy spectrum was found by comparing the curves $J(x)$ measured with the magnetic field switched on ($J_H(x)$) and with it switched off ($J_0(x)$). The detector was placed at point A in the undeflected beam for the $J_0(x)$ measurements (see Fig. 1). The square of the dispersion σ_E^2 was defined as the difference $\sigma_H^2 - (k\sigma_0)^2$, where σ_H and σ_0 are the dispersions of the functions $J_H(x)$ and $J_0(x)$. The factor k characterizes the focusing action of the magnetic field and is approximately equal to $(1 - \theta^2/2)$ (θ being the angle through which the beam is bent in the spectrometer). For the value of θ which we chose ($\theta \simeq 16°$), k hardly differed from unity ($k = 0{\cdot}96$). In the experiments which were carried out without the helium pipe the ratio σ_E/σ_0 was 0·5. The mean energy spread of the beam was determined to an accuracy of about 0·6 MeV, but it was not possible to say anything definite about the shape of the energy spectrum. When helium was used, the ratio σ_E/σ_0 increased by a factor of 2·5 and this enabled us to make a reasonably accurate determination of the dispersion σ_E and of the shape of the spectrum. The function $F(x)$ describing this spectrum may be found by solving the integral equation

$$J_H(x) = \int_0^\infty J_0(k[x - \xi])F(\xi)\,d\xi. \qquad (1)$$

The energy spectrum of the protons in the extracted beam turned out to be symmetrical; it is well described by a Gaussian curve

$$\Phi(E) \simeq \exp\{-(E - \bar{E})^2/2\,\Delta_E^2\}. \qquad (2)$$

At a proton energy $\bar{E} = 665$ MeV, Δ_E was found to be $3{\cdot}0 \pm 0{\cdot}6$ MeV (for measurements made without helium) and $2{\cdot}7 \pm 0{\cdot}3$ MeV (for measurements made with helium). The weighted mean of these values is

$$\Delta_E = (2{\cdot}8 \pm 0{\cdot}3)\ \text{MeV}.$$

The spectrometer was calibrated, i.e. the quantities $\bar{E}$ and $d\bar{E}/dx$ were determined, by the flexible wire method (see below).

The magnitude of the dispersion Δ_E which we found in our work differs from the value $\Delta_E \simeq 4$ MeV obtained by ZRELOV and STOLETOV[2] who used the ionization chamber method described by MATHER and SEGRÉ.[3] The reason for this discrepancy is that MATHER and SEGRÉ[3] over-estimated the magnitude of Δ_E owing to their neglect of the effects caused by proton scattering.

The method described above was also used to measure the spectra at lower energies, the beam energy being reduced by passing it through a polyethylene absorber (see Fig. 1). The energy spread of the beam increases as the proton energy is reduced because of the increased ionization loss and the dispersion due to straggling (caused by fluctuations in the ionization energy loss). Consequently the relative dispersion Δ_E/E rose rapidly as the proton energy was reduced, making the measurements very

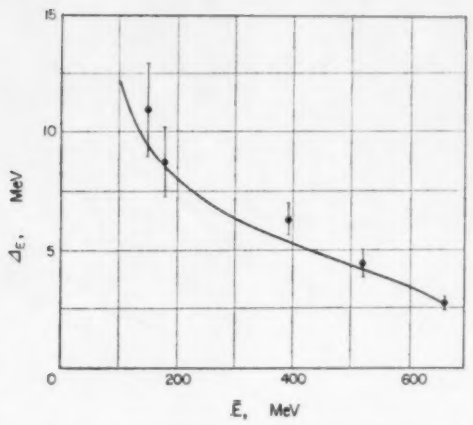

FIG. 3.—Beam dispersion Δ_E for different proton energies E. The curve represents the theoretical energy dependence of the dispersion.

much easier to do. For example, at $\bar{E} = 185$ MeV it is already possible to dispense with the use of helium and one can also use a wide collimator slit (0·7 cm) since even in this case $\sigma_E/\sigma_0 = 2{\cdot}5$. At low proton energies the functions $J_H(x)$ and $J_0(x)$ are Gaussian curves and it follows that, at low energies, the proton spectra $\Phi(E)$ also have a Gaussian shape.[2]

The experimentally determined dispersions Δ_E are shown in Fig. 3. In plotting the theoretical curve the effects due to the increased ionization loss and straggling[4] were allowed for. As can be seen from this figure, this curve is in agreement with the experimental points.

3. MEAN PROTON ENERGY

The most generally used method of determining the average energy of the extracted beam is the measurement of the range of the protons using an ionization chamber.[3] However, in the form described by MATHER and SEGRÉ,[3] this method leads to a systematic error (2–3 MeV for $E \simeq 650$ MeV) because, when applying the Bragg curve, no allowance is made for proton scattering. Errors due to this cause can be eliminated by calculating the effects due to scattering but, even so, the ionization chamber method does not yield energy measurements with an accuracy better than 5 MeV at $E \simeq 650$ MeV. This is because the ionization potential, which enters into the range-energy relation, is at present not known with sufficient accuracy.

The velocity of the protons can be determined directly by measuring the angle of emission of the Cherenkov[2,3] radiation. This method is most accurate at low proton energies, the accuracy falling as the proton energy is increased, until at $E = 650$ MeV it is 2 MeV. The optical equipment required to measure the angle of the Cherenkov radiation is rather complex and this limits the usefulness of the method.

In order to measure the mean proton energy with an accuracy better than 1 MeV, we used the flexible wire method[5] with the arrangement shown in Fig. 1. A thin flexible wire was fastened to a ring placed at the centre of the detector (in front of the scintillator) and passing through the magnet and collimators, it traced out the path of the proton beam. The wire was kept taut by passing it over a pulley at point B and attaching a weight T. A current i was passed through the wire and if T, i and the average proton momentum P satisfy the relation

$$P = cgT/i \qquad (3)$$

then the projections of the wire and the proton beam on a horizontal plane coincide. Here, c is the velocity of light and g is the acceleration due to gravity.* If the detector is placed so that the scintillator is at the maximum of the curve $J(x)$, then one can find the momentum P by measuring the current i through the wire which makes it coincide with the centre of the slit in collimator K_0.

Relation (3) is valid only if the wire is perfectly flexible and weightless. The fact that the wire has elasticity introduces a moment which acts in a direction opposite to that of the magneto-electric forces and, consequently, the momentum calculated from equation (3) differs from the true value (for a copper wire) by the amount $5 \cdot 10^3 Pd^4/T$, where d is the diameter of the wire in centimetres. In addition to this correction, one should also allow for the sag of the wire due to its own weight, this being characterized by the quantity $d^2l_0^2/T$ (where l_0 is the length of the wire). The wire therefore takes up a position which is below the beam trajectory and, if the magnetic field is inhomogeneous, the displacement is considerable. The appropriate correction may be determined experimentally by moving the wire vertically. In our experiments this correction was less than 0·05 per cent. From the form of the correction for sagging it follows that the wire length l_0 should be made as short as possible but, on the other hand, shortening the wire reduces the accuracy of the measurements since,

$$\Delta y/\Delta P \approx L(l_0 - L - R)\sin\theta/Pl_0 \qquad (4)$$

(see Fig. 1). The shortest length l_0 which should be used is determined by the accuracy $\Delta y_{\min}$ to which the wire can be located at the centre of the collimator slit (K_0). In our case $L = 9$ m, $l_0 = 16$ m and $\Delta y_{\min}$

* The value of cg at the latitude of Dubna is 2·9426 if T is expressed in grammes, i in amperes and P in units of MeV/c.

$= 0{\cdot}03$ cm, the adjustment of the wire at the slit centre being made automatically using a contact device. The error $\Delta P/P$ calculated using relation (4) was 0·03 per cent. The errors rapidly diminish as the wire diameter is reduced; for example, with $T = 100$ g and $d = 0{\cdot}01$ cm the correction for the elasticity of the wire was 0·4 per cent (for $P = 1300$ MeV/c) while with $d = 0{\cdot}013$ cm this correction was 1 per cent. The magnitude of this correction can be reduced by a factor of two or three if, before making the measurements, the wire is annealed by passing a large current through it.

In our experiments $d = 0{\cdot}01$ cm and $T = 100$ g and the current i was varied between the limits $0{\cdot}2 - 1{\cdot}0$ amp. T and i were measured with accuracies better than 0·05 per cent, enabling the mean beam energy $\bar{E}$ to be determined with less than 1 MeV error. At the exit port of the accelerator vacuum tank

$$\bar{E} = (667{\cdot}1 \pm 0{\cdot}7)\ \text{MeV}.$$

The proton mass was taken as 938·21 MeV. Attainment of this degree of accuracy is restricted to protons having energies $E > 250$ MeV; at lower energies it is necessary to use a thicker wire, a smaller load and to pass a greater current; all of which reduce the accuracy of the measurements.

4. BEAM ENERGY FOR DIFFERENT PROTON ACCELERATION MODES

The mean beam energy depends on the magnetic field strength of the accelerator, on the character of the amplitude spectrum of the radial oscillations and on the beam extraction conditions; that is, it depends on a large number of parameters which characterize the operation of the accelerator. Thus, the energy of the beam is not constant but fluctuates slightly because of uncontrollable changes in a number of accelerator parameters. These fluctuations amount to several MeV:

Date of Measurement	5/7/57	3/8/57
$\bar{E}$(MeV)	$671{\cdot}0 \pm 1{\cdot}5$	$667{\cdot}8 \pm 1{\cdot}0$
Date of Measurement	20/8/57	3/5/58
$\bar{E}$(MeV)	$667{\cdot}1 \pm 0{\cdot}7$	$658{\cdot}8 \pm 1{\cdot}0$
Date of Measurement	5/6/58	29/7/58
$\bar{E}$(MeV)	$669{\cdot}3 \pm 1{\cdot}0$	$666{\cdot}3 \pm 1{\cdot}0$
Date of Measurement	20/8/58	12/10/58
$\bar{E}$(MeV)	$665{\cdot}3 \pm 1{\cdot}0$	$663{\cdot}2 \pm 1{\cdot}0$

To some extent it is possible to change the magnitude of $\bar{E}$ by choosing suitable accelerator operating modes. Such adjustments are also necessary in experiments where it is necessary to keep the beam energy constant. Our experiments have shown that the energy $\bar{E}$ depends very largely on the magnetic field strength of the accelerator (Fig. 4) and small adjustments of $\bar{E}$ can also be made by changing the beam extraction conditions (Fig. 5). Using these methods it is possible to raise the beam energy to the value

$$E_{\max} = 683\ \text{MeV}.$$

However, the intensity of the beam falls by three orders of magnitude when these changes are made. In addition, changing the accelerator operating mode and the beam extraction conditions slightly affects the magnitude of the dispersion Δ_E. Thus, when the current I_0 is increased from 4100 to 4250 amp, the dispersion Δ_E is reduced by $0{\cdot}5 \pm 0{\cdot}3$ MeV and,

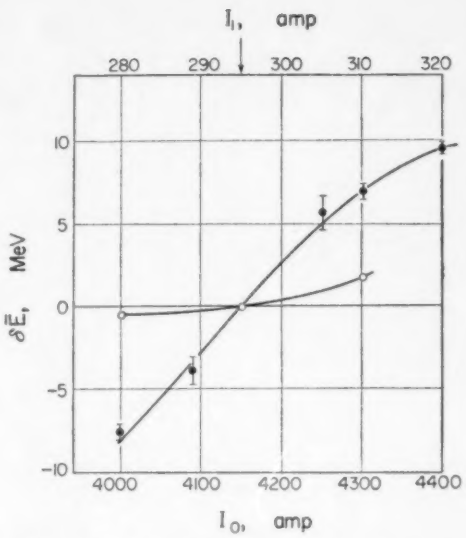

FIG. 4.—Variation of the mean energy of the proton beam caused by changing (a) the accelerator magnet current I_0 (●) and (b) the current I_1 (○) through the coil which adjusts the position of the median plane of the magnetic field. The arrows indicate the values I_0 and I_1 used under normal operating conditions.

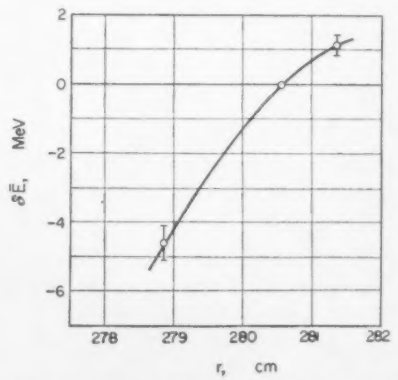

FIG. 5.—Variation of the mean energy of the proton beam caused by changing the regenerator position (r).

changing the position r of the beam extraction regenerator, has the following effect on the dispersion Δ_E:

At $r = 281{\cdot}35$ cm $\quad \Delta_E = 3{\cdot}7 \pm 0{\cdot}5$ MeV

At $r = 280{\cdot}35$ cm $\quad \Delta_E = 2{\cdot}8 \pm 0{\cdot}3$ MeV

At $r = 278{\cdot}85$ cm $\quad \Delta_E = 1{\cdot}8 \pm 0{\cdot}5$ MeV.

As we have already mentioned, beam energy measurements are particularly important in experiments where absorbers are used to moderate the energy of the protons (a common experimental requirement). In such experiments one must either determine the energy of the moderated protons (which is inconvenient because the absorber thickness is usually changed several times during the experiment) or one must first measure accurately the energy of the unmoderated beam and then use an energy loss–absorber thickness relation to find the energy of the moderated beam. We determined such a relation for polyethylene $(CH_2)_n$ over the energy range 150–665 MeV (Fig. 6) (this relation is applicable also for paraffin absorbers). We would point out that one should not use the ordinary range–energy relation in place of the energy loss–absorber thickness relation since this procedure gives rise to large errors, particularly when the energy of the moderated beam is low. For example, if the range–energy relation is known to an accuracy of 2 MeV at $\bar{E} = 650$ MeV (0·4 per cent) then, if the beam is moderated to 100 MeV, the error of the energy determination becomes 6 MeV (6 per cent), i.e. the error is 15 times greater.

Acknowledgements—In conclusion we take this opportunity of thanking TAN SYAO-BEYA and A. A. TIAPKIN for discussion.

Note added in proof: We have applied the same method of measuring the mean energy of the extracted beam to the case where the synchrocyclotron was working in the deuteron and α-particle acceleration modes. At the exit port of the accelerator vacuum tank the energies were $E = (405{\cdot}3 \pm 0{\cdot}5)$ MeV for deuterons and $E = (811{\cdot}3 \pm 1{\cdot}0)$ MeV for α-particles (using mass values $m_d = 1875{\cdot}5$ and $m_\alpha = 3727{\cdot}2$ MeV). The energy spectra of these beams were characterized by the dispersions: $\Delta_E = (1{\cdot}7 \pm 0{\cdot}5)$ MeV for deuterons and $\Delta_E = (3{\cdot}5 \pm 1{\cdot}5)$ MeV for α-particles.

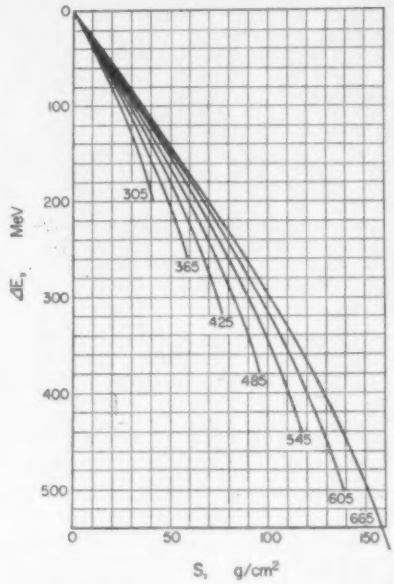

FIG. 6.—Energy loss $\Delta_E = E - E'$ in a polyethylene absorber, thickness S. $\bar{E}$ and $\bar{E}'$ are the mean energies of the proton beam before and after passing through the absorber. The figures attached to the curves give the corresponding values of $\bar{E}$.

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LETTERS TO THE EDITORS

Start-up of a cyclotron with a spatially varying magnetic field*

(*Received* 9 *April* 1959)

In January 1959 a new type of cyclic accelerator was started up at the Laboratory of Nuclear Problems of the Joint Institute for Nuclear Studies, Dubna. This machine is of the spiral ridge type, with $N = 6$, the magnet diameter being 120 cm. The lines of constant magnetic field intensity are Archimedean spirals,[1] with r (cm) $= 16{\cdot}2\varphi$.

The average magnetic field intensity increases in the radial direction in step with the relativistic mass increase of the accelerated ions. The acceleration cycle begins at the centre of the magnet where the betatron oscillation frequencies are $Q_z = 0$, $Q_r = 1$; at $r = 52$ cm these have increased to $Q_r = 0{\cdot}2$, $Q_r = 1{\cdot}01$. The operating point moves into the stability region without crossing the non-linear resonance at $Q_r = 1$. This effect was studied in detail in 1958 on a model having a structure period with $N = 4$ and a radial pitch $2\pi\lambda = 8{\cdot}47$ cm. The lines of constant magnetic field intensity were spirals with r (cm) $= 5{\cdot}4\varphi$.

Theoretical calculations showed that the amount by which the average magnetic field intensity must be increased in order to eliminate the non-linear resonance effect at the centre of the accelerator decreases with increasing n (the order of the non-linear resonance) as $\dfrac{n}{2^n(n-1)!}$ and, for a fixed value of n, decreases as $\left(\dfrac{\lambda_1}{\lambda_2}\right)^{n-2}$ as the radial pitch is increased.

The development of the non-linear resonance effect when $Q_r = 1$ is equivalent to a displacement of the centre of the instantaneous orbit with respect to the centre of the magnetic field. We therefore examined the conditions for the vertical stability of a displaced orbit. We found that the character of the vertical oscillations remained unchanged if the centres of the instantaneous orbits were not displaced by more than λ. The condition that the change in the period of revolution of a particle in a displaced orbit should be small was found to demand a similar restriction.

The results of these calculations were the main factors determining the choice of parameters for the magnetic field having the six-fold spiral structure described above and in which no non-linear resonance effects at the centre of the accelerator could be observed. The required magnetic field variation was obtained by using iron shims having a rectangular cross-section. The best values for the shim parameters, giving the required depth of magnetic field intensity variation, were worked out using an assumed form of shim magnetization curve. Thin cylindrical shims were used to obtain the radially varying magnetic field required to keep the revolution frequency of the ions constant. It was supposed that the magnetization of these shims was uniform when making the necessary calculations. In order to eliminate the effects due to ordinary and parametric resonances ($Q_r = 1$), a small (8 mm diameter) symmetrically placed cylinder was used to cut down the amplitude of the first and second harmonics in the magnetic field structure. All the magnetic field measurements were made to an accuracy of $\pm 1{\cdot}5$ oersted using a nuclear magnetometer.[2] A nuclear stabilizer ensured that the magnetic field remained constant with respect to time.[3]

A resonant quarter-wave system incorporating a single dee was used to accelerate the ions. This system was supplied by a generator having independent excitation and able to provide accelerating voltages up to 40 kV. With a minimum peak dee voltage of 8 kV, the cyclotron accelerated deuterons up to energies of 12 MeV and α-particles up to 24 MeV. Two methods were used to determine the energy of the accelerated particles at the final radius (52 cm): (1) inserting three probes to measure the average orbital radius of curvature and (2) measuring the range of the deuterons in aluminium foil. To avoid radioactive contamination of the interior of the chamber all the measurements were carried out with an internal beam intensity below 1 μA.

Figure 1 gives some autoradiographs of the deuteron beam obtained at various radii and Fig. 2 shows the accelerator chamber of the cyclotron.

The results of these experiments demonstrate that it should be possible to build a cyclotron operating in the relativistic region, to provide protons having energies comparable with those obtained from present day phasatrons (synchrocyclotrons).

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* Translated by D. L. Allan from *Atomnaya Energiya* **6,** 657 (1959).

Self-acceleration of a flux of charged particles by induction*†

Attempts to obtain powerful gas discharges usually lead to the creation of a plasma current in which the electrons have a comparatively small ordered velocity, but a large store of energy in their magnetic self-field. In the present note we shall briefly consider some processes which permit the use of this energy for accelerating a proportion of the charged particles in such a discharge or for attaining large pulsed current densities.

* Translated by N. Kemmer from *Atomnaya Energiya* **6,** 658 (1959).
† Work performed at the end of 1957.

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FIG. 1.—Deuteron beam autoradiographs at various radii.

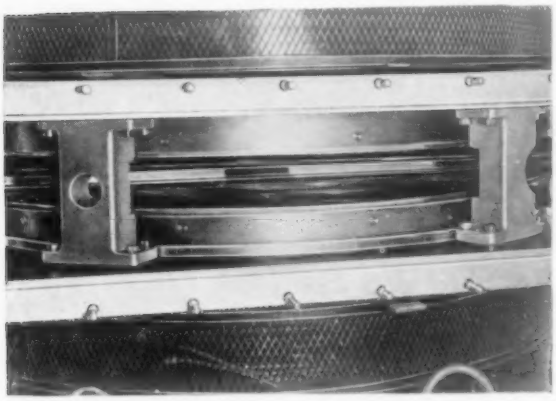

FIG. 2.—Accelerating chamber of the cyclotron.

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Let us assume that a sharp diminution of the directed velocities or of the number of electrons in the strong current takes place as a result of some dissipative processes (elastic and inelastic collisions with the molecules and ions, capture of electrons, change of conductivity due to local change of electron temperature or application of magnetic fields, etc.) This results in the appearance of an inductive field, which accelerates the remaining plasma particles. A rapid decrease of the velocities of the electrons at a surface may occur, for instance if there is a contact interaction between an impinging ring discharge and the surface of a medium or a sharp front of cold and heavy gas. Such an interaction with a surface is associated with the penetration of a vapour front into the interior of the plasma. To diminish the quantity of vapour entering the zone of the remaining discharge one can impart to the current a velocity component along the surface in addition to its normal velocity (as e.g. in a current ring that impinges frontally on a medium and contracts or expands at the same time). If the ring has a fast slipping motion its residual part separates from the region of the vapour burst in the medium and so, with a suitable choice of velocities, one can change the dynamics of the current decrease as well as the movement of new molecules into a given zone of the discharge.

The ionization of the molecules in a vapour front cannot lead to a shunting of the current through the vapour if there is only a thin transition layer in which the electron multiplication can take place. This is because the velocities of the electrons formed are small, and the gas density is increased so that an increase of the electron energy in this part of the discharge is prevented. Further, an induction field is needed for the acceleration of newly-formed or scattered electrons and this field also accelerates the remaining particles. This induction field arises owing to the fact that each 'micro-weakening' of the current due to the mean velocity lost by an electron in ionization is not compensated by the electron produced and the remaining electrons that move with smaller velocities.

In the process of inductive transfer of magnetic field energy an essential role is played by the number of current carriers N per unit length of current and the inductance L per unit length of the remaining current. From the expression for the total energy for moving particles of 'classical radius' r_0

$$W = W_{\text{kin.}} + W_{\text{magn.}} = W_{\text{kin.}}(1 + Nr_0L).$$

It is easy to see that for $Nr_0L \gg 1$ the main energy of the motion resides in the magnetic field. If the plasma of the remaining ring has a sufficiently large number of electrons and if at this stage the total magnetic flux through the circuit may be considered to vary slowly, the process of conductivity decrease at the boundary will be accompanied by an increase of current density in the remaining part of the discharge and an increase in concentration of generated energy. A sharp increase of current density will lead to an increased boundary intensity of the magnetic field encircling the current, to an increased 'pinch' of the current and to strong heating of the remaining plasma. Since the contraction process is rapid the attainable magnitudes of instantaneous pressure and temperature may exceed their quasi-static equilibrium magnitudes ($P_{\text{stat.}} \sim (I^2/a^2)$, where $a(t)$ is the effective cross-sectional dimension of the remaining part of the ring and I the current density in it). For very large current densities which produce a boundary field $H \geqslant 3 \times 10^6$ oersted, self-isolation of the current may take place, the diamagnetic molecules of the surrounding medium being repelled by the magnetic self-field of the current.

If the plasma conductivity is not too great an inductive field may appear as the current decreases, with an intensity sufficient to carry the remaining part of the discharge through the region of strong momentum and energy losses into the region of small losses. The losses will then become less and less significant as the acceleration proceeds.* In such a gaseous betatron with its own alternating magnetic flux electrons can be trapped in circular orbits and 'vertical' stability may be ensured either by Coulomb forces which appear when the quasi-neutrality of the residual ring is destroyed, or by a combination of these forces and an internal or external magnetic field. The possibility that the current may have a 'skin' distribution can only improve conditions for the separation of the accelerated and remaining fluxes of particles. Also, if in some part of the discharge some initial electron acceleration has already taken place, the penetration of vapour molecules will effect these electrons to a much lesser extent than those which have not been accelerated, because of the rapid drop in the interaction cross-section. Both these circumstances reduce the stringency of the conditions on the vapour density gradients of the material causing the current drop.

It is easy to show that a sharp decrease of current can ensure a corresponding magnitude of field intensity. For $E_{\text{ind.}} \approx L_{\text{eff.}}(I_0/\tau)$, where τ is the time of the sharp current drop in the circuit. For instance, for $L_{\text{eff.}} \approx 5 \times 10^{-9}$ H cm^{-1}, $I_0 \approx 30$ kA and $\tau < 10^{-6}$ sec we obtain $E_{\text{ind.}} > 100$ V cm^{-1}, which exceeds the critical field necessary for a directed acceleration of the electrons in a plasma having typical initial electron energies of the order of several eV and ion densities $n \leqslant 10^{14}$ ions cm^{-3}.

These rough estimates show that for a sharp change of current, conditions may be created for some of the electrons to be accelerated in the discharge. The limiting energy attained by each of the remaining electrons will be $\varepsilon_{\max} \sim L_0 I_0 e/c$. For instance, for the values of current and inductance indicated above we get $\varepsilon_{\max} \approx 5$ MeV, so that one would expect the appearance of hard radiation.

The inductive acceleration processes discussed above can also occur if a strong discharge performs radial pulsations which lead to a cooling of the plasma and to a decrease of its conductivity as it expands and also as it makes contact with the walls. If strong axial electric fields occur (of the order of some tens of kV cm^{-1} at rates of current drop of the orders observed on oscillograms, namely about 10^{12} A sec^{-1}) this will lead to acceleration of electrons and ions in the plasma and the appearance of the observed neutron and γ-radiation.[8] The probability of ions being accelerated in the plasma increases because these processes are favoured by diminished plasma density at the expansion stage (in contrast to processes related to sausage-type instabilities or with inductive redistribution of current when the boundary conductivity is suppressed as a result of the magnetic self-field of a current increasing in contraction). The formation of sausage-type instabilities is accompanied by the appearance of bunching, which may also make the contact with the walls easier. We note in passing that the secondary appearance of current near the axis of the discharge can be due to inductive transfer of the current and not to a secondary contraction of the whole plasma.

It is of interest also to study astrophysical examples of the above processes of inductive stage-by-stage acceleration in the case of currents circulating in stellar plasma and of plasma jets

* For different possible types of electron acceleration in a plasma see, for instance, Refs. 1–4.

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ejected by the stars into cosmic space. These effects can apparently cause the appearance of fluxes of accelerated cosmic particles.

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Beam stability in stacked orbits*

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FOLLOWING on the recent suggestion[1] that very high energy nuclear reactions could be produced in the head-on encounter of oppositely-moving beams of relativistic particles, the problem of building up large currents in circular accelerators has been much discussed.[2] The central requirement is to understand the mutual interactions of the large number of particles in a stacked orbit. One well-known phenomenon is the transverse space-charge effect, which is closely connected with the cause of betatron oscillations. It is also important to study the consequences of azimuthal non-uniformity in the beam density, which alters the energy distribution of the particles.

Simple physical considerations show that under certain conditions an azimuthally uniform beam can be unstable. Consider for simplicity a mono-energetic beam of uniform density. If a local density increase occurs, its field will accelerate the particles moving in front of it and decelerate those behind it; if $\frac{d\omega}{dE} > 0$, so that the orbital frequency increases with energy, the fluctuation will be damped out, but if $\frac{d\omega}{dE} < 0$, it will tend to build up. In fact, $\frac{d\omega}{dE}$ is negative for all weak-focusing circular accelerators and for strong-focusing arrangements beyond the transition energy. As the plasma effects in stacking systems are of great physical interest, as well as being important in accelerator theory, we decided to examine this situation in greater detail.†

The energy of a particle in an azimuthal electric field may be described by the Hamiltonian[2]

$$\mathscr{H} = -e\int \mathscr{E}(\phi, t)\, d\phi + \left(v\frac{d\omega}{dE}\right)_s \tfrac{1}{2}W^2, \qquad (1)$$

in which the canonical momentum variable W is related to the energy of a particle of velocity v by

$$W = \int_{E_s}^{E} \frac{dE}{v(E)}. \qquad (2)$$

A suffix s indicates that the value is to be taken at the mean energy of the stacked beam. The canonical variable ϕ, conjugate to W, may be expressed as $\phi = \theta - \omega_s t$, where θ is the so-called generalized azimuth[4] in an arbitrary periodic system. $\mathscr{E}(\theta, t)$ is the azimuthal component of the electric field of the beam.

If we describe the state of the beam by means of a distribution function $F(W, \phi, t)$, which may be written as a sum of two terms thus:

$$F(W, \phi, t) = F_0(W) + f(W, \phi, t); \qquad f \ll F_0, \qquad (3)$$

we are able to discuss small departures from an azimuthally uniform equilibrium state. Now it is physically obvious that in the equilibrium state $\mathscr{E} = 0$, so that the linearized energy equation has the form

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \phi}\left(v\frac{d\omega}{dE}\right)_s W = -\frac{\partial F_0}{\partial W} e\mathscr{E}(\phi, t). \qquad (4)$$

Carrying out a Fourier transformation over azimuth and a Laplace transformation over time,

$$f_p\left[p + ikW\left(v\frac{d\omega}{dE}\right)_s\right] = f(k, 0) - \frac{\partial F_0}{\partial W} e\mathscr{E}_p(k). \qquad (5)$$

To relate $\mathscr{E}(k)$ to the appropriate Fourier component of the charge density we take advantage of the fact that the electric field of the beam is effectively screened by the top and bottom of the vacuum chamber and may for practical purposes be represented by the field of a beam moving in a straight line between two conducting planes. Thus

$$\mathscr{E}(k, t) = -\frac{i\Lambda k e}{R^2\gamma^2}\int_{-\infty}^{\infty} f(W, k, t)\, dW, \qquad (6)$$

with $\gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$ and R the mean radius of the orbit. The same expression may be derived on a rigorous basis by using LEVIN'S[5] formulae for the field of a screened circular current. An exact treatment will be given in a later communication; we remark here only that Λ depends on the ratio of the tranverse dimensions of the beam to the gap height, d, and in a practical case is between 2 and 3. Equation (6) presupposes that the non-uniformity being considered is much larger in dimension than the chamber aperture, i.e. that $k \ll R/d$.

By solving (5) and (6) in the usual way it is not difficult to show that the asymptotic behaviour of the perturbation is determined by that root of

$$\frac{\Lambda e^2 k i}{R^2\gamma^2}\int_{-\infty}^{\infty} \frac{\left(\frac{\partial F_0}{dW}\right) dW}{\left[p + ikW\left(v\frac{\partial\omega}{dE}\right)_s\right]} = 1 \qquad (7)$$

which has the greatest real part. As we are concerned with the existence of an unstable solution, we are interested in roots for which $\mathrm{Re}\,(p) > 0$ and may perform the integration simply along the real axis. Consider now two actual distributions $F_0(W)$:

(i) the uniform distribution

$$F_0 = \begin{cases} N/4\pi W_0, & |W| < W_0, \\ 0, & |W| > W_0. \end{cases} \qquad (8)$$

After elementary manipulations, (7) gives

$$\frac{p}{kW_0\left|\frac{d\omega}{dE}\right|_s} = (\xi - 1)^{\frac{1}{2}}, \qquad (9)$$

$$\xi = \frac{2N\Lambda r_0}{\pi R\beta^2\gamma^3}\frac{\gamma^2 - 1}{\alpha\gamma^2 - 1}; \quad r_0 = \frac{e^2}{mc^2}, \qquad (10)$$

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* Translated by R. D. LOWDE from *Atomnaya Energiya* **7**, 549 (1959).

† After this note had been submitted, a paper by NIELSON and SESSLER[3] appeared in which they also consider the stability of a supercritical beam.

δ is the relative energy spread in the beam and α the expansion coefficient of the orbit. It is apparent from (9) that there can be instability [Re(p) > 0] only (a) in the supercritical region having $\alpha\gamma^2 > 1$, and (b) when the energy spread is sufficiently small that

$$\delta^2 < \frac{2N\Lambda r_0}{\pi R\gamma^3}\,\frac{\gamma^2 - 1}{\alpha\gamma^2 - 1}\,. \tag{11}$$

(ii) Now consider

$$F_0 = \frac{NW_0}{2\pi^2}\,\frac{1}{W^2 + W_0^2}\,; \tag{12}$$

it is found that

$$\frac{p}{kW_0\left|v\,\dfrac{d\omega}{dE}\right|_s} = \xi^{\frac{1}{2}} - 1. \tag{13}$$

The same conclusions follow as for (9); instability can exist only if $\dfrac{d\omega}{dE} < 0$ ($\xi > 0$) and (11) is satisfied. In both examples the condition $\xi < 0$ results in the beam density undergoing small stable oscillations at a frequency determined by Im(p). Whether these oscillations can be damped out is a question that must be examined separately.

The formulae may be illustrated by the example of an electron storage ring for which $\gamma = 60$, $\alpha = 5 \times 10^{-2}$, $R = 150$ cm, $\Lambda = 3$ and $N = 10^{14}$. If $\delta = 5 \times 10^{-3}$, the growth time of instability is $T \doteqdot p^{-1} = 40\ \mu$sec. It should, however, be remembered that the growth of the perturbation destroys the mono-energetic nature of the particle beam, and this must ultimately lead to conditions limiting the size of the instability.

We conclude by pointing out that these effects are absent for azimuthally-restricted bunches in accelerators where there is a high-frequency accelerating field, if the growth time of the instability is much greater than the period of synchrotron oscillations. For in such cases the effect of the perturbing field, averaged over an oscillation period, is zero.

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A ring cyclotron with a vertically increasing magnetic field*

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Recently attempts have been made[1,2,3] to use accelerators with vertically increasing magnetic fields instead of those in which the fields are symmetrical about a plane. In the former type a particle moves upwards as its energy increases; the perimeter of its orbit remains constant and for relativistic energies $\omega(h)$ = constant, where ω is the frequency of revolution of the particle. The constancy of this frequency makes it possible to obtain particle currents which at present can be achieved only in cyclotrons. In[1,2,3], however, it was not possible to show that such an acceleration can be stable.

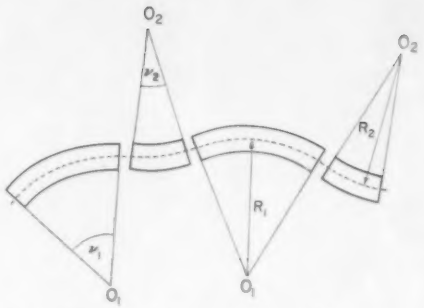

Fig. 1.—Arrangement of sectors and orbit in a ring cyclotron.

Here we shall consider an accelerator with a vertically-increasing guiding magnetic field constant in time (a ring cyclotron or FFAG) and we shall investigate the stability of the motion of particles therein.

The magnetic system consists of a number of similar periods, each of two sectors separated by field-free straight sections. The directions of the magnetic fields in adjacent sectors are opposite, so that the curvature of the orbit changes sign. The magnitude of the field in each sector increases vertically, with $H \sim z^n$. The alternation of sign of the field ensures, in certain conditions, the focusing of the particles. At the same time the vertical increase of the field has the result that, with increasing energy, the particle orbit moves upwards, its perimeter remaining unchanged.

This type of accelerator can be constructed in various ways. We shall consider only the simplest case (Fig. 1), where the orbit is a plane curve consisting of arcs of circles with curvatures of alternate sign and segments of straight lines $R_1 = R_2 = R$ = constant. If the angular dimensions ν_1 and ν_2 of the sectors are large compared with the straight sections and the radial gap, edge effects may be neglected.

Let the magnetic field near such an orbit be (Fig. 2):

$$\left.\begin{aligned} H_z &= \pm H_0[1 + ny + \tfrac{1}{2}n^2(y^2 - x^2)],\\ H_r &= \pm H_0\, nx[1 + (n+1)y],\\ H_\theta &= 0, \end{aligned}\right\} \tag{1}$$

where $x = \rho/R$, $y = \Delta z/R$ are the dimensionless deviations from the orbit and n the parameter which defines the magnetic

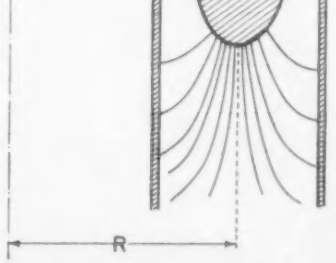

Fig. 2.—Field shape in a ring cyclotron.

* Translated by J. B. Sykes from *Atomnaya Energiya* **8,** 552, (1960).

field configuration.* (Here and below, the upper sign refers to the section ν_1 and the lower sign to the section ν_2.) Linearization of the equation of motion in the field (1) gives

$$\begin{aligned} d^2x/d\vartheta^2 + x \pm ny = 0, \\ d^2y/d\vartheta^2 \pm nx = 0, \end{aligned} \tag{2}$$

where the angle ϑ is measured from the centre of curvature. The solution in the two sectors may be written

$$x = \sum_{i=1}^{4} x_i \exp(\omega_i \vartheta), \quad y = \sum_{i=1}^{4} y_i \exp(\omega_i \vartheta), \tag{3}$$

where $\omega_i = \pm\sqrt{[-\frac{1}{2} \pm \sqrt{(\frac{1}{4} + n^2)}]}$ and the constants x_i and y_i, which are determined by the boundary conditions, are related by

$$y_i = -nx_i/\omega_i^2. \tag{4}$$

Joining the solutions (3) at the ends of the sectors of the mth period in the absence of straight sections, we have the matrix equation

$$\mathbf{x}_{m+1} = M\mathbf{x}_m, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \tag{5}$$

whose solution is of the form

$$\mathbf{x}_m = \Lambda^m \mathbf{x}_0. \tag{6}$$

The value of Λ is given by the characteristic equation

$$\Lambda^4 - S_1\Lambda^3 + S_2\Lambda^2 - S_3\Lambda + S_4 = 0, \tag{7}$$

where S_i is the sum of all possible diagonal minors of M of order i. By Liouville's theorem $S_4 = 1$ in the system considered, and it can be shown that S_i is real ($i = 1, 2, 3$) and $S_1 = S_3$. Thus equation (7) is a symmetrical equation of the fourth order with real coefficients. The solution of equation (6) is stable if all Λ_i are complex. This gives the condition $4S_2 - S_1^2 = 8$.

As an example, the range of stability of such an accelerator with $N = 30$, $n \simeq 10$ has been determined. In this case there is a fairly wide range of stability, with $1{\cdot}21 < \nu_1/\nu_2 < 1{\cdot}33$. The gain factor of the apparatus varies between about 7 and 10, depending on the choice of the operational parameters. The mean radius of the orbit is determined by the final energy.

When three-dimensional orbits are considered we may hope to obtain (by analogy with the 'symmetrical' ring synchrocyclotron [4,5]) a 'symmetrical' ring cyclotron, in which it will be possible to accelerate electrons in opposite directions simultaneusly and to bring about collisions.

In consequence of the radiation emitted by the revolving electrons, there is a series of limiting orbits given, when the phase distribution of the particles is uniform, by

$$\gamma = E/E_0 \simeq 35(eV_0 \cos\phi_0 R)^{\frac{1}{4}},$$

where the increase in energy per revolution (eV_0) is measured in eV and the limiting values ϕ_0 are determined by the magnetic field tolerances. By grouping the electrons near a certain phase (e.g. by a preliminary phase bunching or by making the revolution frequency phase-dependent), the energy spread of the accelerated beam may be reduced, leading in practice to a stacking of particles.

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* The index n of the vertical increase of the magnetic field obviously depends on the choice of the origin of z.

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Some properties of orbits in accelerators in conditions of similarity*

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THE expression 'dynamical similarity' means that the condition that the mode numbers for betatron oscillations should be independent of the energy of the accelerated particles applies. This is because, during an acceleration cycle, the frequencies of betatron oscillations must not take dangerous resonance values. The similarity condition is particularly important in strong-focusing accelerators with constant fields, where the orbit parameters undergo considerable changes during the acceleration.

It has been shown in[1] that the condition for dynamical similarity of plane orbits is that the magnetic field should have the particular form

$$H_z = f(\theta)r^{n_0}, \tag{1}$$

where H_z is the field component perpendicular to the orbital plane, $f(\theta)$ an arbitrary periodic function, r and θ the cylindrical co-ordinates of a point in the orbit and n_0 = constant. In this paper we shall consider some general properties of the motion of particles in such systems. These systems have been extensively developed of late on account of the problems of stacking and 'synchro-crash.'

Let a plane closed orbit $\mathbf{r}(\sigma)$, σ being arc length, correspond to a particle momentum p. The linearized equations for the deviations from this orbit along the normal (ρ) and the binormal (z) are well known:

$$\begin{aligned} \rho'' + K^2R^2(1 - n)\rho = 0, \\ z'' + K^2R^2nz = 0, \end{aligned} \tag{2}$$

where the field index n is measured along the normal to the orbit, K is the curvature and the differentiation is with respect to the generalized azimuth $\vartheta = \sigma/R$, R being the mean radius of the orbit. The radius vector of another orbit corresponding to momentum $p + \Delta p$ may be written

$$\mathbf{r}_1(\sigma) = \mathbf{r}(\sigma) - \psi(\vartheta)\mathbf{n}\,\Delta p/p, \tag{3}$$

where $\mathbf{n}$ is a unit vector along the normal to the orbit, and $\psi(\vartheta)$ is a periodic solution of the equation

$$\psi'' + K^2R^2(1 - n)\psi = KR^2 \tag{4}$$

(the differentiation being again with respect to ϑ).

The invariance condition for equations (2) is evidently

$$\left[\frac{\partial(KR)}{\partial p}\right]_{\vartheta = \text{constant}} = 0, \quad \left[\frac{\partial n}{\partial p}\right]_{\vartheta = \text{constant}} = 0. \tag{5}$$

Since the arc length of the curve $\mathbf{r}_1(\sigma)$ is

$$\sigma_1 = \sigma + \frac{\Delta p}{p}\int_0^{\sigma} K\psi\, d\sigma, \tag{6}$$

* Translated by J. B. SYKES from *Atomnaya Energiya* **8,** 553 (1960).

we easily obtain, on differentiation of (3) and using the Serret-Frenet equations and the first condition (5),

$$K(\alpha - 1) - nK^2\psi - K'\left[\int_0^\theta K\psi\, d\theta - \alpha\theta\right] = 0, \qquad (7)$$

where

$$\alpha = \frac{1}{2\pi}\int_0^{2\pi} K\psi\, d\theta$$

is the instantaneous-orbit factor. In formula (7) n may be expressed in terms of n_0 by means of the identity

$$\begin{aligned} n_0 &= -\frac{r}{H_z}\frac{\partial H_z}{\partial r} \\ &= -\frac{r}{H_z}\left(\frac{\partial H_z}{\partial \rho}\frac{\partial \rho}{\partial r} + \frac{\partial H_z}{\partial \sigma}\frac{\partial \sigma}{\partial r}\right) \\ &= -K(\mathbf{r}\cdot\mathbf{n})n - K'(\mathbf{r}\cdot\boldsymbol{\tau})/KR, \end{aligned} \qquad (8)$$

where $\boldsymbol{\tau}$ is the tangent vector.

For a given shape of orbit, equations (4) and (7) form an inhomogeneous set of equations with periodic coefficients, linear in the unknowns ψn_0 and ψ. It follows from general theorems[2] that such a system has, in the non-resonance case, only one periodic solution, with the same period as the coefficients. It is easily verified that such a solution is

$$n_0 = \text{constant}, \quad \psi = -\frac{1}{1 - n_0}\mathbf{r}\cdot\mathbf{n} = -\alpha\mathbf{r}\cdot\mathbf{n}, \qquad (9)$$

and this is not related to any particular shape of orbit, i.e. it remains valid for any azimuthal dependence of the magnetic field.

The condition of dynamical similarity affects not only the geometry of the orbits but also the dynamics of the accelerated particles. In some cases similar orbits exhibit an analogy with circular orbits. This is particularly important as regards radiation effects in constant-field accelerators.

It has been shown by various authors that the reaction of radiation leads to a positive or negative damping of betatron and synchrotron oscillations with decrements[3]

$$\left.\begin{aligned} \xi_r &= \tfrac{1}{2}\,\overline{\Gamma[1 - (1 - 2n)K\psi]} \quad \text{(radial oscillations)}, \\ \xi_z &= \tfrac{1}{2}\,\overline{\Gamma}\ \text{(vertical oscillations)}, \\ \xi_s &= \tfrac{1}{2}\,\overline{\Gamma[2 + (1 - 2n)K\psi]} \quad \text{(synchrotron oscillations)}, \end{aligned}\right\} \qquad (10)$$

where the bar denotes an averaging over the periodicity element and Γ is the dimensionless intensity of the radiation. The sum of the decrements is always $2\overline{\Gamma}$, whatever the form of the magnetic system.

If the orbit consists of arcs of constant curvature as in ordinary weak-focusing and strong-focusing synchrotrons, we have the well-known formulae

$$\xi_r = \tfrac{1}{2}(\alpha - 1)\overline{\Gamma},\ \xi_z = \tfrac{1}{2}\overline{\Gamma},\ \xi_s = \tfrac{1}{2}(4 - \alpha)\overline{\Gamma}, \qquad (11)$$

whence, in particular, it follows that, in a strong-focusing magnet ($\alpha \ll 1$), the radial betatron oscillations become unstable.[3]

For accelerators with similarity orbits of variable curvature this conclusion is not so obvious, but we shall show that it remains valid. The fundamental importance of this result should be noted: it implies that the damping systems used in ordinary synchrotrons[3,4] cannot be used under similarity conditions. The proof consists in substituting the expression (9) for ψ in (10). Then

$$\xi_r = \tfrac{1}{2}\,\overline{\Gamma[1 + K\alpha\mathbf{r}\cdot\mathbf{n} - 2\alpha(n_0 + \frac{K'}{RK}\mathbf{r}\cdot\boldsymbol{\tau})]}. \qquad (12)$$

Since $\Gamma \sim K^2$, we have

$$2\,\overline{\frac{\Gamma K'}{RK}\mathbf{r}\cdot\boldsymbol{\tau}} = -\,\overline{\frac{\Gamma}{R}(\mathbf{r}\cdot\boldsymbol{\tau})'} = -\,\overline{\Gamma} - \overline{K\Gamma\,\mathbf{r}\cdot\mathbf{n}}. \qquad (13)$$

Substitution of (13) in (12) gives

$$\xi_r = \tfrac{1}{2}\,\overline{\Gamma[1 + 2\alpha n_0 - \alpha]} = \tfrac{1}{2}\,\overline{\Gamma(\alpha - 1)} = \tfrac{1}{2}n_0\overline{\Gamma}/(1 - n_0). \qquad (14)$$

The expressions for the remaining decrements are likewise the same as (11). It is important to note that these expressions are in no way related to the azimuthal dependence of the magnetic field, which may be arbitrary; they have been derived only from the similarity condition.

Thus, when the similarity condition holds, it is impossible to set up a damping system based on a coupling between radial oscillations and synchrotron oscillations; hence the only remaining possible method of overcoming radial instability is to set up an artificial coupling with the vertical oscillations. Using the formulae derived in[3] we find the damping decrement for coupled oscillations in a similarity system:

$$\xi_{r,z} = \tfrac{1}{2}\,\overline{\Gamma}\{\alpha \pm (2 - \alpha)\delta/\sqrt{(\delta^2 + \Delta^2)}\}, \qquad (15)$$

where δ is the distance from the difference resonance and Δ a parameter characterizing the coupling of the oscillations. Numerical estimates show that, to bring about an effective damping in a ring electron synchrocyclotron as described in[5], a longitudinal magnetic field of about 10 per cent of the guiding field may be used.

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BIBLIOGRAPHY ON MAGNETO-FLUID-DYNAMICS

FOREWORD

We have prepared this bibliography on Magneto-Fluid-Dynamics in the AGARD Secretariat to provide a source of information to the scientists in this field.

There has been considerable semantic uncertainty as to the title to be given to this new branch of science. Dr. VON KÁRMÁN suggested that the term 'Magneto-Fluid-Dynamics' (MFD), which seems to have been agreed upon by a large number of workers in this field, be used.

The co-operation of experts and organizations from several countries has given us the opportunity of making it as complete and up-to-date as possible. We are grateful to all of them.

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Prof. L. NAPOLITANO of the University of Naples greatly assisted the work by screening the material and preparing a list by subject.

A collection of abstracts made available from various sources is included as an Appendix.

The printing of the bibliography has been provided by the PERGAMON PRESS, and the printed collection of abstracts by the SCUOLA d'INGEGNERIA AERONAUTICA, Rome.

We will appreciate all comments and information which could help us in improving this bibliography in the future.*

A. G. VANNUCCI
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* A completely revised and expanded edition of the Bibliography is in preparation by AGARD and will be published later by Pergamon Press.

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LIST OF ABBREVIATIONS

ABBREVIATIONS	TITLE	COUNTRY
C. R. Acad. Sci., Paris	*Académie des Sciences, Paris* (*Comptes rendus*)	F
Accad. Lincei, Roma	Accademia Nazionale dei Lincei, Roma	I
Acad. Sci. Pol.	Académie Polonaise des Sciences	Pol
Acc. Sci., Torino	Accademia Scientifica, Torino	I
Acad. Sci. USA	National Science Academy	USA
Addison	Addison-Wesley Publishing Corp.	USA
Advanc. Phys.	*Advances in Physics* (Journal)	UK
AEC	Atomic Energy Commission, Washington	USA
AERE	Atomic Energy Research Establishment, Harwell	UK
Aeronaut. Engng. Rev.	*Aeronautical Engineering Review*	USA
Aerotecnica	*L'Aerotecnica, Torino* (Journal)	I
AFCRC	AF Cambridge Research Center	USA
AFOSR	U.S. Air Force Office of Scientific Research	USA
AGARD	Advisory Group for Aeronautical Research and Development, Paris	NATO
Amer. Astronaut.	American Astronautical Society	USA
Amer. J. Phys.	*American Journal of Physics*	USA
Amer. Math.	*American Math. Monthly*	USA
Amer. Phys. Soc.	American Physical Society, N.Y.	USA
Ann. Astronaut.	*Annales d'Astronautique*	F
Ann. Mat. Para. Appl.	*Annali di Matematica Pura e Applicata*	I
Ann. Phys.	*Annals of Physics*, N.Y.	USA
Appl. Sci. Res.	*Applied Science Research*	Ne
Arch. Mech. Anal.	*Archive for Rational Mechanics and Analysis*	Ge
Ark. Fys.	*Arkiv för Fysik, Stockholm*	S
Ark. Mat. Astr. Fys.	*Arkiv. Mat. Astron. Fysik.*	S
ARC	Aeronautical Research Council	UK
ARDC	Air Research and Development Command	USA
ARS	American Rocket Society, N.Y.	USA
ASTIA	Armed Services Technical Information Agency	USA
Astronautics	*Astronautics*	USA
Astrophys. J.	*Astrophysical Journal*, Chicago	USA
Astr. Soc. Ne.	Astr. Soc. Netherlands	Ne
Austral. J. Phys.	*Australian Journal of Physics*	Austral
AVCO Res. Lab. Rep.	AVCO Research Laboratory Report	USA
Av. Week	*Aviation Week*	USA
CADO, Dayton	Central Air Documents Office, Dayton	USA
Cah. Phys.	*Cahiers de Physique*, Paris	F
Calcutta Math. Soc.	Calcutta Mathematical Soc.	India
Canad. J. Phys.	*Canadian Journal of Physics*	Canada
Coll. Aero.	College of Aeronautics, Cranfield	UK
Conf. Fluid Mech.	Conference on Fluid Mechanics, Ann Arbor	USA
Conf. Dyn. Ion.	Conference on the Dynamics of Ionized Media	USA

ABBREVIATIONS	TITLE	COUNTRY
Congress Appl. Mech.	9th International Congress of Appl. Mechanics	USA
Congress Astronaut.	Astronautics Congress, 1958	
Cornell U. Aero. Lab.	Cornell University, Aeronautical Laboratory	USA
K. Danske Vidensk. Selske, Mat-fys-Medd	*Kgl. Danske Viendenskabernes Selskabs, Mathematiak, Fysiske Meddelesles*, Copenhagen	D
Dokl. Akad. Nauk SSSR	*Proceedings of the Academy of Sciences of the USSR*	USSR
Douglas Res.	Douglas Research, Santa Monica, Cal.	USA
DRTE, RPL	Defence Research Telecomunications, Radio Physics Laboratory	Canada
DVL	Deutsche Versuchsanstalt für Luftfahrt E.V.	Ge
Fluid Mech. Inst.	Heat Transfer and Fluid Mechanics Institute	USA
Fortschr. Phys.	*Fortschritte der Physik*	Ge
Geofys. Publ.		
IAS	Institute of Aeronautical Sciences	USA
Indian J. Phys.	*Indian Journal of Physics*	India
Indian J. Meteorol. Geophys.	*Indian Journal of Meteorology and Geophysics*, New Delhi	India
Izv. Akad. Nauk SSSR	*Bulletin of the Academy of Sciences*	USSR
Izv. Akad. Nauk Lat. SSR	*Bulletin of the Latviiskoy Ac. Sciences*	USSR
Izv. Vys. Uch. Zav.	*Bulletin of Higher Educational Institutions, Radiophysics Section*	USSR
K. Tek. Högskol., Handl.	*Kungliga Tekniska Högskolans Sweded Handligar* (Royal Institute of Technology, Sweden)	S
J. Acoust. Soc. Amer.	*Journal of Acoustical Society of America*	USA
J. Aero/Space Sci.	*Journal of the Aero/Space Sciences*, N.Y.	USA
J. Aero. Sci.	*Journal of the Aeronautical Sciences*	USA
J. Appl. Phys.	*Journal of Applied Physics*, N.Y.	USA
J. Appl. Mech.	*Journal of Applied Mechanics*	USA
J. Amer. Rocket Soc.	*Journal of American Rocket Society*	USA
Jet Prop.	*Jet Propulsion*, New York	USA
J. Fluid Mech.	*Journal of Fluid Mechanics*, London	UK
J. Franklin Inst.	*Journal of Franklin Institute*	USA
J. Geophys. Res.	*Journal of Geophysical Research*	USA
J. Math. Mech.	*Journal of Mathematics and Mechanics*	UK
J. Nucl. Energ.	*Journal of Nuclear Energy*	UK
J. Phys. Earth	*Journal of Physics of the Earth*	Japan
J. Phys. Rad.	*Journal of Physics of Radiations*	USA
J. Phys. Soc. Japan	*Journal of the Physical Society of Japan*	Japan
J. Rat. Mech. Anal.	*Journal of Rational Mechanics and Analysis*	USA
J. Roy. Aero. Soc.	*Journal of the Royal Aeronautical Society*	UK
J. Sci. Inst.	*Journal of Scientific Instruments*, London	UK
Mathematika	*Mathematika*	UK
MIT	Massachussetts Institute of Technology	USA
Mon. Not. Roy. Astr. Soc.	*Monthly Notices of Royal Astronomical Society*	UK
NACA	National Advisory Committee for Aeronautics	USA
NASA	National Aeronautical Space Agency	USA

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ABBREVIATIONS	TITLE	COUNTRY
Nature, Lond.	*Nature, London*	UK
NRL	Naval Research Laboratory, US Navy	USA
Odin Ass.	Odin Association	USA
ONERA	Office National d'Etudes et de Recherches Aéronautiques, Paris	France
Phil. Mag.	*Philosophical Magazine*	UK
Phys. Fluids	*Physics of Fluids*	USA
Phys. Rev.	*Physical Review*	USA
Physics To-day		
Plasmadyne	Giannini Plasmadyne Corp., Cal.	USA
Prikl. Mat. Mekh.	*Prikladnaya Matematika i Mekhanika* (*Applied Mathematics and Mechanics*)	USSR
Proc. Camb. Phil. Soc.	*Proceedings Cambridge Philosophical Society*	UK
Proc. Phys. Soc.	*Proceedings Physical Society*	UK
Proc. Roy. Astr. Soc.	*Proceedings of the Royal Astronomical Society*, London	UK
Proc. Roy. Soc.	*Proceedings of the Royal Society*	UK
Progr. Sci. Phys.	*Progrès des Sciences Physiques*	F
Progr. Theor. Phys. Jap.	*Progress of Theoretical Physics*, Kyoto	Japan
Quart. Appl. Maths.	*Quarterly of Applied Mathematics*	USA
Quart. J. Mech. Appl. Maths.	*Quarterly Journal of Mechanics and Applied Mathematics*, London	UK
Ramo-Wooldridge	The Ramo-Wooldridge Corp., Los Angeles	USA
Republic Av. Rep.	Republic Aviation Corp. Report, Farmingdale	USA
Rev. Gen. Elect.	*Revue générale d'électricité*	F
Rev. Mod. Phys.	*Reviews of Modern Physics*, N.Y.	USA
Rev. Sci. Instr.	*Review of Scientific Instruments*	USA
J. Soc. Automot. Engrs.	*Journal of the Society of Automotive Engineers*, N.Y.	USA
Sandia Corp.	Sandia Corp., Albuquerque, New Mexico	USA
Sci. Amer.	*Scientific American*, N.Y.	USA
Smithsonian Inst.	Smithsonian Institution, Astrophysical Observatory	USA
Sov. J. Atom. Energ.	*Soviet Journal of Atomic Energy*	USSR
Sov. Phys.	*Soviet Physics, Tech. Phys.*	USSR
Space/Aero.	*Space/Aeronautics*	USA
Space Res. Lab.	Physical Research Laboratory, Space Research Laboratory	USA
Symp. gaseous masses	Symposium on the Motion of Gaseous Masses, Paris	F
Technology	*Technology* (Review)	USA
Tellus	*Tellus*, Stockholm	S
TIL/MOS	Technical Information and Library Services, Ministry of Supply, London	UK
UTIA	Toronto Univ., Institute of Aerophysics	Canada
Boll. Unione Mat.	*Bollettino Unione Matematica Italiana*	I
Volta Conf.	Volta Conference, Roma	I
WADC	US AF Wright Air Development Center, Dayton	USA
Z. Astrophys.	*Zeitschrift für Astrophysik* (*Journal of Astrophysics*)	Ge

ABBREVIATIONS	TITLE	COUNTRY
Zh. Eksp. Teoret.	*Zhurnal eksperimentalnoi i teoreticheskoi* (*Journal de Physique Expérimentale et Théorique*)	USSR
Z. Angew. Math. Mech.	*Zeitschrift für angewandte Mathematik und Mechanik*	Ge
Z. Angew. Math. Phys.	*Zeitschrift für angewandte Mathematik und Physik*	Ge
Z. Naturforsch.	*Zeitschrift für Naturforschung*	Ge
Z. Phys.	*Zeitschrift für Physik*	Ge
Zh. Tekh. Fiz.	*Zhurnal tekhnicheskoi Fiziki* (*Journal Technique de l'Académie des Sciences de l'URRS*)	USSR
Zh. Teoret. Eksp. Fiz.	*Zhurnal Teoreticheskoĭ i Experimentalnoi Fiz.*	USSR
J. Theor. Exp. Phys.	English translation of above Journal	USA

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INSTABILITY WAVES IN MAGNETICALLY CONFINED PLASMAS

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Abstract—In most of the treatments of hydromagnetic stability, the perfect conductivity approximation has been made which ignores the Hall e.m.f. For small amplitude perturbations with frequency $\omega/2\pi$, the solutions for ω^2 are always real, and for an unstable discharge ($\omega^2 < 0$) there are no oscillations or wave motion. Experimentally, however, wave motion is observed. It is pointed out in this paper that if the Hall e.m.f. is retained, wave motion is then obtained theoretically which is in good agreement with experiment. The full set of hydromagnetic equations is not solved; instead a physical picture of the wave motion is obtained by considering the property of 'freezing in' of magnetic lines of force in the plasma. When the Hall e.m.f. is retained the magnetic field is trapped to the mean motion of the electrons. The approximate wave velocity obtained is the component of electron drift velocity perpendicular to the instability.

INTRODUCTION

IN the many theoretical treatments of the hydromagnetic stability of magnetically confirmed plasma, such as the pinch discharge (KRUSKAL and SCHWARZSCHILD, 1954; TAYLER, 1957; SHAFRANOV, 1956; and others) it has been the general practice to take for the electric conduction equation the so-called hydromagnetic approximation

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{j} \tag{1}$$

and further, in most cases the electrical resistivity η has been assumed zero. ($\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, $\mathbf{j}$ the conduction current density and $\mathbf{V}$ the mass velocity.) In the case of the pinch discharge, the solutions of the hydromagnetic equations for small disturbances are then of the form

$$A = A_0 + A_1(r) \exp i(\omega t - m\theta - kz) \tag{2}$$

where ω^2 is always real. (Cylindrical co-ordinates are used and A represents any of the variables. A_0 is the equilibrium value.) When the discharge is unstable ($\omega^2 < 0$) the perturbation will therefore grow continuously with time and there are no oscillations or wave motion.

On the other hand, the few experimental observations which have been made on the motion associated with such instabilities have indicated that a wave motion was involved. Thus multi-frame photography of the growth of a wriggle ($m = 1$) instability by Payne and Allen of this Laboratory showed that the helical disturbance moved in the z-direction with a velocity of the same order of magnitude as the radial growth velocity (ALLIBONE *et al.*, 1958). A fully-grown instability of this type was observed by ALLEN (1957) to be a rotating helix and spectroscopic Doppler measurements showed that the rotation was a wave motion. Lastly, BAKER *et al.* (1958) found, in the presence of an applied axial magnetic field, a regular rotating helical disturbance, which Doppler measurements again showed to be a wave motion. In all cases the helical wavelength was equal to, or a little greater than, the discharge tube diameter. The wave velocities in the z-direction were in the range 10^5 to 10^6 cm sec^{-1}. Strictly speaking, since these experimental observations were made on large amplitude disturbances, they should not be compared with the predictions of small amplitude theory. However, it is the purpose of this paper to point out that if a more accurate electric conduction equation for a low pressure plasma is used, wave motion is predicted even for small disturbances.

THE ELECTRIC CONDUCTION EQUATION FOR A LOW PRESSURE PLASMA

The two main terms which have been omitted from equation (1) and which do not vanish in the absence of collisions, are the Hall e.m.f. and the electron pressure gradient e.m.f. and the more accurate form of the equation is (SPITZER, 1956)

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} - \frac{1}{ne}\mathbf{j} \times \mathbf{B} + \frac{1}{ne}\nabla . \mathbf{p}_e = 0 \tag{3a}$$

or alternatively, using the momentum balance equation

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} - \frac{1}{ne}\nabla . \mathbf{p}_i - \frac{M}{e}\frac{d\mathbf{V}}{dt} = 0 \tag{3b}$$

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where $\mathbf{p}_e$ and $\mathbf{p}_i$ are electron and positive ion pressure tensors, M the ion mass and n the electron density.

In a steady state low pressure plasma the third term in (3b) is generally equal to E and the second and fourth terms zero. In small amplitude instability theory it is necessary to consider the change in magnitude of each term from its equilibrium value. BERNSTEIN *et al.* (1958) have pointed out that the perturbation to the fourth term is small compared with $\mathbf{V} \times \mathbf{B}$ if

$$\omega \ll \omega_{ci} \equiv \mathrm{Be}/M \tag{4}$$

where M is the ion mass, and they deduce that the relative magnitude of the third term is also small if in addition,

$$\omega^2 L^2 \gg kT/M \tag{5}$$

where L is the characteristic length over which the physical quantities change. In the experiments referred to above, the observed frequencies satisfy the relation (4), but the wave velocity ωL was about one third of $(kT/M)^{1/2}$ so that the perturbation to $1/ne\ \nabla p_i$ was of the same order of magnitude as $\mathbf{V} \times \mathbf{B}$.

It is true that the relation $\mathbf{E} + \mathbf{V} \times \mathbf{B} = 0$ has been obtained as a first order solution to the Boltzmann equation for no collisions by using a series expansion in the parameter M/e (CHEW, GOLDBERGER, and LOW, 1956, and others). However, for the above discharges and many others this does not yield an accurate approximation since the ion Larmor radius is not negligibly small compared with the length $p/(dp/dr)$.

Even if the magnitude of both the extra terms were small, this would not necessarily justify their omission. Thus, taking for example the well known solution obtained for the simple case where m and B_z are zero (the 'sausage' instability), the radial acceleration predicted is a maximum at the crests of the perturbation, but the radial component of $\mathbf{V} \times \mathbf{B}$ is zero. Similarly dp/dz is a maximum where the z component of $\mathbf{V} \times \mathbf{B}$ is predicted to be zero. Thus, some of the components of the two extra terms are out of phase with the corresponding vector components of $\mathbf{V} \times \mathbf{B}$. In addition the extra terms change the order of the system of differential equations and imply additional boundary conditions at interfaces.

The danger of neglecting such terms can be seen by analogy with an induction motor. If the resistance of the rotor is small compared with its inductance, the component of current in phase with the voltage induced in the rotor would be small compared with the out of phase component. If the former component of current were neglected the equations would wrongly predict that the rotor would never rotate.

THE FREEZING OF MAGNETIC LINES OF FORCE IN A PLASMA

The presence of the extra terms in equation (3) makes the solution of the hydromagnetic equation considerably more difficult. However, the physical basis for the wave motion can be demonstrated by considering the property of 'freezing-in" of magnetic lines of force in the plasma. If the approximate equation (1) (with $\eta = 0$) is combined with the Maxwell equation for magnetic induction and the continuity equation, the freezing in of the magnetic field is given by

$$\frac{d}{dt}(\mathbf{B}/n) = (\mathbf{B}/n) \cdot \nabla \mathbf{V} \tag{6}$$

(see for example COWLING, 1957). (It should be noted that d/dt is the mobile operator $\partial/\partial t + \mathbf{V} \cdot \nabla$). Thus for motion in which the right-hand side of equation (6) is zero, the consequent vanishing of the left-hand side shows that the magnetic field will behave as if it is trapped in the plasma and moving with the mass velocity. If, on the other hand, the more accurate equation (3a) is used, the trapping of the magnetic field satisfies (SCHLUTER, 1956)

$$\left(\frac{d}{dt}\right)_e (\mathbf{B}/n) = (\mathbf{B}/n) \cdot \nabla \mathbf{v} - \frac{1}{n^3 e} \nabla n \times \nabla \cdot \mathbf{p}_e + \frac{1}{n^2 e} \nabla \times (\nabla \cdot \mathbf{p}_e) \tag{7}$$

where $(d/dt)_e \equiv \partial/\partial t + \mathbf{v} \cdot \nabla$ and $\mathbf{v}$ is the mean electron velocity. (Equation (7) is obtained by taking the curl of equation (3a), substituting for $\nabla \times \mathbf{E}$ from the appropriate Maxwell equation, and eliminating $\nabla \cdot \mathbf{v}$ by means of the electron continuity equation. The electron density has been assumed equal to the ion density n, and it must be remembered that $\mathbf{V} \times \mathbf{B} - 1/ne\ \mathbf{j} \times \mathbf{B}$ is an approximation for $\mathbf{v} \times \mathbf{B}$).

In the case when $\nabla \cdot \mathbf{p}_e \ll \nabla \cdot \mathbf{p}_i$ the second and thirds terms on the right-hand side of (7) can be neglected and the magnetic field is then frozen to the electrons and moves with the mean electron velocity $\mathbf{v}$. In the case of a small amplitude instability the main component of $\mathbf{v}$ is the unperturbed electron drift velocity $\mathbf{v}_0$. Since in general $\mathbf{v}_0$ will have a component $(\mathbf{v}_{0\perp})$ normal to the perturbation in the magnetic field, the perturbed field will be carried along with velocity $\mathbf{v}_{0\perp}$. Because of the gradient of the unperturbed part of $(\mathbf{B}/n)$ the perturbed component of $\mathbf{v}_\perp$ will also cause a contribution to the velocity for the magnetic field. When $\nabla \cdot \mathbf{p}_e$ is not negligible the magnetic field is no longer accurately frozen to the electrons, but in most cases this will be a better approximation than freezing to the mass velocity.

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THE WAVE VELOCITY

The wave motion proposed here will be illustrated by considering the instability of a pinch discharge with no magnetic shear [i.e. $\partial/\partial r\,(B_\theta/rB_z) = 0$]. The most likely modes are the interchange instabilities which satisfy

$$mB_\theta + krB_z = 0,$$

i.e. instabilities for which the helical crests and troughs of the perturbation are parallel to $\mathbf{B}_0$, and for which it can be shown $\mathbf{B}_1$ is parallel to $\mathbf{B}_0$. (Here, and in subsequent formulae the subscripts 0 and 1 refer to the equilibrium and perturbed parts of the variable.) Assuming $\nabla \cdot \mathbf{p}_e$ can be neglected the field trapping equation (7) reduces to (see Appendix)

$$i\omega\beta_1 + v_{r1}\frac{\partial\beta_0}{\partial r} - i\left(\frac{m^2}{r^2} + k^2\right)^{1/2} v_{0\perp}\beta_1 = 0 \quad (8a)$$

or alternatively,

$$\left(\frac{d}{dt}\right)_e \beta = 0 \quad (8b)$$

where β ($\equiv \beta_0 + \beta_1\alpha$) can represent either B_θ/rn or B_z/n, and α is the exponential in equation (2). As usual, the products of small quantities have been neglected.

From equation (8b) β is a constant of the mean electron motion and hence

$$\beta_1 = -\boldsymbol{\xi}_{e1} \cdot \nabla\beta_0 = -\xi_{er1}\frac{\partial\beta_0}{\partial r}$$

to the first order of small quantities. Hence, dividing equation (8a) by β_1

$$i\omega\,\xi_{er1} - v_{r1} - i\left(\frac{m^2}{r^2} + k^2\right)^{1/2} v_{0\perp}\,\xi_{er1} = 0 \quad (9)$$

which is a component of the identity $\mathbf{v} = (d/dt)_e\boldsymbol{\xi}_e$. This equation follows directly from the definition of $\boldsymbol{\xi}_e$ and is true even in the special case when $\beta_1 = 0$. To predict the relative importance of the three terms in (9) it would be necessary to solve the complete set of hydromagnetic equations. Here we rely on experiment to indicate their relative magnitudes. The ratio of the magnitudes of the second and third terms in (9) is

$$\frac{v_{r1}\lambda}{v_{0\perp}\,2\pi\,\xi_{er1}}$$

where λ is the wavelength of the instability measured perpendicular to the wavefront, namely $2\pi/(m^2/r^2 + k^2)^{1/2}$.

The instability observed by BAKER *et al.* (1958) was approximately of the interchange type with $m = 1$ and $k = 2\pi/15$ cm^{-1} and, for the particular instability studied, the results suggest that $v_{r1} \ll v_{0\perp}/2$, but that ξ_{er1} is appreciably greater than $\lambda/2\pi$ (= 2 cm). If this is so, then for this particular experiment the above ratio is small compared with unity and the second term in (9) can be neglected. In this case equation (9) gives the approximate wave velocity perpendicular to the helical wavefront to be

$$V_w = \frac{\omega}{(m^2/r^2 + k^2)^{1/2}} \simeq v_{0\perp} \quad (10)$$

i.e. the instability configuration is carried along with the component of electron drift velocity perpendicular to $\mathbf{B}$. Since in the initial unperturbed discharge $v_{0\perp}$ will not in general have the same radial variation as $(m^2/r^2 + k^2)^{-1/2}$ this relation implies that a given mode can exist only over an annular region which is thin in the radial direction. Several modes with different ω, k and m might develop simultaneously, each at a different radius. However, once a given mode has grown to appreciable amplitude, it will generate magnetic field perturbations with the same wave velocity in the plasma at neighbouring radii. These field fluctuations will tend to trap the electrons in these regions so that one of the non-linear effects to be expected is that the electron drift velocity will become modified so that $v_{0\perp}$ is proportional to $(m^2/r^2 + k^2)^{-1/2}$. It is suggested that this is what has happened in the experiment of BAKER *et al.*, since only one mode is observed which pervades the whole discharge.

To the order of magnitude considered in equation (10) there is no imaginary part of ω and this agrees with the slow variation of amplitude with time when the above measurements were made. The name, instability wave, is still justified since to reach the observed large amplitude the wave must have grown as the result of an initial instability. Non-linear effects are probably the cause of stability at large amplitude.

If it is assumed that the positive ions make a negligible contribution to the equilibrium current, then

$$V_w \simeq \frac{j_{0\perp}}{n_0 e} = \frac{(\mathbf{j}_0 \times \mathbf{B}_0) \times \mathbf{B}_0}{n_0 e\, B_0} = \frac{\nabla p_0 \times \mathbf{B}_0}{n_0 e\, B_0^2} \quad (11)$$

The apparent velocity with which the helical wave will move in the z-direction is

$$V_w{}^z = V_w\frac{B_0}{B_{0\theta}} = \frac{1}{n_0 e\, B_{0\theta}}\frac{dp_0}{dr}. \quad (12)$$

In the helical wave observed by BAKER *et al.* (1958) the order of magnitude of n, B_θ and dp/dr were 4×10^{14} cm^{-3}, 200 gauss and -400 dynes cm^{-3}. The corresponding value of $V_w{}^z$ is therefore approximately -3×10^5 cm sec^{-1}. The observed wave velocity was about -2×10^5 cm sec^{-1}, so that both the correct sign and order of magnitude have been predicted.

The angular frequency of the helical wave will be

$$\frac{\omega}{2\pi} = \frac{V_w^{\theta}}{2\pi r} = \frac{1}{2\pi r\, n_0 e\, B_{z0}} \frac{dp_0}{dr}. \qquad (13)$$

With typical values taken from the results of BAKER *et al.* (1959), equation (13) gives the correct order of magnitude for the frequency and correct direction of rotation. It also predicts qualitatively the correct variation of frequency with applied axial magnetic field and initial pressure. Since $1/n_0\, dp_0/dr$ will decrease with increase in initial pressure, the frequency in (13) will decrease. For large applied magnetic fields the magnetic field in the discharge (B_{z0}) will increase with applied field, and since $(1/n)\, dp_0/dr$ will, if anything, decrease, the frequency in (13) will decrease. At low applied fields the contribution to B_{z0}, due to the instability itself, will exceed the applied field, and B_{z0} will become independent of the applied field. Also, at these low applied fields in a metal discharge tube, there will be a reverse axial magnetic field near the tube wall. This reverse field leads to magnetic shear which will have the effect of slowing down the wave giving a lower frequency. Thus a maximum frequency can be expected at an applied field which is roughly equal to the B_z field produced by the instability itself.

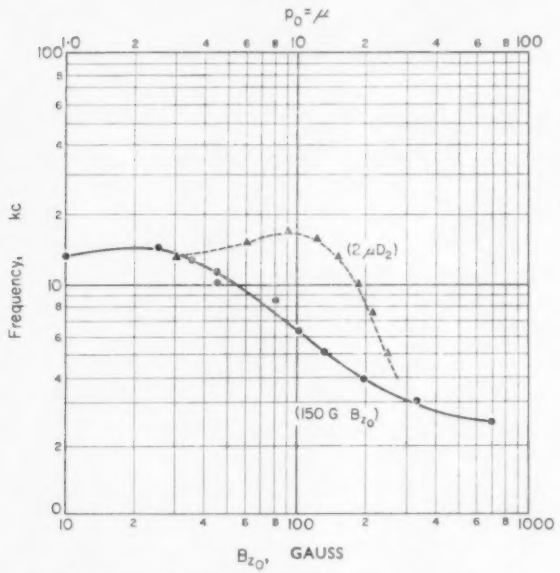

FIG. 1.—Experimental variation of the frequency of the helical wave with applied magnetic field (broken curve) and with initial pressure (full curve) in the experiments of BAKER *et al.*

Fig. 1 shows the experimental results observed by BAKER *et al.* (1958) for variation of frequency with pressure and magnetic field (private communication). It can be seen that the above predictions are in qualitative agreement. It should be noted that the decrease of wave velocity with magnetic field for large fields is in marked contrast to the dependence of wave velocity on magnetic field for other types of hydromagnetic waves.

CONCLUSIONS

It has been shown that when the Hall e.m.f. term is included in the electric conduction equation for a low pressure plasma, the lines of magnetic force behave approximately as if they were frozen to the electrons and not to the positive ions. Because of this, the perturbations associated with hydromagnetic instabilities will, under certain conditions, propagate as waves with a wave velocity approximately equal to the component of electron drift velocity perpendicular to **B**.

The few experiments on discharge instability which were capable of discerning the type of motion involved, all showed the presence of a wave motion. The theory presented here predicts the correct direction of propagation of these waves and the correct order of magnitude for the wave velocity. Finally the theory predicts the observed decrease of wave velocity with magnetic field. This property is in direct contrast to that for other types of hydromagnetic waves.

In view of the importance of the terms omitted from present instability theory, it is suggested that the theory should be re-examined with these terms included. In particular, since it appears that the inclusion of the extra terms leads to the prediction that the electrons and ions will have different velocities in the instability motion, the energy principle (BERNSTEIN *et al.*, 1958) needs to be modified to include separate terms for the thermal energies of the electrons and ions.

Acknowledgements—The author wishes to thank Dr. T. F. STRATTON of the Los Alamos Scientific Laboratory for a copy of unpublished experimental results and for permission to publish Fig. 1. In addition the author acknowledges the benefit of discussions with Messrs. B. S. LILEY and R. HERDAN of this Laboratory, with DRS. W. B. THOMPSON and R. J. TAYLER of A.E.R.E. and with PROF. T. G. COWLING, F.R.S., and thanks DR. D. R. CHICK of this Laboratory for his help and criticism in the author's absence in the United States. Part of this work was done under contract to the Atomic Energy Research Establishment, Harwell and thanks are due to DR. T. E. ALLIBONE, F.R.S., Director of the Research Laboratory, Associated Electrical Industries, Aldermaston Court, for permission to publish this paper.

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Note added in proof: Since this paper was written it has come to the author's attention that several Russian workers have obtained solutions for the $m = 0$ instability in a discharge with $B_z = 0$ using equations which essentially retain the extra terms in the above conduction equation (3a). Their results (TSERKOVNIKOV, 1957; KADOMTSEV, 1959; and RIDAKOV and SAGDEEV, 1959) include instability modes which have the same wave velocity as given in equation (11).

APPENDIX

When $\nabla \cdot \mathbf{p}_e \ll \nabla \cdot \mathbf{p}_i$, since $\mathbf{j} \times \mathbf{B} \simeq \nabla \cdot (\mathbf{p}_i + \mathbf{p}_e)$, the last term in equation (3a) can be neglected and equation (7) becomes

$$\frac{\partial}{\partial t}(\mathbf{B}/n) + \mathbf{v} \cdot \nabla(\mathbf{B}/n) = (\mathbf{B}/n) \cdot \nabla \mathbf{v} \tag{A1}$$

Following the usual procedure for normal mode analysis the variables are assumed to have a small harmonic component superimposed on their equilibrium values. For the cylindrical discharge the variables will take the form

$$\mathbf{B} = (B_{r1}\alpha,\ B_{\theta 0} + B_{\theta 1}\alpha,\ B_{z0} + B_{z1}\alpha)$$
$$\mathbf{v} = v_{r1}\alpha,\ v_{\theta 0} + v_{\theta 1}\alpha,\ v_{z0} + v_{z1}\alpha)$$
$$n = n_0 + n_1\alpha$$

when α is the exponential in equation (2). Neglecting the products of small quantities the radial component of (A1) reduces to

$$\frac{i\omega B_{r1}}{n} - \frac{imv_{\theta 0}B_{r1}}{rn} - \frac{ikv_{z0}B_{r1}}{n} = -\frac{iv_{r1}}{n}\left(\frac{mB_{\theta 0}}{r} + kB_{z0}\right) = 0. \tag{A2}$$

(The right-hand side is zero from the assumptions that there is no magnetic shear and the instability is parallel to $\mathbf{B}_0$.) Hence,

$$\left(\frac{d}{dt}\right)_e (B_r) = 0 \tag{A3}$$

and since B_r is zero initially it will always remain zero.

The θ-component of (A1) therefore reduces to

$$i\omega\left(\frac{B_\theta}{n}\right)_1 + v_{r1}\frac{\partial}{\partial r}\left(\frac{B_{\theta 0}}{n_0}\right) - \frac{imv_{\theta 0}}{r}\left(\frac{B_\theta}{n}\right)_1 - ikv_{z0}\left(\frac{B_\theta}{n}\right)_1$$
$$= -\frac{imB_{\theta 0}v_{\theta 1}}{n_0 r} + \frac{B_{\theta 0}v_{r1}}{n_0 r} - \frac{ikB_{z0}v_{\theta 1}}{n_0} = \frac{B_{\theta 0}v_{r1}}{n_0 r} \tag{A4}$$

where $$\left(\frac{B_\theta}{n}\right)_1 = \frac{B_{\theta 1}}{n_0} - \frac{n_1 B_{\theta 0}}{n_0^2}.$$

Whence, dividing by r and rearranging

$$i\omega\left(\frac{B_\theta}{rn}\right)_1 + v_{r1}\frac{\partial}{\partial r}\left(\frac{B_{\theta 0}}{rn_0}\right) - i\left(\frac{mv_{\theta 0}}{r} + kv_{z0}\right)\left(\frac{B_\theta}{rn}\right)_1 = 0 \tag{A5}$$

where $$\left(\frac{B_\theta}{rn}\right)_1 = \frac{1}{r}\left(\frac{B_\theta}{n}\right)_1.$$

Equation (A5) is identical with (8a) since

$$v_{\theta\perp} = \left(\frac{mv_{\theta 0}}{r} + kv_{z0}\right)\Big/\left(\frac{m^2}{r^2} + k^2\right)^{1/2}.$$

Similarly, the z component of equation (A1) reduces to an identical equation with (B_z/n_1) replacing $(B_\theta/rn)_1$.

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REGULAR OSCILLATIONS IN A TOROIDAL DISCHARGE

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Abstract—Regular oscillations have been detected under a limited range of conditions in a 12-in. bore torus. The electric and magnetic fields associated with these have been investigated and indicate a helical notch of reduced density travelling on the pinched current channel with a uniform velocity. A possible explanation of the phenomenon is given. The oscillation is considered to be growing as a sound wave in a partially-ionized medium. The mode is cited as an example of a pinched discharge with poor confinement.

1. INTRODUCTION

LARGE irregular fluctuations have been observed in the magnetic fields of high-current pinched discharges (see, for example, BUTT *et al.*, 1958). It has recently been found that the electric field in such a discharge also exhibits large fluctuations; in general, these are completely irregular, but under a certain limited range of conditions a regular reproducible oscillation is obtained, consisting of a series of well-separated sharp peaks. In these peaks the electric field reaches values an order of magnitude greater than the driving electric field of the discharge. Electric fields of such a size are of great interest in connexion with the rapid diffusion of plasma across a containing magnetic field, which has been observed, for example, by ELLIS *et al.* (1960) and by BURTON and WILSON (1960). The work to be described in this paper was carried out to elucidate the structure and origin of the regular oscillation and to determine whether the large electric fields could lead to a loss of plasma from the discharge.

2. EXPERIMENTAL RESULTS

The work was carried out in a toroidal discharge tube known as the Mark IV; the principal parameters of this apparatus are given in Table 1.

In Table 2 we show some observed properties of the oscillations and the ranges of conditions under which they occur. The electric field peaks are correlated with more nearly sinusoidal magnetic field fluctuations, similar to those reported by SAWYER, SCOTT and STRATTON (1959) (hereinafter referred to as S.S.S.)

In Fig. 1 a typical oscillogram of the electric and magnetic field variations is shown. The electric field was measured with an electric probe consisting of two platinum electrodes, each of area 0·28 cm², inserted into the plasma about 1 cm apart; the electrodes were connected directly to the balanced input of an oscilloscope, whose input impedance was 1 MΩ, to display the potential difference between the probes. The magnetic field was measured with search coils and integrators similar to those described by HARDING *et al.* (1958). The oscillations were also observed with no probes inserted into the discharge, using a bank of eight photomultiplier tubes viewing the light output of the discharge through a transverse slit in the torus wall.

Most of this work was carried out in discharges in hydrogen; similar oscillations have been observed in helium and argon discharges with magnetic probes, but in argon they are less regular and the electric field peaks, if present, are masked by irregular fluctuations.

In hydrogen, the electric probe signals are very smooth between the peaks and show high frequency hash, up to at least 1 Mc/sec, during the peaks. The oscillation normally follows a period of irregular hash at the beginning of the current pulse; it starts at about

TABLE 1.—THE MARK IV TORUS

Bore	12″
Aspect ratio	4:1
Torus body	Aluminium
Liners	Aluminium, lobster shell*
Cores	Iron
Gaps	2
Gas current I	1–50 kA
Applied electric field E_θ	0·2 to 1·0 V/cm
Rise time	200–400 μsec
Duration of pulse	1 msec
Applied magnetic field B_θ	0–800 gauss
Normal operating pressure	1–2 μ H_2
Pre-ionization	R.F.

* For a description of this type of liner, see BUTT *et al.* (1958).

TABLE 2.—PROPERTIES OF THE OSCILLATIONS

Frequency range	2–15 kc/sec
Peak electric field	2–5 V/cm
Range of gas current	1–10 kA
Range of applied magnetic field	10 to >100 gauss
Range of pressure in hydrogen	0·3 to >2·5 microns

* Royal Institution of Technology, Stockholm.

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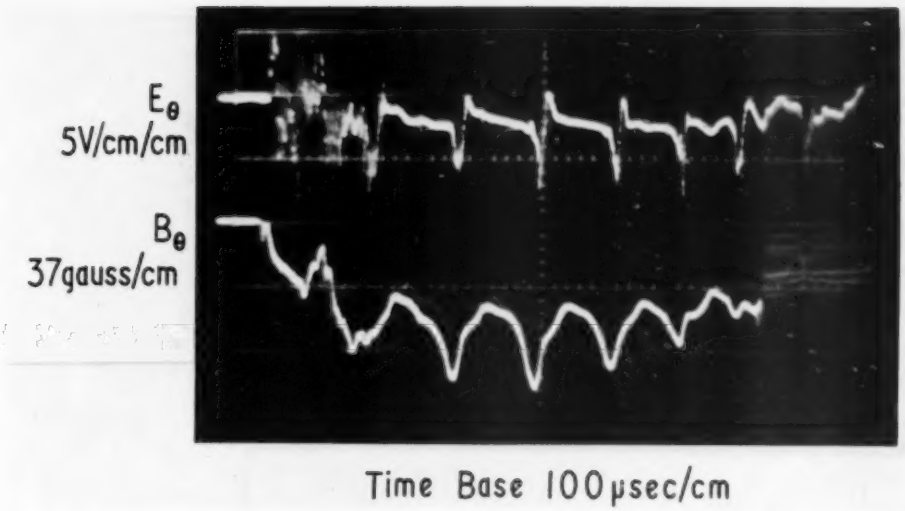

FIG. 1.—Correlation of electric and magnetic fields, showing regular oscillations.

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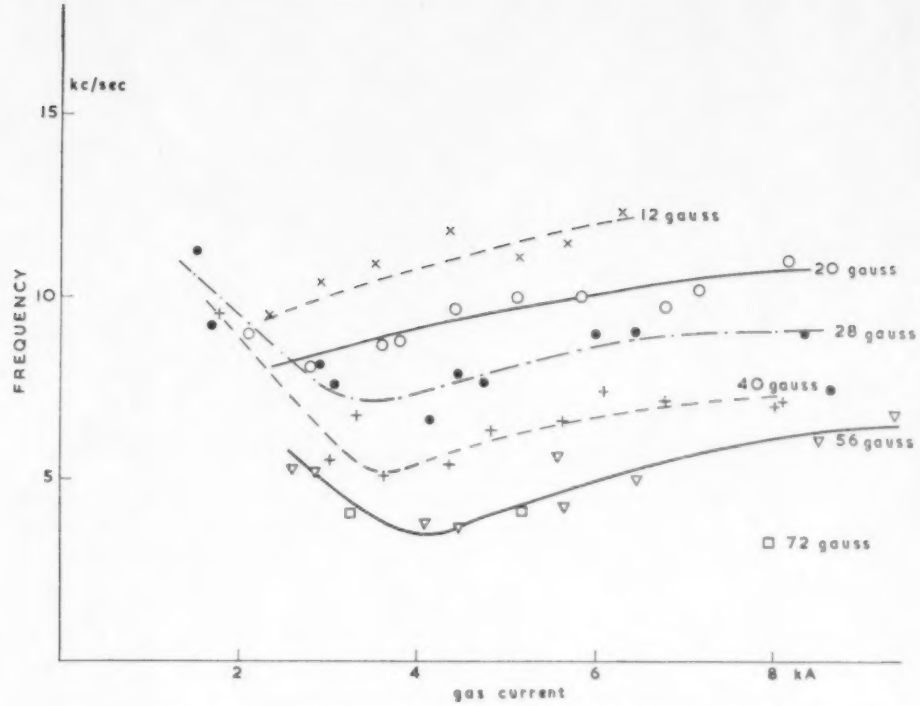

FIG. 2.—Frequency against gas current, parametric in applied magnetic field.

150 μsec and continues for the remainder of the pulse as shown in Fig. 1. The dependence of frequency on conditions is shown in Fig. 2. The high frequencies at low currents probably represent a different mode of oscillations; the electric fields here are of the same order of magnitude but more nearly sinusoidal in form. The frequency range is limited at the high-current end by the disappearance of the regular oscillation into irregular hash; the lowest frequencies become uncertain because their period is sensibly the same as the length of the current pulse. At constant current the frequency falls with increasing applied field B_0.

Measurements of the phase relation of the oscillation at different points along the discharge tube were made with a T-shaped probe containing four electric probes spaced at 8-cm intervals. These showed that the disturbance was propagated as a wave along the discharge tube; the velocity is independent of B_0 and varies only slightly with gas current, as shown in Fig. 3.

The direction of propagation is opposite to that of conventional current flow.

The variations of magnetic field measured across the discharge for a condition in which the regular oscillation is well developed are shown in Fig. 4. In this figure the mean magnetic field **B** is plotted separately from the fluctuating component **b**. The presence of a large b_0 at the centre is characteristic of a fundamentally $m = 1$ perturbation, where m is the axial symmetry number of the perturbation. Simultaneous measurements with two electric probes on opposite sides of the discharge show that the peaks occur in antiphase, so that the fundamental m-number must be odd. The voltage peak is found to become less sharp and eventually sinusoidal in the centre of the torus.

The mean magnetic field configuration was found to be approximately shear-free (i.e. $\mu = \dfrac{B_\theta}{rB_z}$ is approximately constant) near the centre, but the field helix in this region does not coincide with the perturbation helix, though the wavelengths of the two are roughly proportional to each other. The wavelength of the perturbation was found to be a function only of the magnetic parameter $\theta = \dfrac{2I}{aB_0}$ where a is the discharge tube radius; (LEES and RUSBRIDGE, 1960). This is shown in Fig. 5. This is a good indication that the perturbation helix is determined by the field configuration; it appears that the pitch of the helix represents some average value of the pitch of the magnetic field lines over the whole discharge.

The phase relation between the electric and magnetic field fluctuations shows that the peak occurs when the current channel is displaced away from the probe, i.e. the electric field occurs in a region of reduced current

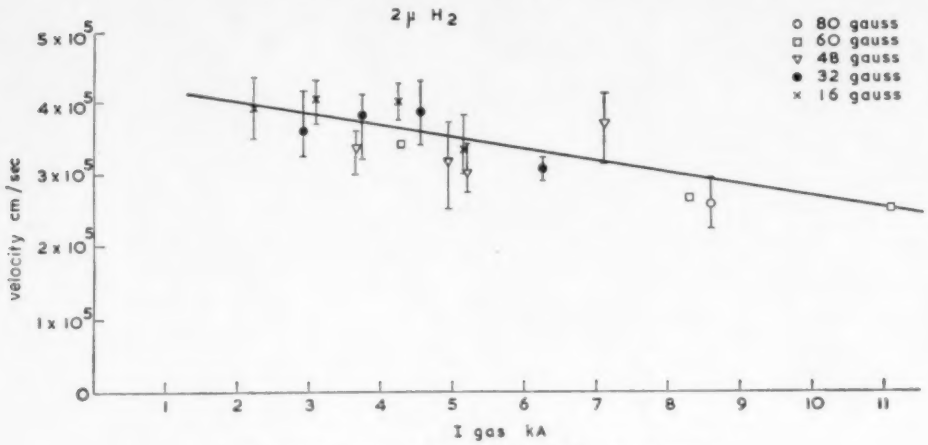

FIG. 3.—Wave velocity as a function of gas current.

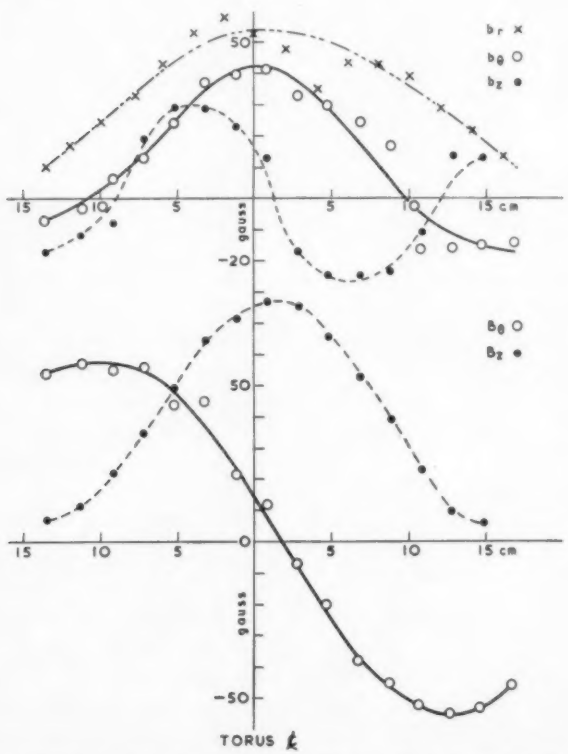

FIG. 4.—Magnetic field configuration. **B** is the mean magnetic field and **b** the amplitude of the fluctuating component.

density. S.S.S. found that their observations of magnetic field fluctuations indicated a helical notch of reduced current density; the observed frequencies, the ranges of current and applied field and the measured velocity are very similar to those obtained in the present work.

A probe was designed to measure simultaneously all components of E and B, in order to measure the velocity given by $\frac{\mathbf{E} \times \mathbf{B}}{|B|^2}$. The electric probes were made of platinum and spaced 1·4 cm apart; one pair could be used as an equal area double Langmuir probe to measure electron temperature and ion density. The radial component of velocity $\frac{(\mathbf{E} \times \mathbf{B})_r}{|B|^2}$ was evaluated as a function of radius and time; a large inward directed velocity was found during the E peak reaching 2–5 $\times$ 10^6 cm/sec. Between the peaks there is a small outward velocity of the same order of magnitude as the pinching velocity. Figure 6 shows a

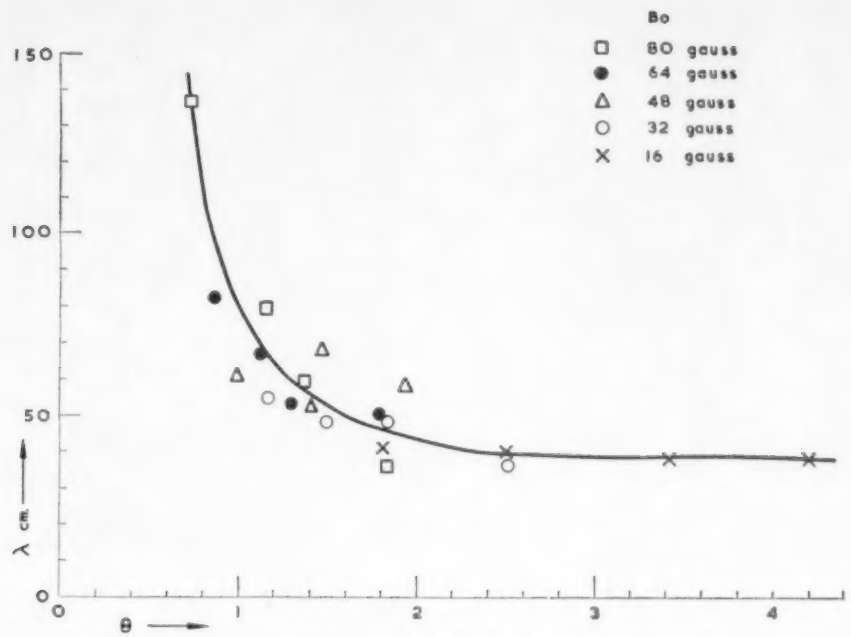

FIG. 5.—Wavelength of perturbation as a function of the magnetic parameter θ.

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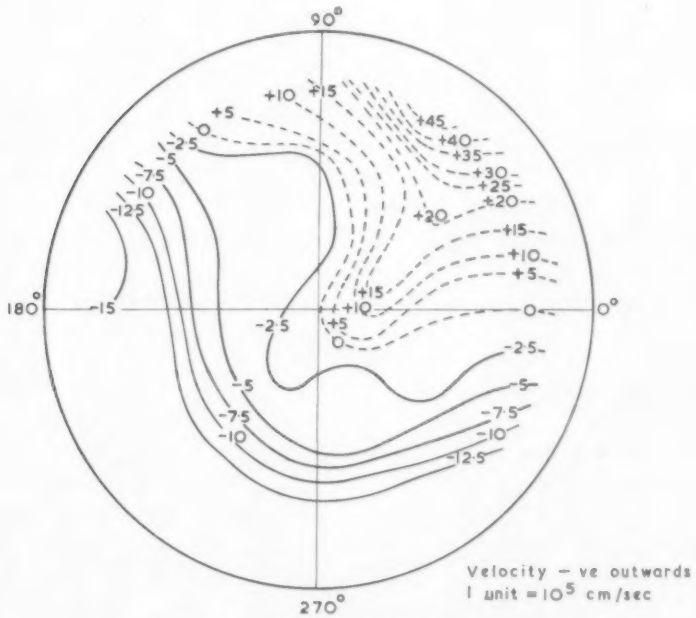

FIG. 6.—Contour diagram of radial velocity.

contour diagram of radial velocity obtained in this way. It should be emphasized that the azimuthal axis represents time and not space since axial symmetry cannot be assumed. The diagram indicates a slightly twisted ingoing channel, but this may be due to experimental error.

A contour diagram of ion density is shown in Fig. 7. This was obtained using both a d.c. biased and a swept Langmuir probe system (JONES and SAUNDERS, 1960). The electron temperature was found to be approximately constant at 3 eV, any systematic variation being less than 15 per cent.

The experimental results, therefore, indicate the presence of a helical notch of low density and absence of current on the surface of the current channel, across which there exist strong electric fields. The helix

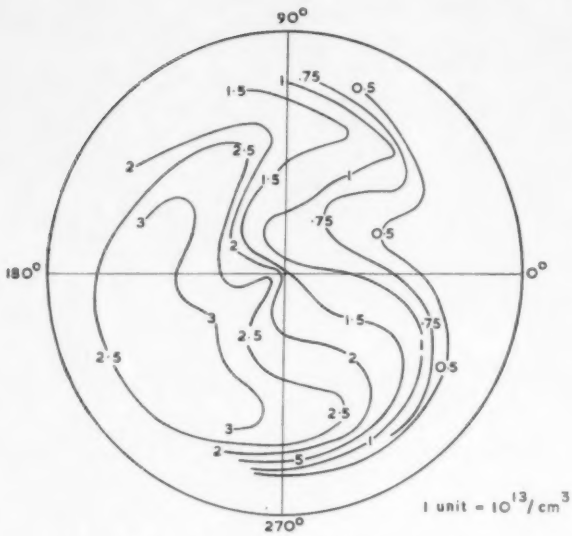

FIG. 7.—Contour diagram of ion density.

travels down the tube with a velocity of $3{\cdot}5 \times 10^5$ cm/sec.

3. INTERPRETATION OF ELECTRIC PROBE SIGNALS

A number of additional effects besides the electric field in the plasma may contribute to the observed potential differences between the electrodes of a double probe. These are as follows:

(a) Variation of local wall potential. An electrode inserted into a plasma takes up a local wall potential with respect to the plasma, given by (VON ENGEL, 1955):

$$V_{\text{wall}} = \frac{kT_e}{2e} \ln \frac{T_e m_i}{T_i m_e}.$$

Thus gradients of electron temperature give rise to potential differences. Assuming $T_i = 300°\text{K}$, the observed upper limit on electron temperature variations leads to an upper limit of 0·2 V/cm for the apparent electric field.

(b) Magnetic pick-up. The area of the loop enclosed by the electrodes and plasma is about 10 cm^2; with the observed rates of change of magnetic field, this gives a maximum pick-up voltage corresponding to about 0·05 V/cm.

(c) Photo-emission and secondary emission. The electrodes are made as nearly identical as possible and they are sufficiently near each other that photo-emission should be sensibly the same at each surface. Secondary emission, on the other hand, may vary because of the great variation in electron density; any effect due to this will be proportional to the electron density gradient.

(d) Other sources of potential gradient in the plasma. In addition to the electric field in the plasma which we seek to observe, there may arise electric fields set up by charge separation to balance density and temperature gradients. Such fields do not drive current and cannot be responsible for motion of plasma across a magnetic field. Expressions for them may be obtained from MARSHALL (1958):

$$E_{s1} = -\frac{kT_e}{ne} \nabla n$$

$$E_{s2} = -1{\cdot}2 \frac{k}{e} \nabla T_e$$

(the numerical coefficient of E_{s2} is slightly dependent on magnetic field). From the observed temperature and density distributions we find that E_{s1} may reach 0·5 V/cm, while E_{s2} is less than 0·03 V/cm.

Thus, apart from possible effects due to secondary emission, all the additional effects do not suffice to explain the observed fields. Moreover, all these effects of an appreciable size are proportional to the gradient of temperature or density; the variation of density and electric field with time at a constant radius are shown in Fig. 9 and it is clear that the observed field is not proportional to the density gradient. However, in many cases the electric field peak is not quite symmetrical as in the case shown in Fig. 9, but is accompanied by a small peak in the opposite direction of the order of 0·2–0·4 V/cm; the case shown in Fig. 1 is an example. This may clearly be interpreted as the sum of a symmetrical peak, as in Fig. 9, and one or more of the additional effects listed above. However, the peak field will be only slightly altered by these effects and we have not attempted to correct for them.

4. THE ORIGIN OF THE OBSERVED ELECTRIC FIELD

A possible explanation of the appearance of an electric field across the notch is that it is an ohmic field set up by a non-uniform conductivity. Fig. 8 shows the variation of conductivity with electron density for partially-ionized hydrogen gas for the following conditions:

(a) The electron temperature is constant at 3 eV.

(b) The total gas density is constant and equivalent to 2 microns pressure, i.e. $1{\cdot}4 \times 10^{14}$ atoms/cm^3. We assume that no gas is lost from the discharge tube; the constancy of the oscillation frequency through the pulse suggests that no continuous pump-out is occurring.

(c) All un-ionized gas is molecular; neutral atoms are assumed to escape, recombine on the walls and be re-injected.

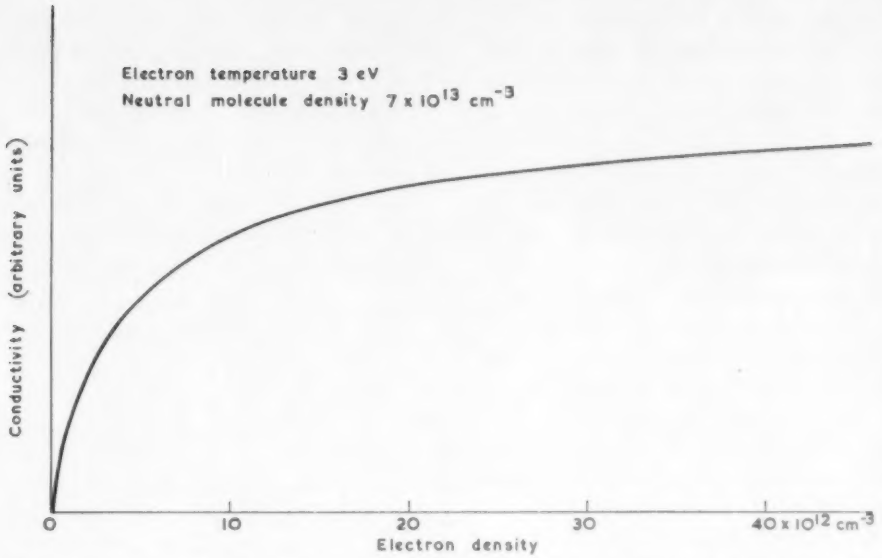

FIG. 8.—Conductivity of partially-ionized hydrogen as a function of electron density for conditions given.

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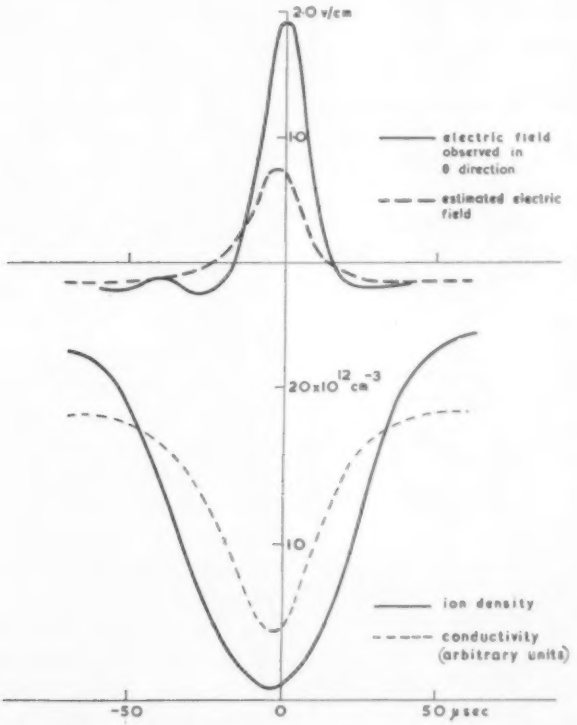

FIG. 9.—Estimated electric field produced by conductivity variation compared with observed field.

(d) The collision time for electrons with neutral hydrogen is assumed independent of velocity; a value of 0·1 μsec for the above conditions was obtained from information supplied by ROBERTS (private communication) following VON ENGEL (1955).

From Fig. 8 we see that below a density of about 5×10^{12} electrons/cm^3, the conductivity is reduced by collisions with neutral gas; with the observed density variation this leads to conductivity variations of about a factor of 4. Fig. 9 shows the density variation at a radius of 10 cm, the corresponding conductivity variation and an estimate of the observable ohmic field to which this would lead, compared with the actual field. We assume in this case that the current flow is

unperturbed by the notch. We see that although the estimated field is of the right shape (in contrast, for example, to the effects discussed in Section 3) it is neither large enough nor sharp enough to explain the observed field.

The observations of S.S.S. show that in their case current is diverted from the region of the notch; a radial component of current flowing inwards on one side and outwards on the other appears in their results, while the current density in the notch itself is reduced to about 10 per cent of that elsewhere. The present results are not sufficiently accurate to give current densities, but if we now assume that a radial current flow exists in our case also, that it at least partly flows through a region of reduced conductivity and that the effective conductivity is still further reduced because the current flows across the magnetic field, we find that qualitatively the potential drop across the notch can be accounted for by the ohmic electric field associated with the radial current flow. A radial electric field of the right order was indeed observed. A field set up in this way should be broadly proportional to the applied electric field E_0; the observation confirms this and show that the sense of the field is predicted correctly.

However, we do not understand the process which reduces the current in the notch to the low value required by this explanation. Evidently the calculated reduction in conductivity is insufficient and the magnetic field cannot be responsible, since it is not in general parallel to the notch. A more careful consideration of conditions in the notch suggests two possible explanations:

(a) The mean free path in the notch is of the same order as the width, so that the notch is approximately collision-free. The drift velocity is probably of the order of 2×10^7 cm/sec and the electron thermal velocity is 10^8 cm/sec. Since the ion temperature is probably much less than the electron temperature, conditions may be such that ion wave oscillations can grow (BERNSTEIN *et al.*, 1960). These might be connected with the observed high frequency hash in this region.

(b) Locally, runaway electrons may occur since the parameter $\frac{ET_e}{N_e}$ (GIBSON, 1957) reaches values greater than 10^{-8}. These may lead to plasma oscillations and effectively prevent the current flowing. In this case the hash might represent turbulence in the inflowing plasma.

In any case, the large velocities given by $\frac{\mathbf{E} \wedge \mathbf{B}}{|B|^2}$ will not necessarily be realized, since there is probably insufficient current to accelerate the plasma and maintain the velocity against neutral friction. Crude estimates suggest, however, that plasma energies up to about 3 eV may occur, giving velocities of the order of 10^6 cm/sec. Even so, during the peak, the plasma may move up to 10 cm which is of the same order as the tube radius.

5. INTERPRETATION OF THE RESULTS

S.S.S. have suggested that in their case the oscillation may be classed as a hydromagnetic wave. There are, however, several points which prevent this interpretation being extended to the present work:

(a) The measured ion density corresponds to about 15 per cent ionization. Estimates of the Alfvén speed with this density give about 3×10^6 cm/sec; even if strong coupling between plasma and neutral gas is assumed (LEHNERT, 1959), an increase in total gas density of about a factor of 20 would be required to account for the observed velocity of $3{\cdot}5 \times 10^5$ cm/sec.

(b) The observed velocity decreases slowly with increasing gas current. From data given by LEES and RUSBRIDGE (1960) it may be shown that in this type of discharge an average magnetic field is approximately proportional to gas current I for sufficiently high values of θ. The average density must then increase at least as I^2 to agree with experiment; the Langmuir probe results, however, show that the ion density increases only slowly with current above about 2 kA.

(c) The observed electric fields are about a factor of 10 greater than those expected in a hydromagnetic wave.

We suggest that the oscillation starts as a sound wave in the partially-ionized gas, which grows in amplitude because of the increased rate of ionization at the peak of the wave. Such a growing wave has been predicted by THONEMANN (private communication). The predicted velocity is about 2×10^6 cm/sec, much greater than the observed velocity, but this may be modified since the propagation is at a large angle to the magnetic field where the notch is strongest. As the wave grows the density in the troughs will fall until a point is reached when the conductivity locally starts to fall; an electric field is developed which drives the plasma inwards and thus further reduces the density. Hence the conductivity decreases still further until runaways and growing oscillations occur in the notch; these reduce the current in this region to a low value and generate the hash. The electric field rises to its observed value and perturbations occur in the magnetic fields, produced by the diversion of the current (so that the magnetic field fluctuations are an entirely secondary phenomenon). Only in the latter phase is the wave observable.

This interpretation suggests, then, that the mode of oscillation should not occur if the ionization is greater than about 50 per cent since growth will be limited by reaching full ionization in the peaks of the wave. This is consistent with the experimental fact that the gas is approximately 30 per cent ionized at 10 kA gas current, where the oscillatory mode can no longer be detected. S.S.S., however, found a much greater degree of ionization in their case.

6. IMPLICATIONS OF THE RESULTS

If this mode is a feature of a partially-ionized gas, it may be of importance in the ionization phase of a fully-ionized plasma (e.g. ZETA) and may be a reason for the rapid diffusion of magnetic field reported under certain conditions (LEES and RUSBRIDGE, 1960).

The containment of a distorted current channel such as this is worse than that of a similar axially symmetric configuration, since the density gradient is given for axial symmetry by

$$\frac{nE_0B_\theta}{B^2} = -D \text{ grad } n$$

where D is the ambipolar diffusion coefficient. In this oscillatory mode, between the notches, E_0 is opposed by the observed field and the two effects are of the same order of magnitude. Hence grad n may be much reduced with no abnormal value of D.

The effect may be visualized as comparable to the insertion of an insulating sheet into the plasma, thus increasing the current path length and adding extra resistance; this increases the rate of decay of currents and gives rise to increased field diffusion.

Finally it may be noted that between the notches the plasma appears completely quiescent and might therefore be useful for experiments demanding a quiet plasma of reasonably high density.

Acknowledgements—We wish to thank MR. R. S. PEASE, DR. P. C. THONEMANN and DR. K. V. ROBERTS for several helpful discussions and also the two last named for access to their unpublished work, and MR. A. G. HEARN for the observations with the bank of photomultipliers.

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THE LIFETIME OF FAST IONS IN OGRA*

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Abstract—The lifetime of injected H_2^+ ions and trapped H^+ ions in OGRA has been measured. It is found that the mean free path of molecular ions is about 1·5 km for a magnetic field configuration which gives the optimum velocity of azimuthal drift. The maximum lifetime of atomic ions of energy 100 keV is 9·3 ± 0·9 msec at a pressure 1×10^{-7} mm mercury. The lifetime of atomic ions of energy 50 keV is 3·2 msec at a pressure $2{\cdot}5 \times 10^{-8}$ mm mercury.†

The maximum density of atomic ions after injection of 20 mA of H_2^+ ions of energy 200 keV is about $1{\cdot}5 \times 10^7$ cm^{-3}. The dependence of the lifetime on the ion energy and residual gas pressure shows that in OGRA there are no mechanisms of loss of captured ions, other than charge exchange with the residual gas, up to characteristic times of the order of 10 msec.

1. INTRODUCTION

THE measurement of the lifetime of fast ions captured in a magnetic trap is the most direct and reliable means of examining its retaining properties. At the present time the density of fast H^+ ions trapped in OGRA does not exceed $\sim 10^7$ cm^{-3}, and the pressure of the residual gases does not fall below $2{\cdot}5 \times 10^{-8}$ mm mercury. Under these conditions the lifetime of fast ions must be determined entirely by charge exchange with the residual gas if there are no other mechanisms which cause an additional loss of ions from the trap. Such additional loss of fast ions may occur for the following reasons.

(1) The motion of the individual particles may cease to be adiabatic, either because there is a perturbation of the magnetic field in OGRA which is not symmetrical about the axis or because the magnetic field varies considerably within the Larmor circle of an ion.

The symmetry of the magnetic field about the axis is destroyed by the magnetic channel through which molecular ions are injected into OGRA and by various ferromagnetic bodies outside the chamber. In the neighbourhood of the stoppers, in order to ensure the necessary velocity of azimuthal drift, there is set up such a rapid increase in the magnetic field that the centre of the Larmor circle moves through ~30° during one reflection. Thus the motion of the ions near the stoppers is certainly not adiabatic.

(2) According to an analysis by Kadomtsev, hydromagnetic instability may develop in OGRA at ion densities above 10^6 cm^{-3}, and leads to a rapid dissipation of the plasma. As the plasma density increases, so does the danger of an increased escape of fast ions to the walls and by collective processes.

In this paper we describe measurements of the lifetime of trapped fast H^+ ions as a function of the magnetic field configuration, the residual gas pressure, the pressure of the helium and argon admitted to the chamber, and the current and energy of the injected ions. We have also measured the lifetime of molecular ions, which is in one-to-one relation with the mean path traversed by molecular ions before their destruction by the channel. This mean path determines the critical current which ensures the burn-out of the residual gas and the building up of a dense plasma (KURCHATOV, 1958; GOLOVIN, 1959 and ARTEMENKOV *et al.*, 1959).

The density of fast H^+ ions was determined from their lifetime, the lifetime of the injected molecular ions and the cross-section (estimated from data in the literature) for dissociation of H_2^+ ions by the residual gas.

2. APPARATUS

Changes in the fast-ion density were recorded by means of detectors of the fast neutral particles formed by charge exchange of fast ions with the molecules of the residual gas. The detectors were placed on the walls of the chamber and caused no perturbation of the plasma.

The detectors were sets of stainless-steel prisms and plates (Fig. 1) placed along the magnetic field. A positive potential (~20 V) was applied to the plates in order to collect secondary electrons ejected by neutrals from the surfaces of the prisms. The current in the prism circuit, which emits electrons, was measured. This circuit contained no potentials except that produced, by the current being measured, across the input resistance of the amplifier circuit. To eliminate any possibility of cold plasma reaching the

* Translated by J. B. SYKES.
† See *Note added in proof*, p. 114.

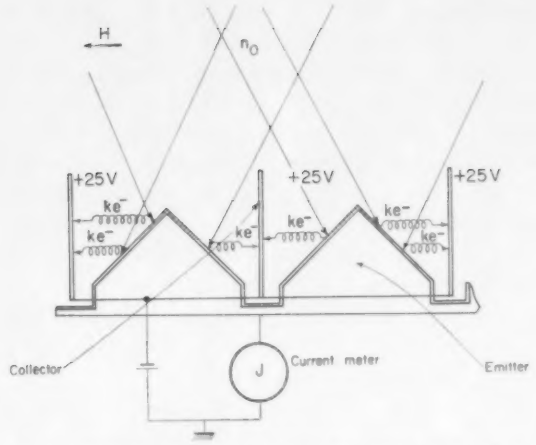

FIG. 1.—Diagram of element of secondary-emission detector of neutrals.

prisms, the detector was screened. Control measurements of the lifetime by means of a detector with a thin (0·12 μ) aluminium foil, under conditions where the cold-plasma density was one hundred times the fast-ion density, showed that the cold plasma does not distort the results of measurements of lifetime when secondary-emission detectors of fast neutrals are used.

The detectors were arranged in two groups placed symmetrically about the central plane of the chamber at 2 m from the centre. Each group consisted of four detectors (length 45 cm, width 5 cm), placed uniformly round the circumference of the chamber. The arrangement is shown in Fig. 2, which also gives the distribution of fast ion density along the axis of the chamber, as measured by a mobile detector of neutrals (10 × 10 cm) at 1 × 10^{-6} mm mercury pressure.

With this arrangement, the total current in the detector circuit in each group is, with sufficient accuracy, proportional to the fast-ion density averaged over the volume. This is of importance, since with the injection method used, the local density distribution of molecular and atomic ions is quite complicated.

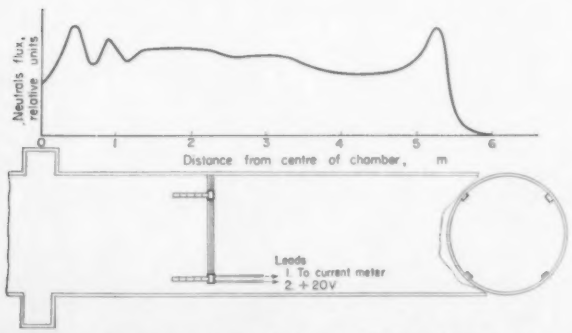

FIG. 2.—Arrangement of detectors in OGRA chamber and fast-ion density distribution along chamber axis.

In addition, an insulated screen surrounding the magnetic channel was used as a detector in measuring the lifetime of molecular ions.

The lifetime was determined from oscillograms showing the decay of the detector current after cutting off the injection current. The current was cut off by extinguishing the source arc after about 5 μsec.

The signals from the detectors passed to a two-beam oscillograph with a d.c. amplifier which gave a beam sensitivity of 1·4 mV/mm and a linear deflection up to 70 mm. The upper limit of the transmission band was 200 kc/s. We also used a preamplifier with a gain factor of up to 150 and transmission band from 0·1 c/s to 200 kc/s. The input resistances of the preamplifiers were short-circuited by fast relays. These were opened and the input resistances switched in by a pulse synchronized with the instant of extinction of the source arc. The pulse which brought in the preamplifier input resistances could be shifted by 3 msec each way, and the time during which the relay remained open could be varied from 2 to 25 msec. This system enabled us to separate the decay curve of current from the neutrals detector without difficulty into components with various characteristic times and to measure accurately the slow component against the initially stronger fast component.

3. THE LIFETIME OF MOLECULAR IONS

In the initial stage of build-up, when the fast ion density in OGRA is less than the residual gas density, atomic ions are captured as a result of dissociation of molecular ions by the residual gas. The molecular ions are brought into the trap through a magnetic channel, and the configuration of the magnetic field is made such that the instantaneous centre of the Larmor circle of an ion drifts round the axis of the chamber with a velocity which maximizes the path of molecular ions before their destruction. To find such a field configuration, Semashko and Balebanov have selected, on the basis of an electron model, a field configuration which gives a sufficiently high drift velocity. This configuration was then refined on OGRA. The criterion of optimum field configuration was based on the readings of detectors of the fast neutrals formed by charge exchange of fast ions with the residual gas. The fast-neutral flux is proportional to the fast-ion density, and this is the greater, the longer the path of molecular ions before their destruction by the channel. Figure 3 shows the magnetic field configuration which gives the optimum drift velocity, determined mainly by the very steep increase of the magnetic field near the stopper. The steep increase of the field is effected by a special coil in which the current is in the opposite

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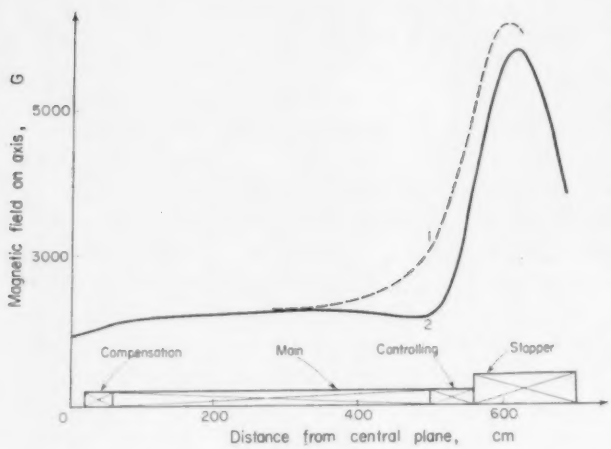

FIG. 3.—Magnetic field configurations in OGRA:
1—Configuration giving small drift
2—Configuration giving optimum drift of H_2^+ ions.

direction to that in the central and stopper coils. Figure 3 also shows a configuration which gives a very low drift velocity. The main difference between this field configuration and the drift field is that there is no drop before the final rise and accordingly a less steep increase of the field at the stopper.

In the first measurements the path of molecular ions before their destruction by the channel was found from the gas pressure at which half the current of molecular ions injected into the trap returned to the channel (GOLOVIN, 1959 and ARTEMENKOV *et. al.*, 1959). It was found that the path L lies between 10^4 and 10^5cm. In the present work we have measured the mean lifetime of molecular ions before their destruction by the channel, which is related to the mean path by $L = \tau_{\text{ll}} v$.

The mean lifetime of molecular ions was determined from oscillograms of the decay of the channel and neutrals-detector currents after stopping the injection.

Figure 4 shows two oscillograms of the decay of the channel current at $1{\cdot}4 \times 10^{-7}$ and 5×10^{-7} mm mercury, after injection of 1 mA of molecular ions of energy 100 keV. Figure 5 shows the same oscillograms on a semi-logarithmic scale. The oscillograms have a very steep initial slope, a clearly marked step, and then an almost exponential decay, which sometimes shows a break representing a broadened second step.

These properties of the oscillograms reflect the fate of molecular ions with the injection system used in OGRA. The initial drop corresponds to the destruction of some ions immediately after the first reflection from the stopper, the step represents a drift of the remaining ions through an angle $\sim 2\pi$, and the second break represents the destruction of some of these ions when they again pass the channel. The residual ions are subsequently destroyed by the channel in a random manner, since the beam is by then greatly broadened.

The lifetime of molecular ions can be determined by a simple graphical integration of the oscillograms. The time thus found is fixed by the destruction of ions in the gas and at the channel. Since the ions destroyed at the first reflection make almost no contribution to the build-up of atomic ions if the vacuum is good, only those ions which pass the channel at the first reflection are taken into account in the reduction of the oscillograms. Their lifetime $\tau_{\text{av}} = \tau_2$ is shown in Fig. 5.

To characterize the efficiency of utilization of the injected current, we use the ratio of the current which passes the channel at the first reflection to the total injection current: $a = J_1/J_0$. The product $a\tau_2$ is the mean lifetime of molecular ions, taking into account the total injection current. The ratio a can be determined with sufficient accuracy from the height of the step on the oscillogram. The oscillograms showing the decay of the current to the channel can be approximated by a horizontal step and an exponential decrease. The length of the step does not depend on the gas pressure, but its relative height becomes smaller and the steepness of the exponential decrease becomes greater as the gas pressure increases. These facts can be utilized in order to determine the effective cross-section for total losses of molecular ions in the gas and the true mean lifetime with respect to destruction by the channel. The mean loss cross-section in the gas, as determined from the exponential decrease in three pairs of oscillograms, is $1{\cdot}5 \times 10^{-15}$

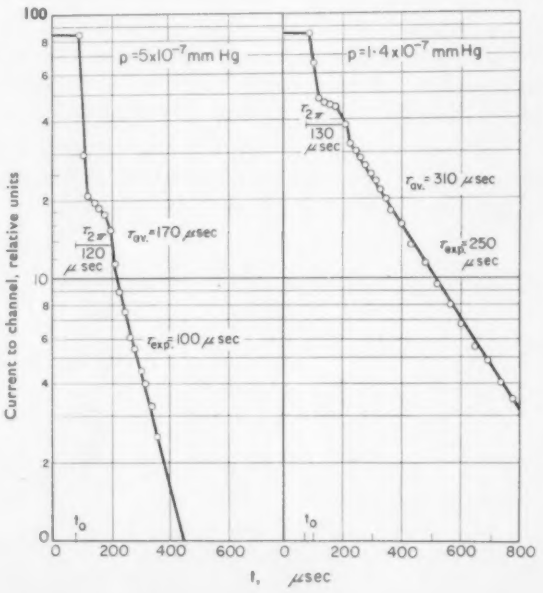

FIG. 5.—Oscillograms from Fig. 4 on semi-logarithmic scale.

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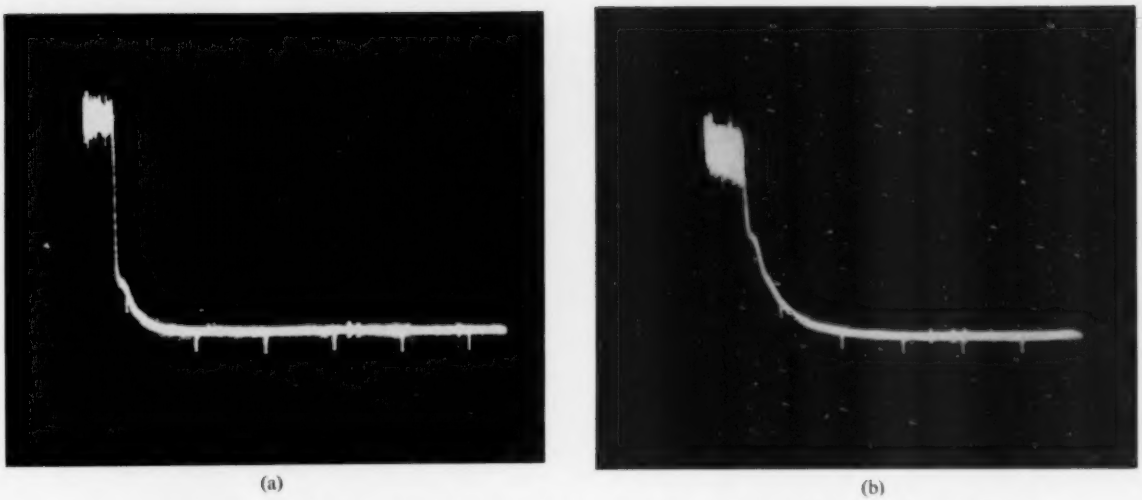
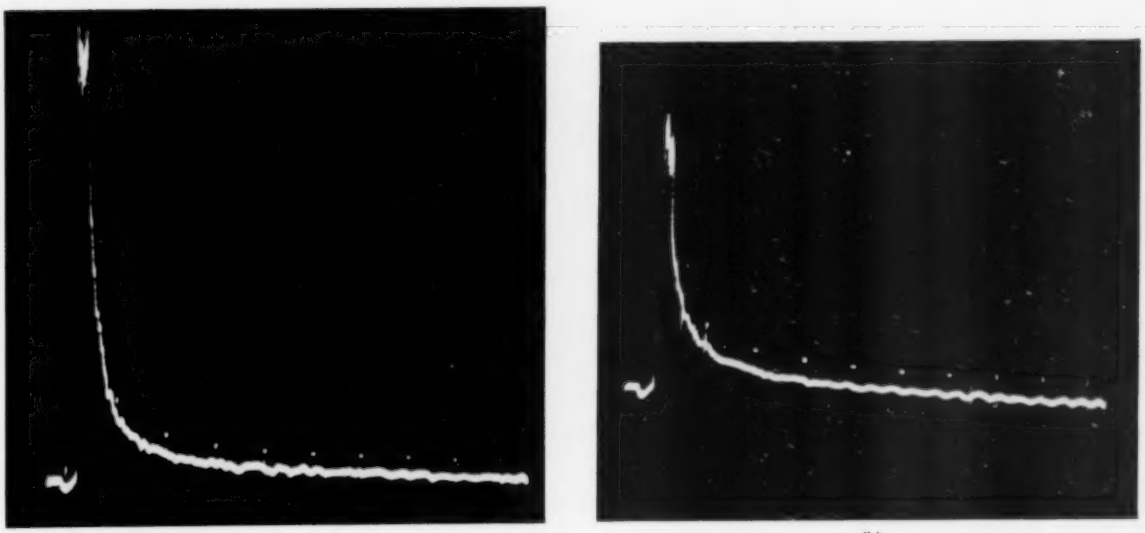


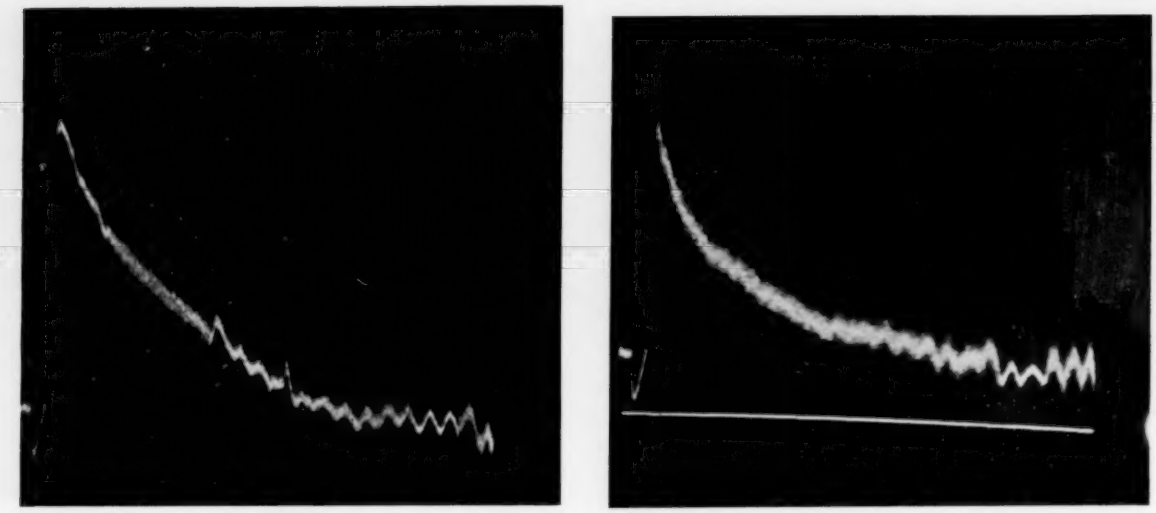
(a) (b)

FIG. 4.—Oscillograms of decay of current to channel after injection of 1 mA of H_3^+ ions of energy 100 keV. Sweep time 3 msec. (a) $p = 5 \times 10^{-7}$ mm Hg, (b) $p = 1{\cdot}4 \times 10^{-7}$ mm Hg.



(a) (b)

FIG. 6.—Oscillograms of neutrals-detector current after injection of 1·5 mA of H_3^+ ions of energy 100 keV. Preamplifier input unshorted 0·5 msec before arc extinction. (a) $p = 1{\cdot}4 \times 10^{-7}$ mm Hg, (b) $p = 9 \times 10^{-8}$ mm Hg.



(a) (b)

FIG. 8.—Oscillograms of neutrals-detector current after injection of 0·5 mA of H_2^+ ions of energy 100 keV. Preamplifier input unshorted (a) 1·5 msec, (b) 0·7 msec after arc extinction. (a) $p = 2{\cdot}5 \times 10^{-8}$ mm Hg, (b) $p = 5 \times 10^{-8}$ mm Hg. Sweep time 10 msec.

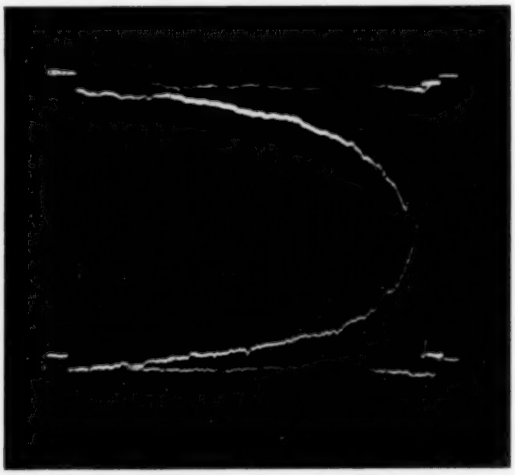


FIG. 10.—Oscillograms of currents with neutrals detectors placed in various parts of the chamber. Recorded simultaneously on a two-beam oscillograph after injection of 0·6 mA of H_2^+ ions of energy 200 keV. Preamplifier input unshorted 1·5 msec after arc extinction.

Pressure 1×10^{-7} mm mercury.

Sweep time 30 msec.

cm². This cross-section and the quantity a can be determined from the relative height of the step. For, if the destruction of current in the gas before the first collision is neglected (as is permissible for a sufficiently high vacuum), then the current $\Delta J_1 = J_0\,(1 - a)$ goes to the channel when the beam first approaches it. During the time corresponding to a drift angle $\sim 2\pi$, the current J_1 decreases, owing to destruction in the gas, by a factor $\exp(n_0\sigma x)$, where $x = \tau_{2\pi} v$. Hence the current to the channel at the second approach is increased by $\Delta J_2 = aJ_0(1 - a)\exp(-n_0\sigma x)$, and at the kth approach by $\Delta J_k = J_0(1 - a)[a \exp(-n_0\sigma x)]^{k-1}$. The sum of all ΔJ_k gives the total current to the channel in a steady state,

$$Q_0 = \sum_k \Delta J_k = J_0(1 - a)\,\frac{1}{1 - a\exp(-n_0\sigma x)}.$$

The current to the channel immediately after cutting off the beam after the first reflection is

$$Q_1 = Q_0 - \Delta J_1 = J_0(1 - a)\,\frac{a\exp(-n_0\sigma x)}{1 - a\exp(-n_0\sigma x)}.$$

Hence we find that the current passing the channel at the first reflection is, in a very high vacuum,

$$J_1 = (Q_1/Q_0)J_0 \exp(n_0\sigma x), \qquad (1)$$

and the residual gas density and the relative step height on the oscillogram are related by

$$Q_1/Q_0 = a\exp(-n_0\sigma x). \qquad (2)$$

From this relation, using two oscillograms taken at different pressures, we can find the total-loss cross-section in the gas and the fraction of the current which passes the channel at the first reflection.

The mean cross-section thus obtained is $1{\cdot}6 \times 10^{-15}$ cm², in good agreement with the value obtained from the exponential decrease. At first sight it appears that a considerable error may occur because one oscillogram is taken at 5×10^{-7} mm mercury pressure, where there is an appreciable contribution from atomic ions formed in the 'shadow' of the channel. A more detailed analysis shows, however, that the contribution of atomic ions must lead to an upward displacement of the current levels in the steady state and of the first step, with no important change in either the slope of the exponential part of the oscillograms or the relative height of the step. The reason is that the drift velocity of the atomic ions is only half that of the molecular ions. With such a drift velocity, our measurements show that more than 80 per cent of the ions moving in the shadow of the channel are destroyed at the first passage. Since the atomic ions are formed only by the dissociation of molecular ions, the mean lifetime of ions formed in the shadow cannot exceed the time taken by molecular ions to drift through an angle $\sim 2\pi$. The atomic ions as it were mask the destruction of molecular ions in the gas, and this is equivalent to a fictitious increase in the molecular-ion density or injection current. The latter does not appear in the relations which are used to determine the cross-sections. Thus the atomic ions do not, apparently, cause any error in the results obtained. Nevertheless, the error in the absolute value of the cross-section may be considerable, since the absolute values of the pressure are determined with an accuracy no better than 50 per cent. This, however, is not important in determining the lifetime with respect to destruction by the channel.

The mean decay time of the exponential parts of three oscillograms taken at $1{\cdot}4 \times 10^{-7}$ mm mercury pressure is 230 μsec. When account is taken of destruction in the gas, using the cross-section $1{\cdot}5 \times 10^{-15}$ cm², the value obtained is 460 μsec. To this must be added a quantity somewhat exceeding $0{\cdot}5\tau_{2\pi}$, since the presence of the second bend means that the exponential part of the curve intersects the level of the first step at about $0{\cdot}5\,\tau_{2\pi}$. Thus we find that the true mean lifetime of ions which pass the channel at the first reflection, with respect to destruction by the channel, is 530 μsec. The fraction of these ions among all those injected is, from formula (2), about 75 per cent, and the mean lifetime, taking into account the total injection current, is about 400 μsec. The corresponding path lengths traversed before destruction by the channel are 1·6 and 1·2 km. Preliminary measurements of the mean lifetime of molecular ions of energy 200 keV have given about the same value.

The lifetime of molecular ions may also be determined from the curves of decay of the current measured by the detectors of the neutrals formed by charge transfer of fast ions.

Figure 6 shows two oscillograms of the fast-neutral detector current, taken at two different pressures of the residual gas. In Fig. 7 these oscillograms are plotted on a semi-logarithmic scale and resolved into two components. The fast component of the current is due to the flux of neutrals formed by charge exchange of the molecular ions and some of the atomic ions, whose lifetime is determined mainly by destruction by the channel. This current component may be written in terms of the mean ion densities* and their

* We shall use the term 'ion density' to denote the ion density averaged over the volume: $n_i = (1/V)\int n_{i1}(r,\phi,z)dV$, where n_{i1} is the local ion density, whose distribution in the volume V is fairly complicated.

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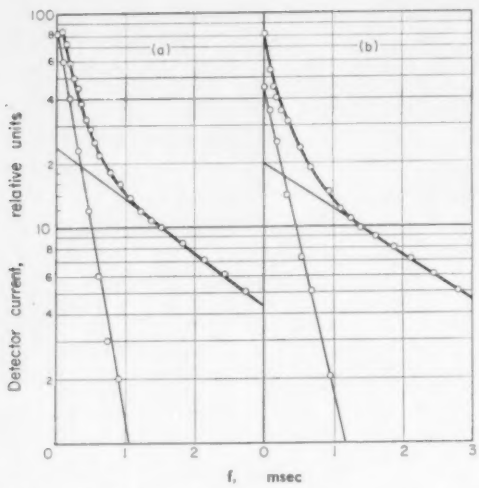

FIG. 7.—Oscillograms of neutrals-detector current on semi-logarithmic scale and their resolution into two components.
(a) $p = 1{\cdot}4 \times 10^{-7}$ mm Hg, $\tau_1 = 1700$ μsec, $\tau_2 = 250$ μsec;
(b) $p = 9 \times 10^{-8}$ mm Hg, $\tau_1 = 2000$ μsec, $\tau_2 = 300$ μsec.

cross-section for interaction with the residual gas:

$$I_f = \frac{VS}{S}\left(\kappa_1 n_1' \sum_i n_{i0}\sigma_{in1} v + \kappa_1 n_2 \sum_i n_{i0}\sigma_{np} v + \right.$$
$$\left. + \kappa_2 n_2 \sum_i n_{i0}\sigma_{in2} v\right), \quad (3)$$

where κ_1 and κ_2 are the secondary-emission coefficients of the detector material under the action of the fast neutrals, σ_{in1} and σ_{in2} the cross-sections for charge exchange between the atomic and molecular ions and the ith component of the residual gas, n_1' and n_2 the ion densities, v the velocity of the neutrals, σ_{np} the cross-section for the process $H_2^+ \rightarrow H + H^+$, n_{i0} the density of the ith component of the residual gas, V the volume occupied by the plasma, S its lateral surface area and s the detector surface area. The first term is due to charge exchange by atomic ions formed in the shadow of the channel (density n_1'), the second to the flux of atoms formed by dissociation of molecular ions (density n_2), and the third to the charge exchange by molecular ions.

The ion densities may be expressed in terms of the current injected per unit volume of the trap and the lifetime of the ions:

$$n_2 = aj\tau_2, \quad n_1' = abj\tau_2 n_0 \sigma_d v \tau_1' = abj\tau_2\tau_1'/\tau_d, \quad (4)$$

where a is the fraction of molecular ions which pass the channel at the first reflection (see (2)), b the fraction of ions which dissociate in the neighbourhood of the channel and form atomic ions which are destroyed by the channel (an estimate of the geometrical conditions shows that b is about 0·35), and σ_d the cross-section for proton formation by dissociation of H_2^+ ions.

Substituting these expressions in (3) and replacing $\Sigma n_{i0}\sigma_i v$ by the reciprocals of lifetimes of ions before destruction by the corresponding process, we can write

$$I_f = \frac{saJ_0}{S}\left(\kappa_1 b \frac{\tau_2}{\tau_d}\frac{\tau_1'}{\tau_1} + \kappa_1 \frac{\tau_2}{\tau_{np}} + \kappa_2 \frac{\tau_2}{\tau_{n2}}\right), \quad (5)$$

where J_0 is the total injection current.

It has been shown above that the lifetime of atomic ions destroyed by the channel does not exceed the time for molecular ions to drift through an angle 2π, i.e. it is about 120 μsec. Hence, in a high vacuum, the first term in (5) is small in comparison with the other two. For example, at 10^{-7} mm mercury pressure the lifetime of trapped atomic ion is 2·5 msec, that of molecular ions is about 300 μsec, τ_d is about 800 μsec and the contribution of atomic ions to the fast component of the neutrals-detector current does not exceed 2 per cent. Consequently the fast component at such pressures is very nearly proportional to the density of molecular ions, and its decay time gives the mean lifetime of these ions. Since the fast component is quite strong, and the amplification chosen must be such that this component is transmitted linearly, the amplitude of deviation of the beam up to the time at which it decays is small. Hence the part of the curve which corresponds to the slow component (trapped atomic ions) is difficult to draw accurately. This difficulty, however, is easily overcome, since this part can be recorded simultaneously by means of a second group of detectors and a high-gain preamplifier whose input is unshorted 1·5 msec after the extinction of the arc (Fig. 8). The slopes of the curves in Fig. 6 were checked in this way.

Table 1 shows the mean lifetimes of molecular ions as obtained from a series of oscillograms recorded at two pressures, and the resulting lifetime and path length of molecular ions with respect to destruction by the channel.

The lifetimes shown in Table 1 are, of course, the mean lifetimes of ions which pass the channel at the first reflection. The results are in good agreement with those obtained by measuring the current to the channel.

TABLE 1

P (mm Hg)	τ_2 (μsec)	τ_k (μsec)	L (km)
$1{\cdot}4 \times 10^{-7}$	250	550	1·65
9×10^{-8}	300	550	1·65

4. THE LIFETIME OF TRAPPED ATOMIC IONS

To measure the lifetime of atomic ions which are trapped by the magnetic field of OGRA and which do not strike the magnetic channel, the input resistance of the preamplifier is switched in at 0·7 to 3 msec after the source arc is quenched. In this time the molecular ions are almost entirely destroyed, and therefore only the slow component of the neutrals-detector current is recorded (Fig. 8); it depends on the atomic ion density, the residual gas density and the charge-exchange cross-section, as follows:

$$I_s = \frac{Vs}{S} n_1 \sum_i n_{i0}\, \sigma_{in1} v = \frac{Vs}{S} \frac{n_1}{\tau_1}. \quad (6)$$

The density of trapped atomic ions can be expressed in terms of the injection current, the lifetimes τ_2 and τ_1 of molecular and atomic ions, and the coefficients a and b defined in Section 3 (see formulae (2) and (4)):

$$n_1 = a(1-b) \frac{J}{V} \frac{\tau_2}{\tau_d} \tau_1. \quad (7)$$

From this formula we see that, by measuring both τ_2 and τ_1, we can determine with sufficient accuracy the density of trapped ions. To measure τ_2 and τ_1 simultaneously we used two groups of detectors. The total signal from one group was sent to a preamplifier whose input resistance was switched in at 1·5 msec before the arc was quenched. The input resistance of the preamplifier connected to the second group was switched in at 1·5 msec after the arc was quenched. The density of trapped ions may also be found by measuring the absolute flux of fast neutrals to the chamber walls and the lifetime τ_1:

$$n_1 = \frac{I_{s0}}{\kappa_1} \frac{S}{sV} \tau_1, \quad (8)$$

where I_{s0} is the value of the slow component of the neutrals-detector current, extrapolated to the time when injection ceases, and κ_1 the sensitivity of the detector, which is equal to the secondary emission coefficient of the detector surface under the action of the atomic ions. The accuracy with which the atomic-ion density is determined in this case is fixed by the uncertainty in the effective secondary-emission coefficient and in the effective surface of the detector, since the total current for each group of detectors is proportional to the fast-ion density averaged over the volume (see Section 2 and the footnote to Section 3).

(a) *The lifetime of atomic hydrogen ions as a function of the gas pressure in the chamber.* The existence of mechanisms of the loss of particles from the trap, other than charge exchange with the residual gas, can be seen from the way in which $1/\tau_1$ depends on the residual gas pressure p, provided that the composition of this gas does not vary during the experiment. In this case the function $1/\tau_1 = \sum_i n_{i0}\, \sigma_{in1}\, v = F(p)$ is linear and passes through the origin when $p = 0$ if there are no mechanisms of particle loss which do not involve the gas. If there are such mechanisms, then $1/\tau_1 = \text{constant} + \sum_i n_{i0}\, \sigma_{in1}\, v$.

Unfortunately the composition of the residual gas in OGRA depends on a number of factors which are difficult to control. The chief of these are the following.

(i) The state of the surface of the titanium layer which is applied by evaporation on the part of the chamber surface beyond the stoppers, in order to obtain a high vacuum. The rate of evacuation of the various constituents of the gas (especially hydrogen) decreases with time, and this not only increases the mean pressure but also changes the composition of the residual gas.

(ii) The state of the surfaces which receive the neutrals formed by charge exchange of ions with the residual gas. In OGRA, all the ions injected eventually reach the interior surfaces of the apparatus. A considerable number of the molecular ions do not dissociate, and reach the magnetic channel; the remainder undergo charge exchange and, like the atomic ions which do so, reach the walls of the chamber. To obtain a high vacuum, the OGRA chamber is baked at 400°C. The composition of the gas expelled from the walls by fast neutrals varies with the time which has elapsed since the last heating.

Thus, if some particular gas is admitted, the dependence of $1/\tau_1$ on the pressure of that gas will be linear only if the pressure and composition of the residual gas remain unchanged. The extrapolation of this dependence to the region of extreme vacuum (where $1/\tau_1$ depends on the residual gas, and the slope of the curve may be different, depending on its composition) is not permissible and can not be used to determine the lifetime with respect to losses other than destruction in the gas. Figure 9 shows $1/\tau_1$ as a function of the pressure of helium and argon admitted to the chamber at a low pressure ($2{\cdot}5 \times 10^{-8}$ mm mercury) of the residual gas*. 0·5 mA of molecular ions H_2^+ of energy 100 keV were injected. At this current the residual gas pressure and composition vary fairly slowly, and the experimental points therefore lie fairly well on a straight line.

Figure 9 also shows the lifetimes measured at various pressures of the residual gas. At high pressures

* See *Note added in proof*, p. 114.

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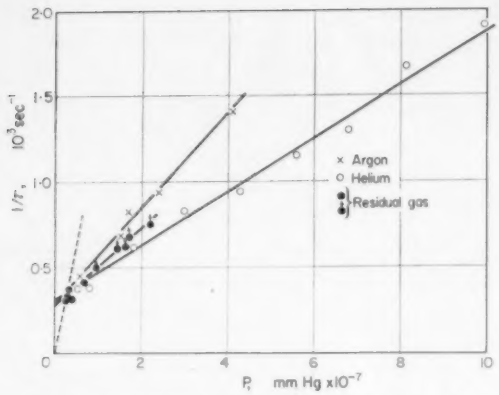

FIG. 9.—$1/\tau_1$ for atomic ions of energy 50 keV as a function of the pressure of residual gas, argon and helium. The points marked with arrows were obtained with an injection current of between 12 and 20 mA, the remainder with a current of 0·5 mA. The charge-exchange cross-sections obtained from the slopes of the lines are: argon $3{\cdot}9 \times 10^{-16}$ cm², helium $1{\cdot}4 \times 10^{-16}$ cm², residual gas $2{\cdot}2 \times 10^{-16}$ cm² per molecule.

(1 to 2 $\times$ 10^{-7} mm mercury), the points lie fairly well on a straight line, since they were obtained in a single series of measurements immediately after the deposition of the titanium. The pressure increased largely because of the increase in the injection current during the series from 10 to 20 mA. The increase in pressure apparently occurred mainly because of the increase in the flow of hydrogen through the channel and the entry of gas from the walls resulting from the high flux of neutrals to the walls from charge exchange at high current. The mean charge-exchange cross-section, as determined from the slope of the curve, confirms this supposition, since the value obtained is close to the cross-section given in the literature (ALLISON, 1958; IL'IN, 1959), for charge exchange of H^+ ions with hydrogen molecules.

At lower pressures the scatter of the points is considerable. This may be explained by the fact that these points were measured in different series of measurements under different conditions, the difference being mainly in the time elapsed since the baking of the chamber and the deposition of the titanium. The composition of the residual gas may also be different for different points. All these points, like the curves for argon and helium, were obtained with a current of 0·5 mA. The mean lifetime of H^+ ions of energy 50 keV (injection of H_2^+ ions of energy 100 keV) is 3·2 msec at the lowest pressure that could be obtained with a current of 0·5 mA.

The curve showing $1/\tau_1$ as a function of pressure does not pass through the origin in any of the three cases. This is not particularly significant, however, as has been explained above. The form of the curve in the extrapolation region is determined by the composition of the gas in that range of pressures. In our conditions (rubber seals and titanium degassing) the heavy hydrocarbons, which may have large charge-exchange cross-sections, become of great importance at extreme vacuum.

To estimate the effect of possible mechanisms of particle loss other than charge exchange, the composition of the gas was measured at a pressure 5×10^{-8} mm mercury by means of a chronotron at the same time as the ion lifetime was measured. Unfortunately, the accuracy of measurement of the gas composition in this experiment was not high. Table 2 shows the measured composition of the gas and the charge-exchange cross-sections of the various components.

TABLE 2

Component	p (mm Hg)	n_0 (cm^{-3})	σ_{n_1} (cm²)
Hydrogen	$4{\cdot}0 \times 10^{-8}$	$1{\cdot}42 \times 10^{8}$	$1{\cdot}5 \times 10^{-16}$
Nitrogen	$6{\cdot}4 \times 10^{-9}$	$2{\cdot}3 \times 10^{8}$	$3{\cdot}6 \times 10^{-16}$
Methane	1×10^{-8}	$3{\cdot}6 \times 10^{8}$	4×10^{-16}
Argon	6×10^{-9}	$2{\cdot}1 \times 10^{8}$	$4{\cdot}4 \times 10^{-16}$

The value of $1/\tau_1$ calculated from Table 2 is 0·18 msec^{-1}; that measured from oscillograms of the neutrals-detector current is 0·38 msec^{-1}. If we assume that the difference is due to losses other than in the residual gas, the lifetime with respect to these losses is $\tau_i = 5$ msec. This estimate is a lower limit (i.e. $\tau_i > 5$ msec), since we have ignored the heavier constituents $(CH_k)_n$, whose charge exchange cross-sections may be large.

Six months after the measurements described above, we measured the lifetime of atomic ions, with molecular ions of energy 200 keV injected. During this interval a leak had appeared in the chamber and when the beam was admitted to the chamber it was not possible to reduce the pressure below 1×10^{-7} mm mercury. Figure 10 shows specimens of oscillograms, and Fig. 11 shows $1/\tau_1$ as a function of the pressure of helium admitted to the chamber at a residual gas pressure of $1{\cdot}6 \times 10^{-7}$ mm mercury and with an injection of 0·6 mA of molecular ions of energy 200 keV. The black circles correspond to various pressures of the residual gas, the white circles to the admission of helium, and the crosses to the evacuation of helium. The slope of the helium curve gives the cross-section for charge exchange of H^+ ions of energy 100 keV, namely $2{\cdot}7 \times 10^{-17}$ cm², which is in good agreement with the value 3×10^{-17} cm² given in ALLISON's survey (ALLISON, 1958). When extrapolated, the helium curve does not pass through the origin. On the other hand, it is very difficult to draw any straight line

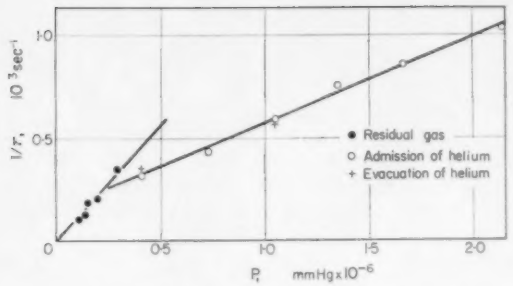

FIG. 11.—$1/\tau_1$ for atomic ions of energy 100 keV as a function of the pressure of residual gas and helium. The slope of the line for helium corresponds to a cross-section $2{\cdot}7 \times 10^{-17}$ cm², and that for the residual gas to 9×10^{-17} cm².

through the points for the residual gas which does not pass through the origin. The maximum lifetime obtained at a pressure 1×10^{-7} mm mercury is $9{\cdot}3 \pm 0{\cdot}9$ msec, the error given being the root-mean-square scatter of the lifetimes obtained from six oscillograms taken in succession in the course of a few minutes. At high pressures this error is still smaller, and for the helium curve it does not exceed 5 per cent. However, the scatter of points about the line is considerably greater on account of uncontrollable changes in the vacuum conditions. The pressure was measured with an Alpert-type ionization gauge, whose accuracy is at best 50 per cent since the composition of the residual gas and the distribution of pressure along the chamber axis were not monitored. Nevertheless, the agreement within ± 20 per cent between our values for the charge-exchange cross-section of H^+ ions of 50 keV with helium and argon and the literature values, and the corresponding figure of ± 10 per cent for 100-keV protons with helium, show that the pressures measured at the time when these curves were recorded are not far from the true values. This does not apply, of course, to the points obtained at extreme vacuum. Here the composition of the gas must be known in order to determine the mean pressure.

(b) *The density of accumulated fast ions.* The measurements whose results are given above were made at a low ion injection current, and therefore at low densities of trapped ions. The density of trapped atomic ions was about 1×10^5 cm^{-3} for molecular ions injected with energy 100 keV, and 3×10^5 cm^{-3} for energy 200 keV. At these densities the particles may be regarded as isolated, and so the maximum lifetimes obtained correspond to the containment of a single particle in a field having the configuration shown in Fig. 2. The points indicated by arrows in Fig. 9 were obtained with the injection of 12 to 20 mA of molecular ions of energy 100 keV. The measured density (4 to 6×10^6 cm^{-3}) of atomic ions with energy 50 keV is proportional to the current of injected ions and independent of the pressure. The fact that these points lie on a line whose slope corresponds approximately to the cross-section for charge exchange of atomic ions with hydrogen indicates that, at density 6×10^6 cm^{-3}, the lifetime of the ions is entirely determined by losses in the gas. No systematic measurements have been made of the lifetime for a large current of molecular ions injected with energy 200 keV. In this case also, however, a few oscillograms taken at an injection current of 20 mA show that at density $\sim 1{\cdot}5 \times 10^7$ cm^{-3} the lifetime is determined only by charge exchange in the gas, although theoretical calculations show that at these densities instabilities may appear.

(c) *The lifetime of trapped ions as a function of their energy.* A measurement of the lifetime of trapped ions as a function of their energy, in magnetic fields which ensure that the ions move in identical paths, has shown that (as we should expect) the relation obtained corresponds to the ion-energy dependences of charge-exchange cross-sections given in the literature. This agreement has been obtained on measuring the energy dependence of the lifetime of trapped ions at partial pressures of helium $1{\cdot}2 \times 10^{-6}$ mm Hg and of the residual gas 2×10^{-7} mm, between 15 and 50 keV (with molecular ions injected at energies from 30 to 100 keV).

A comparison of the lifetimes of H^+ ions of energies 50 and 100 keV shows that the ratio of lifetimes (4·5) is approximately the same as the ratio of cross-sections for nitrogen (3·5) as given by ALLISON (1958). The discrepancy between these ratios, like that between the cross-sections for charge exchange of H^+ ions of energy 100 keV with the residual gas (9×10^{-17} cm²; see Fig. 11) and the published values for nitrogen (5×10^{-17} cm²) and oxygen ($5{\cdot}5 \times 10^{-17}$ cm²) may be explained by a more complex composition of the residual gas.

5. CONCLUSION

In designing the OGRA apparatus, its geometrical dimensions were chosen such that the mean path L traversed by molecular ions before their destruction by the channel should be 1 km (KURCHATOV, 1958). Measurements show that, with the magnetic field configuration chosen, the value of L is about 50 per cent greater than this. Twenty-five per cent of the injected current is destroyed by the channel after the first reflection from the ends, and does not assist in the build-up process.

Measurements of the lifetime of atomic ions have shown that in OGRA there is no mechanism of particle loss, other than charge exchange in the gas, up to

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characteristic times of the order of 10 msec. In the conditions used in these experiments, atomic ions of energy 50 keV undergo about 300 reflections from the stoppers, and those of 100 keV about 1500 reflections. If a dense plasma is to be built up, it is necessary that the ions should be retained in the trap for about 10 seconds and should undergo 10^6 reflections. At present the greatest measured density of atomic ions is $\sim 1{\cdot}5 \times 10^7\ \text{cm}^{-3}$ for an injection of 20 mA of molecular ions of energy 200 keV.

Measurements of the lifetimes of atomic and molecular ions as functions of the pressure of the gases in the chamber make it possible to determine the cross-sections for charge exchange of atomic ions with these gases and the total cross-sections for loss of molecular ions in the gases. Within the limits of error of the measurements of pressure, the cross-sections obtained in this work are in agreement with the values in the literature.

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Note added in proof: Later measurements have shown that the residual-gas pressure was not accurately determined in the experiments with injection of 100-keV molecular ions. These measurements gave a pressure drop of 1·0 to $1{\cdot}5 \times 10^{-7}$ mm mercury between the ends, where the manometers were placed and the titanium was deposited, and the centre of the chamber. For this reason the origin in Fig. 9 must be shifted to the left by 1·0 to $1{\cdot}5 \times 10^{-7}$ mm mercury. The slope of the curves for helium and argon is unchanged, since the rate of evacuation of helium and argon is small and their pressures are approximately the same throughout the chamber. Thus the possible additional losses of H^+ ions are not so great as is shown in Fig. 9.

No correction is necessary in Fig. 11, since the pressure values given there are the averages between the centre and ends of the chamber.

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POLARISATION ROTATOIRE MAGNETIQUE DANS LES PLASMAS—APPLICATION A LA MESURE DE LA DENSITE ELECTRONIQUE

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Résumé—On étudie dans certaines conditions particulières et pour des valeurs étendues des paramètres:

$10^8\ e/m^3 <$ densité électronique $< 10^{24}\ e/m^3$
10^{-4} gauss $<$ champ magnétique $< 10^7$ gauss
10^7 c/s $<$ fréquence de sondage $< 10^{15}$ c/s

la rotation d'une onde polarisée linéairement, se propageant parallèlement à la direction du champ magnétique.

Les rotations théoriques calculées sont présentées sous forme de réseaux de courbes.

Des dispositifs expérimentaux pour la mesure des indices droit et gauche et de leur différence, donnant la rotation du vecteur électrique de l'onde résultante émergente sont ensuite décrits.

Un aperçu des problèmes technologiques posés par cette méthode de mesure est donné, avec le principe de fonctionnement des montages.

Dans le paragraphe 3 quelques résultats sont présentés.

Abstract—The rotation of a linearly-polarized wave, propagating parallel to the direction of a static magnetic field, is studied under certain restrictive conditions and for the following range of parameters:

$10^8\ e/m^3 <$ electronic density $< 10^{24}\ e/m^3$
10^{-4} gauss $<$ magnetic field $< 10^7$ gauss
10^7 c/s $<$ exploring frequency $< 10^{15}$ c/s.

Theoretical results are given by a family of curves.

Two experimental set-ups are described; one measuring the right-hand and left-hand indices, the other their difference (i.e. the rotation of the electric field vector of the emerging wave).

Some experimental results are given in Section 3.

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INTRODUCTION

LE phénomène de polarisation rotatoire magnétique découvert en 1845 par Faraday est bien connu en optique. Un faisceau lumineux à polarisation rectiligne se propageant parallèlement à la direction du champ magnétique appliqué sur un milieu anisotrope voit son plan de polarisation tourner au cours de la propagation.

Cet effet Faraday a été amplement étudié par les opticiens et les électroniciens et appliqué dans de nombreux domaines. Citons pour mémoire les obturateurs rapides magnéto optiques en photographie, le dosage des sucres en saccharimétrie, les gyrateurs à ferrite et les lignes unidirectionnelles dans la propagation guidée hyper-fréquence etc. . . Cette dernière application repose sur la variation de la perméabilité de la substance en fonction du champ magnétique appliqué.

On retrouve une propriété analogue dans les gaz ionisés dont la constante diélectrique est aussi modifiée par un champ magnétique. Ainsi dans le cas d'une onde plane transversale se propageant dans le sens du champ magnétique on constate la biréfringence.

Le phénomène de polarisation rotatoire magnétique dans les gaz ionisés existe aussi dans la nature: Appleton a mis en évidence en 1931 que l'ionosphère devient anisotrope sous l'influence du champ magnétique terrestre.

Le même phénomène a été étudié dans les plasmas aux Etats-Unis par GOLDSTEIN, GILDEN et ETTER (1953) et par SUHL et WALKER (1954), et en France par BONNET, MATRICON et ROUBINE (1955). Dans une application en 1956 de BROWNE, EVANS, HARGREAVES et MURRAY (1956), spécialistes de l'ionosphère, ont utilisé l'effet Faraday pour déduire de leurs expériences d'échos radar sur la lune une mesure de la densité électronique globale de l'ionosphère. Des mesures semblables ont été réalisées à partir des ondes électromagnétiques polarisées linéairement émises par des fusées ou des satellites artificiels (DANIELS, 1958).

Enfin dans une étude de la même année GOLDSTEIN (1958) étudie la propagation de microondes dans un guide cylindrique traversé coaxialement par un tube contenant du plasma et donne des résultats expérimentaux sur la rotation du plan de polarisation dans cette propagation guidée.

Cette revue bibliographique est probablement incomplète mais à notre connaissance, aucun des auteurs cités n'a étudié l'utilisation de l'effet Faraday pour la mesure de la densité électronique des plasmas contenus dans une bouteille magnétique.

Nous nous sommes proposé de faire une étude systématique de la rotation du vecteur champ électrique d'une onde polarisée linéairement se propageant dans un plasma dans certaines conditions particulières afin de déterminer les fréquences de sondage optimum, de discuter les résultats, de décrire les dispositifs expérimentaux réalisés.

1. ETUDE THEORIQUE

Hypothèses

1. Le plasma remplit une colonne de longueur l, de diamètre D, ($D < l$) et de densité uniforme (Fig. 1) placée dans un champ magnétique B statique parallèle á un axe Oz. La densité électronique est nulle à l'extérieur de la colonne et en dehors de l'intervalle $z = 0$, $z = l$.

2. L'onde incidente est une onde plane qui se propage suivant Oz, c'est à dire parallèlement à la direction du champ magnétique. Elle est polarisée linéairement et est parallèle à la surface de séparation du milieu ionisé et du milieu extérieur. Sa longueur d'onde λ est beaucoup plus petite que la dimension transversale D du plasma.

3. On ne tient pas compte du mouvement des ions et on néglige le rayon de Larmor des électrons vis à vis de la longueur d'onde.

4. La fréquence de collision est supposée petite devant la fréquence de l'onde.

Calcul de la rotation du plan de polarisation

L'onde incidente polarisée linéairement se dédouble en deux ondes circulaires tournant en sens inverse. Le plasma se comporte comme un milieu possédant un indice droit n_d pour l'onde droite et un indice gauche n_g pour l'onde gauche.

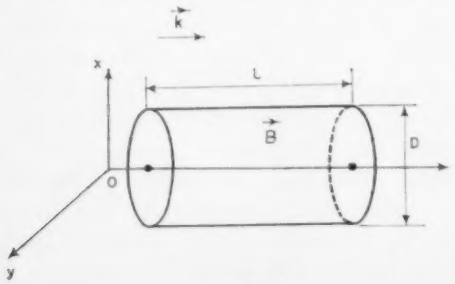

FIG. 1.

$$n_{d,g} = \left(1 - \frac{X}{1 \pm Y_e - jZ}\right)^{1/2} \qquad (1)$$

avec:

$$X = \frac{\omega_p^2}{\omega^2} \qquad Ye = \frac{\omega_{ce}}{\omega} \qquad Z = \frac{\nu}{\omega}$$

$\frac{\omega_p}{2\pi}$ = fréquence de plasma.

$\frac{\omega}{2\pi}$ = fréquence de sondage.

$\frac{\omega_e}{2\pi}$ = fréquence cyclotron électronique.

ν = fréquence de collision.

Si la fréquence de collision est très faible par rapport aux autres fréquences caractéristiques du plasma, la polarisation de l'onde reste sensiblement linéaire et l'angle dont tourne le plan de polarisation pendant la propagation de l'onde est lié au chemin parcouru z, par la relation:

$$\varphi = \mathrm{Re}\left\{\frac{\omega}{2c}\int_0^z (n_g - n_d)\, dz\right\}. \qquad (2)$$

En toute rigueur, les deux rayons droit et gauche subissent une atténuation différente (dichroisme) et l'onde émergente est elliptique. L'évaluation de l'ellipticité de l'onde permet de déterminer l'absorption des deux rayons, tandis que l'orientation du grand axe de l'ellipse nous renseigne sur l'angle φ.

Nous avons calculé l'angle φ à l'aide d'une machine à calculer arithmétique en partant des formules (1) et (2). Les résultats de l'étude sont valables tant que la température électronique reste inférieure à 1 keV. Pour les températures plus élevées, il faut tenir compte d'effets dimensionnels liés au rayon de Larmor des électrons, comme certains auteurs l'ont montré (BERNSTEIN, 1959; DRUMMOND, 1958; SITENKO et STEPANOV, 1956).

La tabulation a été établie avec une précision de sept décimales. Ses limites $10^{-6} < X < 10^5$, $10^{-5} < Y < 10^5$ ont été choisies de telle sorte qu'elles recouvrent toutes les densités électroniques et les grandeurs de champ magnétique que l'on rencontre dans les plasmas de laboratoire et de la nature (ionosphère). Les résultats sont présentés sous forme de réseaux de courbes qui permettent de calculer les rotations attendues pour une fréquence et une densité donnée; ou de calculer le nombre de tours du vecteur champ électrique de l'onde résultante émergente lorsque la densité du plasma évolue entre des limites déterminées.

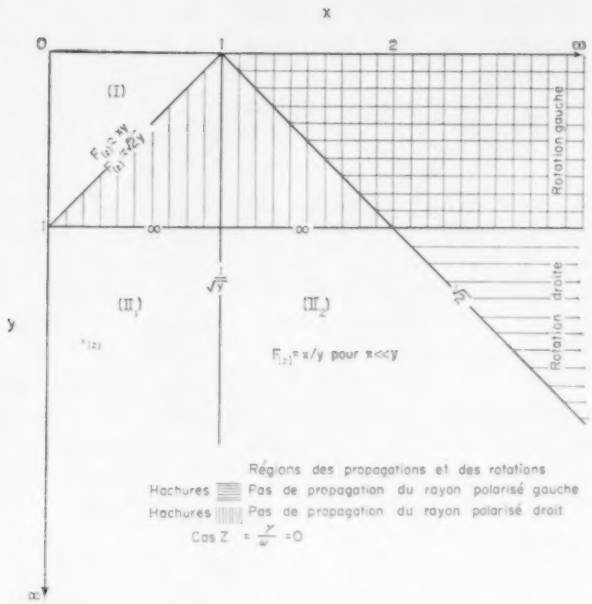

FIG. 2.—Régions de validité des diverses approximations pour F(3).

Ces réseaux représentent dans le système de coordonnées réduites (X, Y) les variations des indices gauche (réseau I, cas $Z = 0$) et droit (réseau II cas $Z = 0$) et de leur différence (réseaux III, IV, V cas $Z = 0, 10^{-2}, 10^{-4}$).

Régions des propagations et des rotations

Les rayons polarisés gauche et droit se propagent dans les régions définies respectivement par les droites $1 + Y - X = 0$ et [$1 - Y - X = 0$, $Y = 1$]—(aires non hachurées dans la Fig. 2).

On voit sur cette même figure qu'il y a rotation dans les régions I et II où les deux rayons se propagent simultanément.

Les propriétés de ces deux domaines sont toutefois différentes.

Dans la région I, la rotation est dans le même sens que l'onde gauche et les densités électroniques mesurables sont inférieures à la densité critique. La densité critique étant celle pour laquelle la fréquence plasma est égale à la fréquence de l'onde de sondage. Dans la région II, la rotation est dans le même sens que l'onde droite. Il est de plus possible de mesurer des densités d'autant plus grandes que la densité critique, que le champ magnétique est plus intense.

Approximations et formules asymptologues

En posant $f(z) = n_g - n_d$ en peut indiquer les valeurs approchées dans les deux régions:

$$\text{pour } \begin{cases} X < 1 \\ Y < 1 \end{cases} F(z) \approx XY \text{ région I}$$

$$\text{pour } Y \gg F(z) \approx \frac{X}{Y} \text{ région II.}$$

On trouve également des valeurs asymptotiques

$n_g - n_d \simeq 2Y$ sur la droite $1 - Y - X = 0$

$n_g - n_d \simeq \infty$ sur la droite $Y = 0$

$n_g - n_d \simeq \frac{1}{Y}$ sur la droite $X = 1$

$n_g - n_d \simeq \sqrt{2}$ sur la droite $1 + Y - X = 0$.

Toutes ces valeurs sont représentées sur la Fig. 2. On en déduit dans le plan (N, B) un diagramme analogue et des formules asymptotiques pour les rotations exprimées directement en tour/m (Fig. 3).

Famille de courbes déduites

Le réseau III est converti en six autres par l'emploi des relations de définition de X et Y:

$N(e/m^3) = 1{\cdot}2419f^2 \times 10^{10}\, X$ où f est en Mcs

$B(\text{gauss}) = 0{\cdot}3576\, Yf$

$\varphi(\text{tour}/m) = 5/3 \times 10^{-3} f(n_g - n_d)$.

En attribuant une valeur déterminée à f, on obtient les courbes de la rotation en fonction de la densité du plasma et du champ magnétique. Nous n'avons présenté dans cet article que les courbes correspondant aux fréquences de sondages suivantes:

$f = 10$ et 5×10^4 Mcs, pour la H.F. et les U.H.F.

$f = 10^{15}$ cs pour le visible.

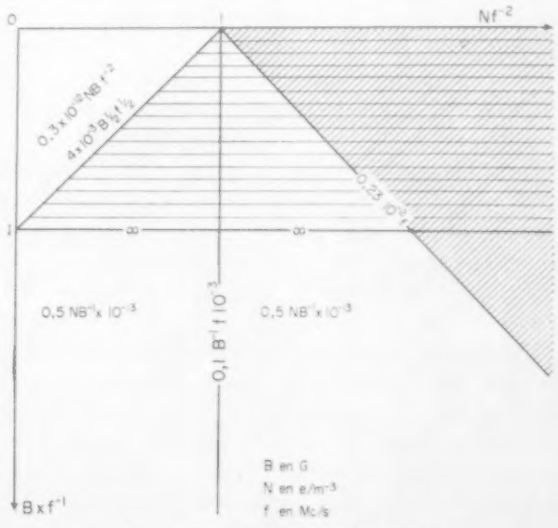

FIG. 3.—Approximations pour les rotations.

On a traité un cas de plasma peu dense—réseau VI-(ionosphère ou décharge H.F.) et un autre de plasma très dense—réseaux VII et (effet Faraday en optique) pour montrer les limites d'applicabilité des réseaux universels.

Il est évident que dans le cas de l'utilisation de rayon de sondage lumineux, l'approximation est excellente, car d'une part la fréquence de collision est plus petite que la fréquence de travail, et d'autre part la longueur d'onde elle-même est beaucoup plus petite que la dimension caractéristique du plasma. On peut ainsi obtenir dans ce cas une très bonne focalisation du pinceau de sondage et explorer le plasma suivant une zone très limitée. On perd cet avantage en U.H.F., puisque l'ouverture du faisceau, par diffraction due au cornet, est plus grande. La diffraction peut quand même être réduite par l'emploi simultané de cornets et de lentilles U.H.F.

Conclusions

Par son principe même, la méthode ne peut servir que pour la mesure de la densité globale longitudinale, c'est-à-dire dans la direction du champ magnétique. Elle constitute une méthode complémentaire aux procédés transversaux (Interférométrie et méthode par reflexion totale). Elle présente sur la méthode interférométrique U.H.F. l'avantage de pouvoir servir à la mesure de la densité électronique dans un plus large intervalle. de densités pour une même fréquence de sondage.

L'usage des courbes permet, connaissant les limites approximatives de variation de la densité électronique d'un plasma, de choisir la fréquence optimale de travail et de prévoir la valeur de la rotation qui en résulte.

Signalons que l'angle de la rotation varie brusquement au voisinage de la fréquence cyclotron. L'application de la méthode devient alors douteuse.

L'examen des réseaux de courbes montre enfin que les rotation en tours par mètre sont très faibles pour les plasmas peu denses à la fréquence de sondage de 10 Mcs. Elles sont quand même mesurables dans l'ionosphère à cause des grandes longueurs de parcours. Pour les plasmas de laboratoire, les fréquences supérieures à 10 kMcs semblent les plus intéressantes.

Enfin, l'existence d'une région de propagation et de rotation dans le cas de champs très intenses $\omega_c \gg \omega_p$, pour densités telles que $\omega_p \gg \omega_{\text{sondage}}$, trouve son application naturelle dans l'étude des plasmas très denses obtenus au cours d'une compression. Si la densité électronique est connue par ailleurs, la méthode semble pouvoir permettre la mesure du champ magnétique interne piégé dans le plasma. Elle se substituera avantageusement à l'emploi de sonde magnétique.

2. DESCRIPTION DU DISPOSITIF DE MESURE

Le schéma du montage optique est représenté par la Fig. 1.

Pour se rapprocher le plus possible des conditions aux limites définies dans l'exposé théorique, il faut que le faisceau électromagnétique soit concentré à l'intérieur du cylindre de plasma. Une première lentille convergente concentre le faisceau électromagnétique cylindrique issu de la lentille émettrice au milieu du cylindre de plasma, une deuxième lentille transforme le faisceau émergent en un faisceau parallèle attaquant la lentille réceptrice. Le diamètre de la tache centrale de diffraction doit être petit par rapport au diamètre du cylindre de plasma.*

Nous avons réalisé deux montages électroniques, l'un servant à la mesure de la rotation du plan de polarisation, l'autre à la mesure des deux indices n_d et n_g et à l'étude des absorptions des deux ondes droite et gauche.

Diagrammes fonctionnels

Le montage de mesure de la rotation du plan de polarisation est représenté par le diagramme fonctionnel II. Il se compose d'un groupe hyperfréquence et d'un groupe changeur de fréquence.

Le groupe hyperfréquence comprend:

Un klystron oscillateur fonctionnant sur une longueur d'onde de 8 mm, protégé par une ligne undirectionnelle.

Un système émetteur constitué par un ensemble cornet lentille électromagnétique.

Un système récepteur constitué par une lentille électromagnétique et un cornet muni d'un dispositif séparateur.

Le groupe changeur de fréquence comprend:

Un premier changeur de fréquence, utilisant un klystron comme oscillateur local, muni d'un amplificateur moyenne fréquence.

Un deuxième changeur de fréquence utilisant un oscillateur piloté par quartz.

Un dispositif de commutation pour la présentation des courbes expérimentales sur l'écran d'un oscilloscope.

* D'une façon générale la construction d'un dispositif d'optique hyperfréquence destiné aux plasmas que l'on réalise actuellement en laboratoire est d'autant plus facile que la longueur d'onde est petite, car les problèmes de focalisation sont simplifiés. Par contre, la puissance des oscillateurs diminue en fonction de la longueur d'onde et des difficultés apparaissent pour détecter l'énergie reçue à partir de longueurs d'onde de l'ordre du millimètre.

Le montage de mesure des indices n_d et n_g est représenté par le diagramme fonctionel III. Il comporte:

Un klystron oscillateur fonctionnant sur une longueur d'onde de 8 mm, protégé par une ligne undirectionnelle.

Un partage par Té magique en deux voies principales E_x et E_y.

La voie principale E_y comprenant elle-même deux bras D et G en parallèle. Chacun de ces deux bras possède un déphaseur et un atténuateur. Un commutateur permet la sélection du bras D ou G au choix.

La voie principale E_x comprenant un coupleur directif destiné à permettre de prélever une partie de l'énergie pour alimenter une voie secondaire R.

La voie secondaire R, qui est utilisée comme bras de référence.

Un système mélangeur M. Ce système permet de composer les champs électriques fournis par les voies principales E_x et E_y et de réaliser une onde plane polarisée circulairement. Cette onde est droite ou gauche suivant que la voie E_x utilise le bras droit ou le bras gauche.

Un système emetteur constitué par une ensemble cornet lentille.

Un système séparateur (analogue à M) permettant sélectionner une composante de l'onde circulaire de reçue.

Un Té magique permettant de faire interférer cette composante avec l'onde délivrée par le bras de référence R.

Principe de la mesure de la rotation du plan de polarisation

L'onde plane polarisée rectilignement subit à la traversée d'un milieu doué d'un pouvoir rotatoire deux effets: rotation de son plan de polarisation et atténuation égale ou inégale des deux ondes droite et gauche; dans ce second cas, l'onde émergente est polarisée elliptiquement et la rotation du plan de polarisation se traduit par la rotation du grand axe de l'ellipse.

Considérons un système d'axes perpendiculaires Ox, Oy. Le champ électrique de l'onde incidente est supposé parallèle à Ox. Le champ électrique de l'onde émergente décrit une ellipse dont le grand axe OX fait un angle 2φ avec Ox.

Si nous décomposons l'onde émergente selon ces deux axes, les composantes du champ électrique s'expriment par les relations suivantes:

$$E_x = (E_d^2 + E_g^2 + 2E_dE_g \cos 2\varphi)^{1/2}$$
$$E_y = (E_d^2 + E_g^2 - 2E_dE_g \cos 2\varphi)^{1/2}.$$

Dans le cas où le milieu ne possède pas de dichroisme, l'onde incidente polarisée rectilignement se décompose en deux ondes polarisées circulairement gauche et droite, l'atténuation que subissent les deux ondes à la traversée du milieu est la même, nous avons donc:

$$E_d = E_g = E.$$

Nous voyons que l'onde émergente est polarisée rectilignement et que son plan de polarisation a tourné de l'angle φ.

La difference des indices $n_d - n_g$ varie en fonction du temps, la courbe représentant l'une ou l'autre des composantes présentera des maxima et des minima pour des valeurs bien déterminées du déphasage. Nous obtenons les relations:

$$\left.\begin{aligned}(E_x)_{max} &= 2E\\ (E_y)_{min} &= 0\end{aligned}\right\} \text{ pour } \varphi = k\pi$$

$$\left.\begin{aligned}(E_x)_{min} &= 0\\ (E_y)_{max} &= 2E\end{aligned}\right\} \text{ pour } \varphi = (2k+1)\,\pi/2.$$

Nous avons aussi

$$E_x = E_y = E\sqrt{2} \quad \text{pour } \varphi = (2K+1)\,\frac{\pi}{4}.$$

Le principe de la mesure consistera donc à présenter sur un oscilloscope l'enveloppe des deux composantes E_x et E_y, ce qui revient à effectuer une détection d'amplitude sur chacune des composantes. La superposition sur l'écran des deux courbes détectées permettra de déterminer des points d'intersection pour lesquels $\varphi = (2K+1)\pi/4$ et des extrema pour lesquels $\varphi = K\pi$. Nous avons donc 8 points de mesure pour une rotation du plan de polarisation de 2π (c'est-à-dire les valeurs O, $\pi/4$, $\pi/2$, $3\pi/4$, . . . etc).

Dans le cas où le milieu possède du dichroisme, l'atténuation que subissent les deux ondes droite et gauche à la traversée du milieu est différente, $E_d \neq E_g$. L'onde émergente est polarisée elliptiquement la rotation du plan de polarisation se traduit par la rotation du grand axe de l'ellipse. Comme précédemment la courbe représentant l'une ou l'autre des composantes sur les axes Ox et Oy présentera des maxima et des minima pour des valeurs bien déterminées du déphasage.

Nous obtenons les relations

$$\left.\begin{aligned}(E_x)_{max} &= E_d + E_g\\ (E_y)_{min} &= E_d - E_g\end{aligned}\right\} \text{ pour } \varphi = k\pi$$

$$\left.\begin{aligned}(E_x)_{min} &= E_d - E_g\\ (E_y)_{max} &= E_d + E_g\end{aligned}\right\} \text{ pour } \varphi = (2k+1)\,\pi/2.$$

Nous avons aussi

$$E_x = E_y = (E_d + E_g)^{1/2}.$$

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Nous remarquons que les minima ne sont plus nuls; le contraste entre maxima et minima dépend de la différence des amplitudes $E_d - E_g$. Le sens de parcours de l'ellipse peut se déterminer si l'on connait les valeurs absolues de E_d et E_g.

Le principe de la mesure est le même que dans le cas précédent, mais la détermination précise des maxima et des minima deviendra d'autant plus difficile que la différence $E_d - E_g$ sera plus petite.

Principe de la mesure des indices n_d et n_g

La mesure de l'indice par rapport à l'air d'un milieu traversé par une onde plane de polarisation rectiligne ou circulaire peut se faire en comparant la vitesse de phase de cette onde à la vitesse de phase d'une onde prise comme référence; le montage utilisé est un montage interférométrique.

Pour mesurer l'indice n_d par exemple, il faut que l'onde se propageant à l'interieur du milieu soit une onde polarisée circulairement à droite. Nous produirons donc une telle onde, nous vérifierons sa polarisation; l'onde émergente du milieu sera décomposées elon deux axes perpendiculaires et l'une des deux composantes interférera avec l'onde de référence.

L'onde incidente devant être polarisée circulairement droite ou gauche, le système émetteur est précédé d'un mélangeur effectuant une addition vectorielle des deux composantes E_x et E_y. La différence de phase entre ces deux composantes est $\pi/2$ pour l'onde gauche et $3\pi/2$ pour l'onde droite. Ce réglage de la polarisation est fait à l'aide de l'ensemble récepteur et changeur de fréquence. La mesure des deux indices n_d et n_g ne nécessite pas l'utilisation du groupe changeur de fréquence ; une des deux composantes E_x ou E_y est prélevée sur le séparateur et interfére dans le Té magique avec l'onde issue du bras de référence R. Bien entendu, le système optique utilisé dans ce cas comporte une lentille émettrice isotrope.

Description du matériel

Nous décrivons ci-dessous, à titre d'exemple, les deux principaux groupes fonctionnels utilisés pour la mesure.

(a) *Groupe hyperfréquence.* L'énergie électromagnétique fournie par un klystron oscillateur fonctionnant sur une longueur d'onde de 8 mm se propage à l'intérieur d'un guide rectangulaire et attaque un système émetteur. Ce système émetteur est chargé de délivrer une onde plane polarisée rectilignement se propageant selon un faisceau pratiquement cylindrique traversant le milieu. Le système émetteur est constitué par un ensemble cornet-lentille électromagnétique. Le rapport λ/D est égal à 7/100, D étant le diamètre utile de la lentille, soit 120 mm; ceci permet d'avoir un angle d'ouverture du faisceau de l'ordre de 2°30″; le faisceau électromagnétique est donc pratiquement concentré à l'intérieur d'un cylindre de diamètre D. La polarisation de l'onde devant être rectiligne et parfaitement définie, une lentille à plaques paralléles est utilisée. Son indice est constant et égal à 0·6. Sa distance focale est de 140 mm; le coefficient de transmission en puissance T est égal à 0·85.

L'onde émergente est reçue à sa sortie du milieu par le système récepteur-séparateur. Cette onde pouvant avoir dans le cas général une polarisation elliptique, le système récepteur devra posséder une symétrie axiale. Il est constitué par une lentille a trous cylindriques de mêmes caractéristiques que la lentille émettrice.

Le role du séparateur est de matérialiser deux axes de références perpendiculaires et d'effectuer la décomposition de l'onde émergente suivant ces deux axes E_x et E_y.

Le cornet cylindrique qui suit la lentille réceptrice alimente une dérivation ayant la forme d'un Y. Dans l'une des branches on sélectionne la polarisation E_x et dans l'autre la polarisation E_y. Il suffit pour cela de placer à l'interieur du guide cylindrique constituant une branche une plaque métallique passant par l'axe du guide et parallèle à la polarisation à éliminer. Une transition effectué le passage du guide cylindrique au guide rectangulaire. Les deux composantes E_x et E_y ainsi définies subissent les mêmes actions de la part du deuxième groupe fonctionnel.

(b) *Groupe changeur de fréquence.* La fréquence de l'onde étant de l'ordre de 35 kMc, il est impossible d'effectuer l'addition vectorielle des deux composantes E_x et E_y avec un oscilloscope classique. D'autre part, la puissance des klystrons oscillateurs fonctionnant dans cette gamme de fréquence est de l'ordre de 100 MW et la puissance reçue après un parcours à l'intérieur du milieu de l'ordre du mètre est inférieure à 1 MW, ce qui entraine la nécessité d'une amplification.

La solution adoptée est un double changement de fréquence: le premier à une fréquence moyenne de 60 Mc, le deuxième à une fréquence de 10 Mc. On sait que le changement de fréquence ne modifie pas en lui-même l'information utile qui est une différence de phase, mais il peut introduire pour des raisons secondaires un déphasage supplémentaire indépendant de temps et de la forme:

$$\varphi = 2k\pi + \psi$$

avec

$$0 < \psi < 2\pi.$$

Un déphaseur ajustable sur l'une des voies permet

d'annuler par réglage préalable ce déphasage supplémentaire.

Le premier changement de fréquence est du type hyperfréquence. Un klystron oscillateur local décalé de 60 Mc par rapport au klystron oscillateur du groupe hyperfréquence attaque un cristal mélangeur. Ce cristal est suivi d'un amplificateur MF.

Les caractéristiques de ce premier groupe sont les suivantes:

Perte par conversion du cristal < 8 dB
gain de l'amplificateur $= 80$ dB
fréquence centrale 60 Mc
largeur de bande $f = 10$ Mc.

L'erreur relative sur la moyenne fréquence consecutive aux dérives thermiques des deux klystrons est de l'ordre de 10^{-5}, moyennant l'utilisation d'alimentations stabilisées classiques et une augmentation de l'inertie thermique des klystrons.

Le deuxième changement de fréquence utilise des circuits électroniques. Un oscillateur à quartz délivre une tension à 50 Mc, la moyenne fréquence résultante est de 10 Mc, la stabilité de l'oscillateur est de 2×10^{-6}.

Les deux tensions à 10 Mc transportant l'information peuvent être utilisées de la façon suivante: si on les applique directement sur les plaques verticales et horizontales d'un oscilloscope, dans le cas ou le pouvoir rotatoire varie lentement en fonction du temps, on voit la courbe de polarisation tourner lentement en se déformant. Si on les détecte pour restituer l'enveloppe des composantes E_x et E_y et si l'on applique les tensions détectées sur les deux voies de l'amplificateur vertical de l'oscilloscope on pourra observer les variations rapides du pouvoir rotatoire en fonction du temps.

Le temps de réponse du montage dépend essentiellement de la bande passante de l'amplificateur moyenne fréquence et du mélangeur du deuxième changement de fréquence. Il est inférieur à la microseconde.

3. PREMIERS RESULTATS EXPERIMENTAUX

La méthode a été vérifiée sur une source de plasma dont les caractéristiques géométriques, ainsi que les valeurs attribuées aux paramètres physiques ont été choisies de telle sorte que les hypothèses de l'étude théorique soient satisfaites.

La sources est une décharge du type réflex à une ou deux cathodes chaudes cylindriques de diamètres $\phi = 8$ cm, de longueur $L = 2$ cm.

Les anodes sont alimentées en tension pulsée par un dispositif électronique et sont distantes de 60 cm. La longueur de l'impulsion est de 100 μsec, le courant de la décharge de 100 ampères, le champ magnétique peut à volonté être rendu sensiblement uniforme sur toute la longueur de la colonne de plasma ou peut avoir la configuration de bouteille magnétique. Son intensité varie de 0 à 1500 gauss, la variation relative le long de la colonne pouvant être rendue inférieure à 1 pour cent.

La pression de travail varie de 10^{-3} à 10^{-4} mm de Hg. Dans ces conditions les densités électroniques obtenues sont de l'ordre de 10^{19} e/m^3.

Le long de la décharge, une sonde de Langmuir et trois dispositifs interférométriques transversaux fonctionnent sur une longueur d'onde de 8 mm, permettent de mesurer et de controler la répartition de la densité électronique dans les trois sections transversales correspondantes (Fig. 4).

Les indications de ces appareils sont comparées aux résultats obtenus par le dispositif basé sur l'effet Faraday.

On a ainsi vérifié qu'aux erreurs de mesures près, la densité est uniforme le long de la colonne, sur une longueur de 60 centimètres, qui représente la distance entre les deux interféromètres transversaux situés entre les deux cathodes et anodes et au voisinage immédiate de celles-ci. Il n'existe pas de gradient longitudinal de la densité jusqu'a un centimètre des cathodes.

D'autre part, on a contrôlé avec la sonde de Langmuir mobile, placée au centre de la décharge, que le cylindre de plasma n'est pas creux. Il existe toutefois entre l'axe de révolution de la décharge et sa frontière, un gradient radial dans la densité. L'ordre de grandeur du gradient-défini par le rapport:

$$\frac{\text{densité périphérique}}{\text{densité sur l'axe}}$$

est 2. Dans la partie centrale de la décharge correspondant au parcours du faisceau électromagnétique focalisé, la densité est quasi uniforme.

On peut faire varier la densité, soit en agissant sur la pression initiale du gaz neutre de remplissage, par l'intermédiaire de la fuite réglable, soit par le courant d'électrons ionisants primaires des cathodes chaudes, elles-mêmes contrôlées par leur courant de chauffage.

Dans les premiers essais de mesures semi quantitatives, on s'est limité à comparer les densités globales radiales et longitudinales mesurées respectivement par une méthode transversale et par effet Faraday.

On a vérifié (photographie *a*) qu'une onde polarisée linéairement avant son entrée dans le plasma, est elliptique à sa sortie. Il y a donc dichroisme, l'angle du grand axe donnant la rotation; l'absorption étant déduite de l'ellipticité. On a enregistré sur la même plaque deux passages du faisceau U.H.F. de sondage,

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l'un immediatement avant la formation du plasma et l'autre durant un intervalle déterminé de la décharge. La faible ellipticité visible sur la photographie est due à un dichroisme résultant de la traversée des deux hublots terminant la source de plasma. On constate sur cette même photographie une rotation d'environ 1/8 de tour. Le montage utilisé pour cette mesure correspond au Fig. 1.

La trace elliptique (même photo *a*) de l'onde émergente est elle-même trouble, elle s'explique par l'origine du plasma. La décharge reflex étant pulsée, avec une fréquence de répétition de 50 fois par seconde, il s'ensuit que d'une impulsion de tension anodique à l'autre le plasma n'est pas rigoureusement identique à lui-même.

Sur les photographies (*b*, *c*) on voit une rotation plus accentuée de 3/4 de tour, mesurée par les deux présentations différentes du Fig. 1.

L'enregistrement (photographie *d*) est l'interférogramme simultané obtenu avec un dispositif transversal.

L'interprétation des deux mesures conduit à une même densité d'environ $5 \times 10^{18}\ e/m^3$, le champ magnétique étant de 430 gauss dans les deux cas.

Enfin sur les photographies *e*, *f*, *g*, on a enregistré les rotations pour une même densité pour trois valeurs différentes du champ magnétique. Ces enregistrements mettent en évidence la variation quasi linéaire et dans le même sens du champ magnétique et des rotations résultantes.

Les premières mesures nous ayant donné des résultats encourageants, les essais seront poursuivis dans une large gamme de valeurs des paramètres densités électroniques, champ magnétique (uniforme et à configuration de bouteille), de la fréquence de sondage et de la température électronique.

Acknowledgements—Nous remercions MM. S. D. WINTER et J. TAILLET pour avoir encouragé cette étude et M. L. LECOMTE pour sa collaboration à l'étude expérimentale.

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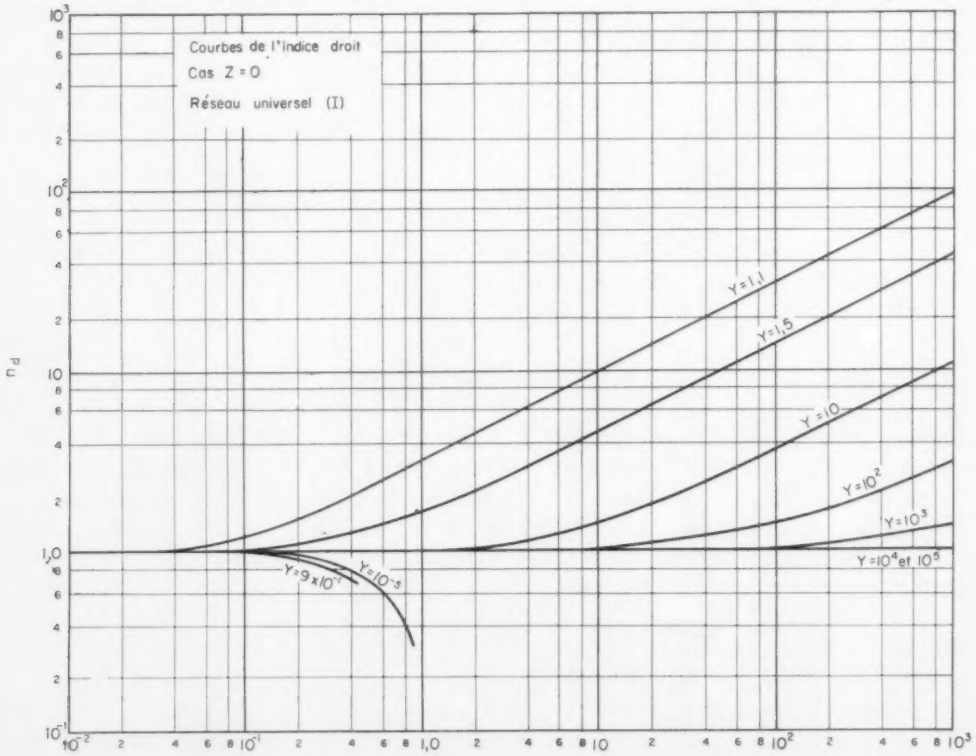

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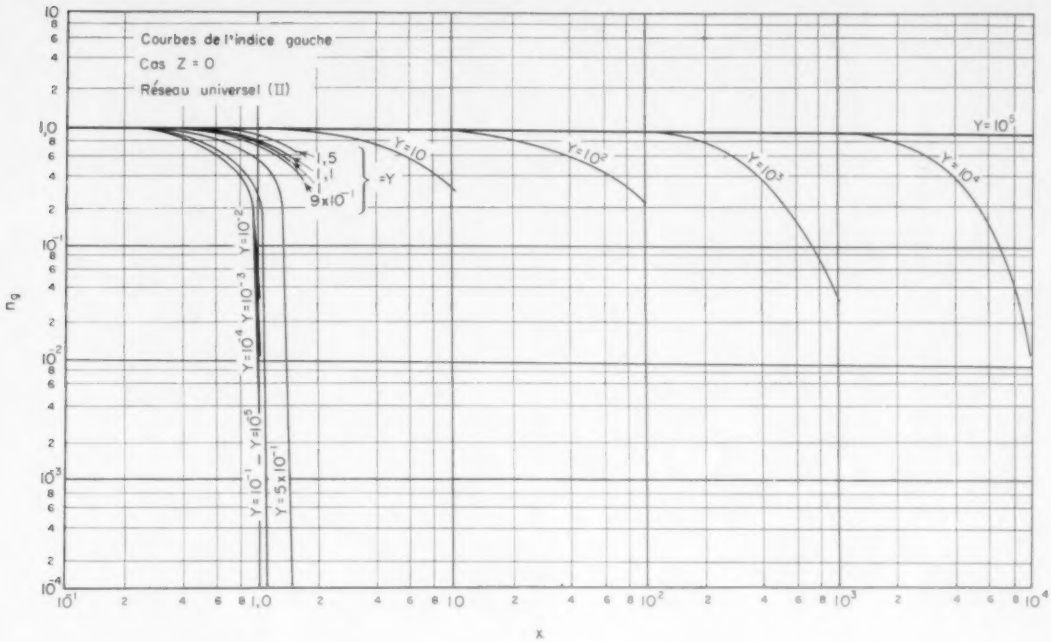

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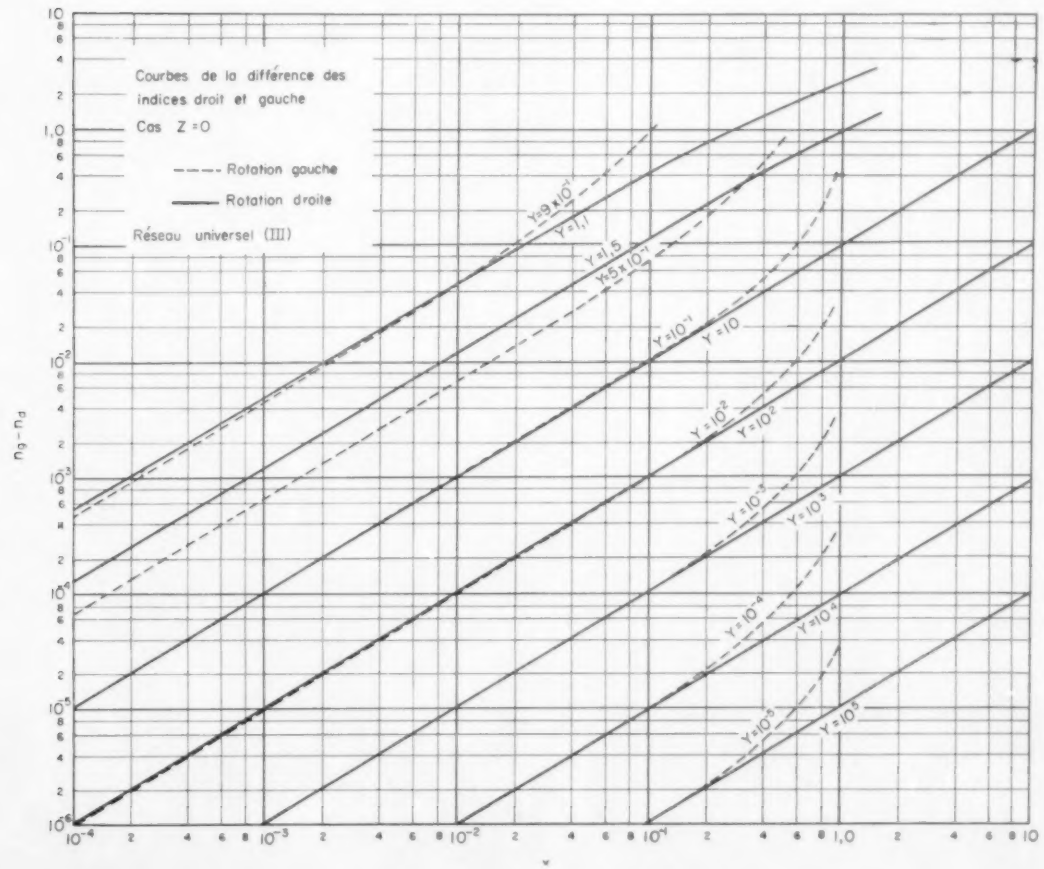

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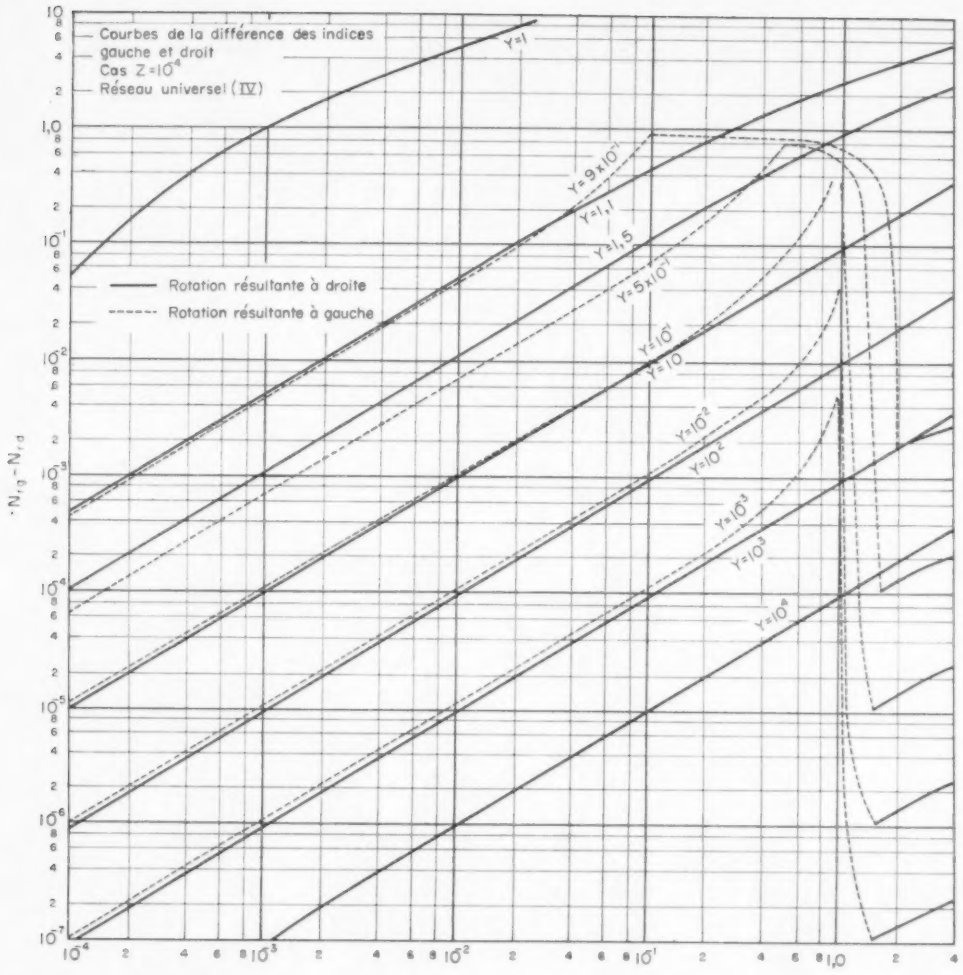
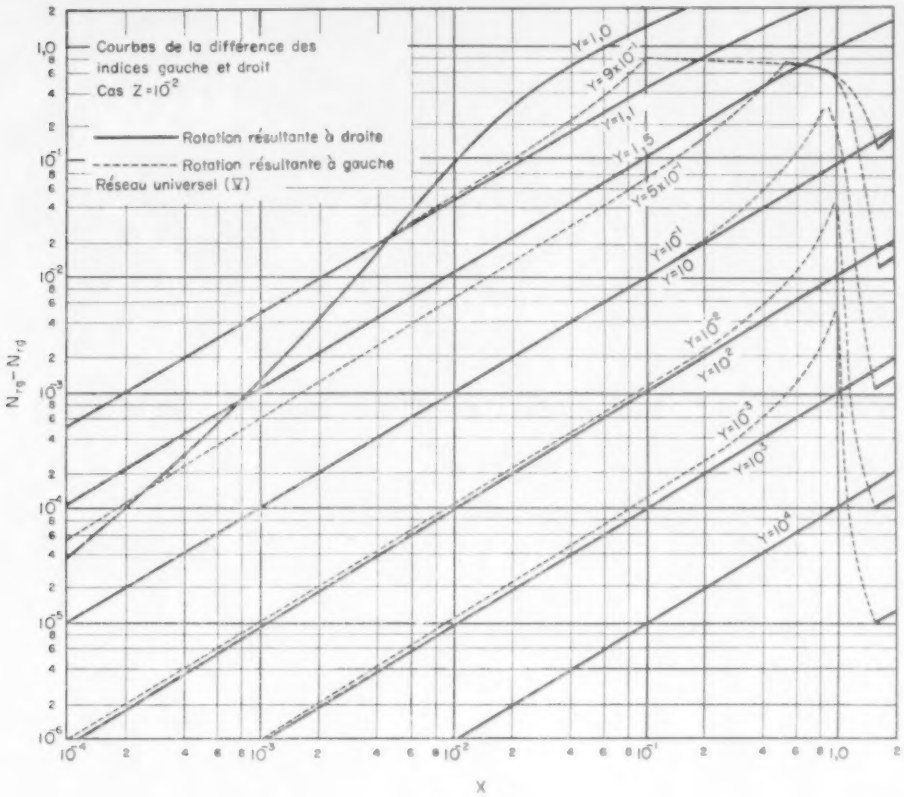

Courbes de la différence des indices gauche et droit
Cas Z=10⁻⁴
Réseau universel (IV)
Rotation résultante à droite
Rotation résultante à gauche
Y=1
Y=9x10⁻¹
Y=1,1
Y=1,5
Y=5x10⁻¹
Y=10¹
Y=10
Y=10²
Y=10⁴
Y=10³
Y=10³
Y=10⁶
N_{i,g} - N_{i,d}
X

RESEAU IV.



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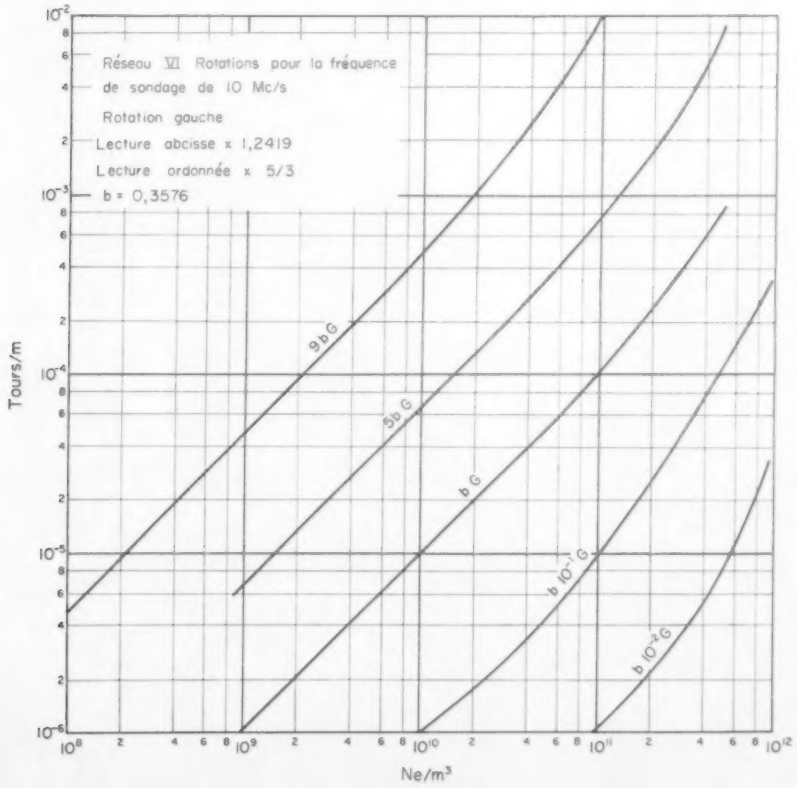

RESEAU VI.

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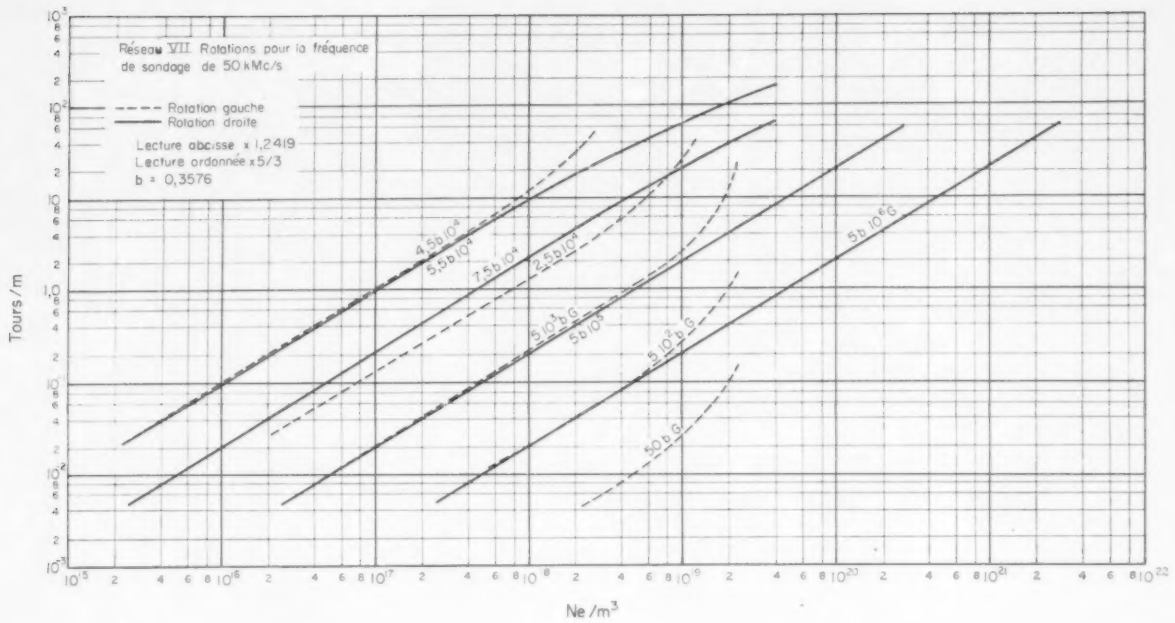

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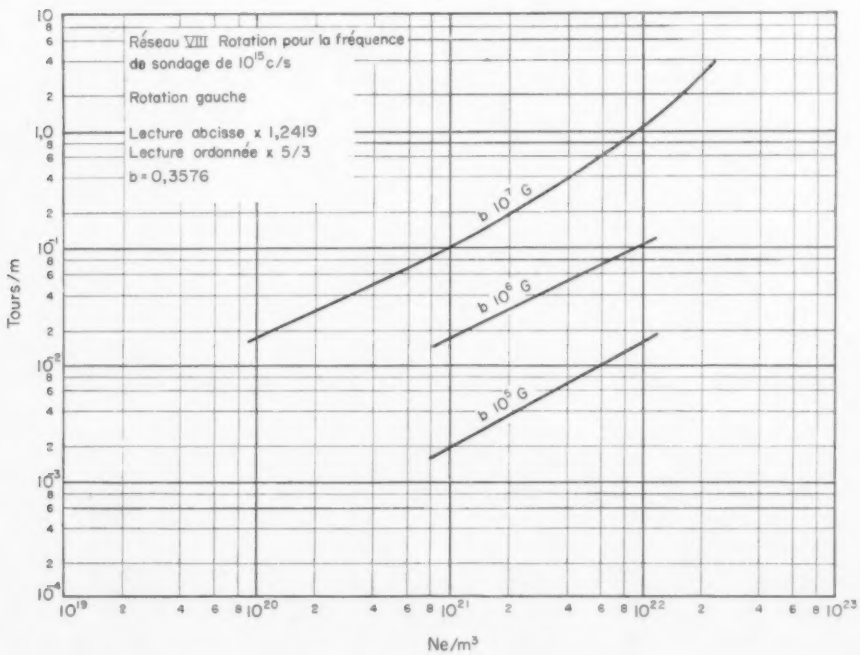

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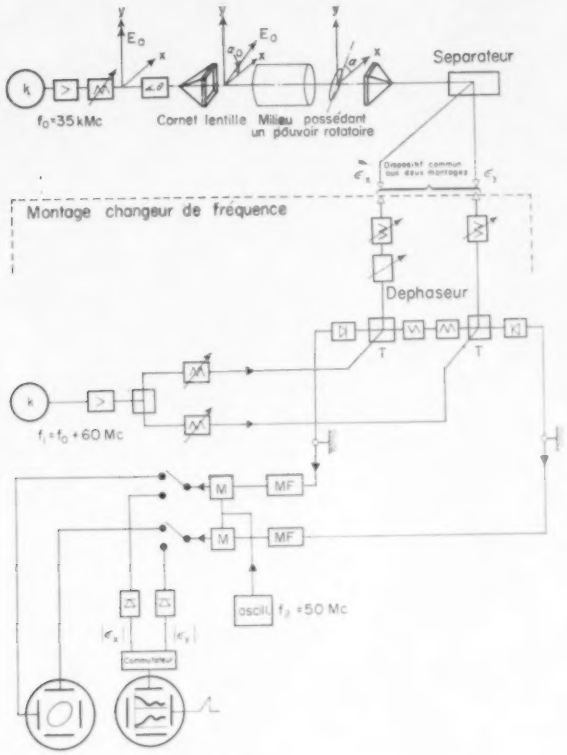
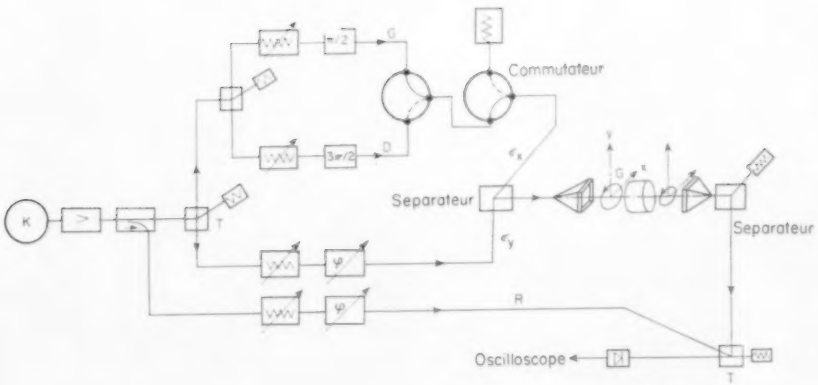


SCHEMA I.



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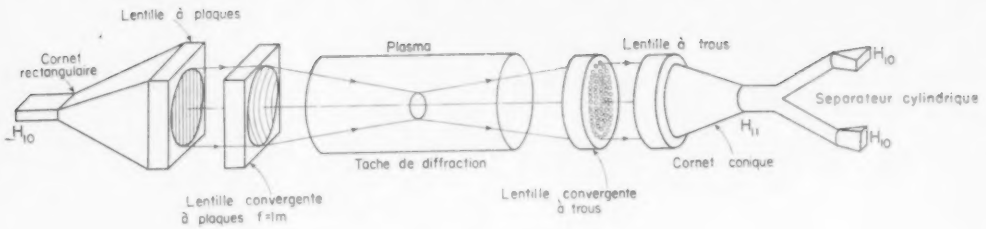

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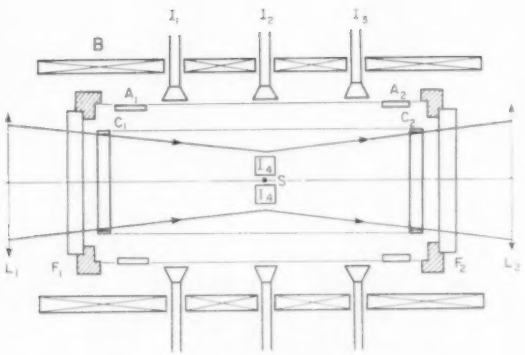

B = Bobine de champ
C_1 = Cathode chaude
C_2 = Cathode froide
A_1 = A_2 = Anodes
F_1 = F_2 = Fenêtres
I_2 = I_1 = I_3 = Interféromètres 8mm
I_4 = Interféromètre 4mm
S = Sonde de longmuir
L_1,L_2 = Lentilles electromagnétiques

SCHEMA IV.

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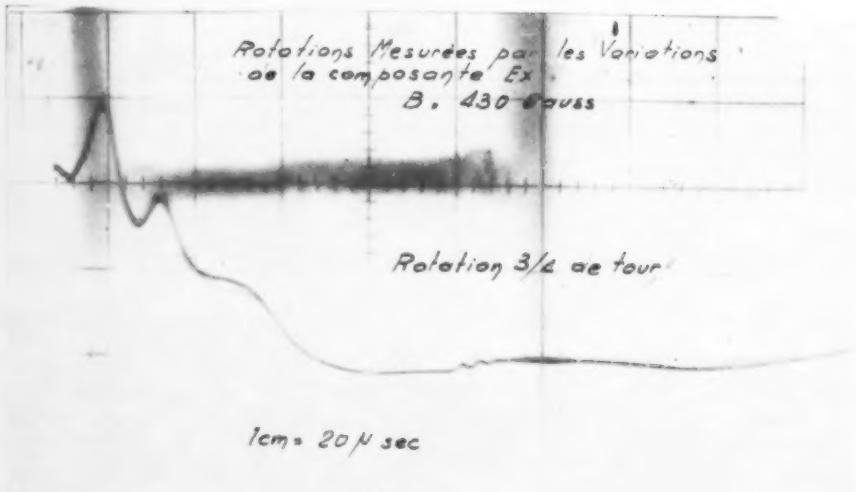
Ellipse de polarisation
Rotation d'1/8 de tour
Ellipse à très faible excentricité = Signal de référence
Ellipse = signal ayant subi la rotation

PHOT. 1.

PHOT. 2.



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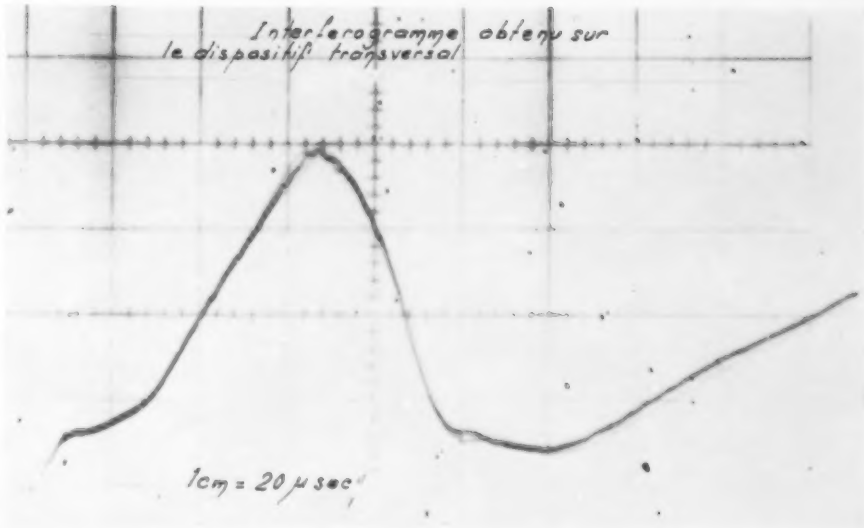

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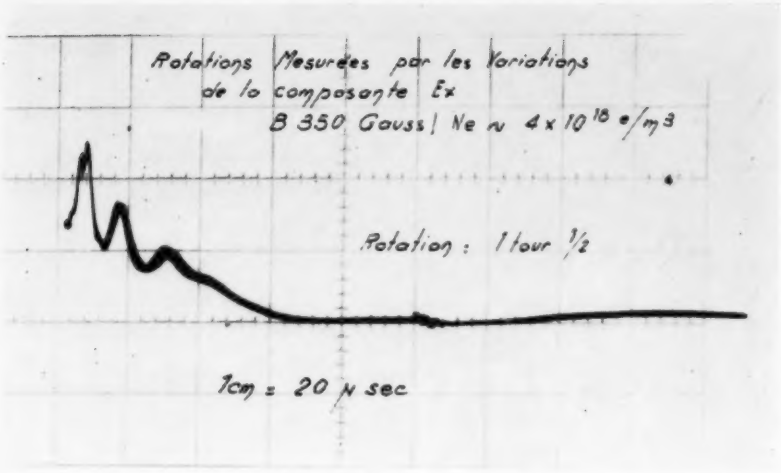
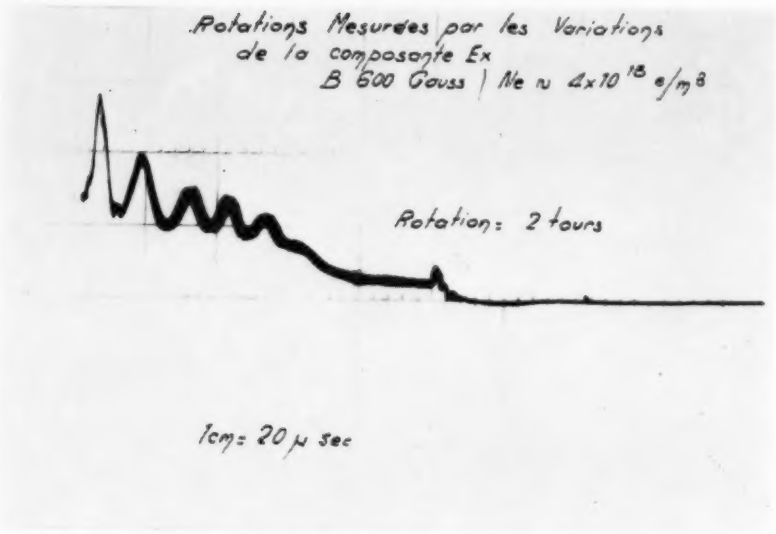


PHOT. 5.



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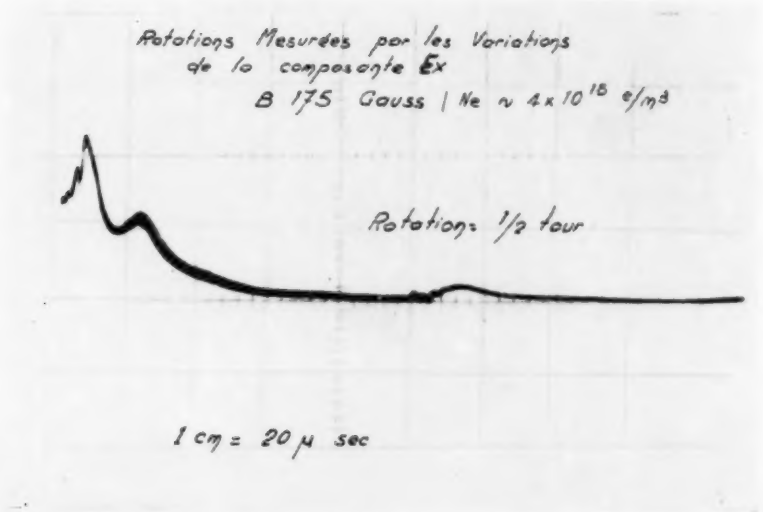

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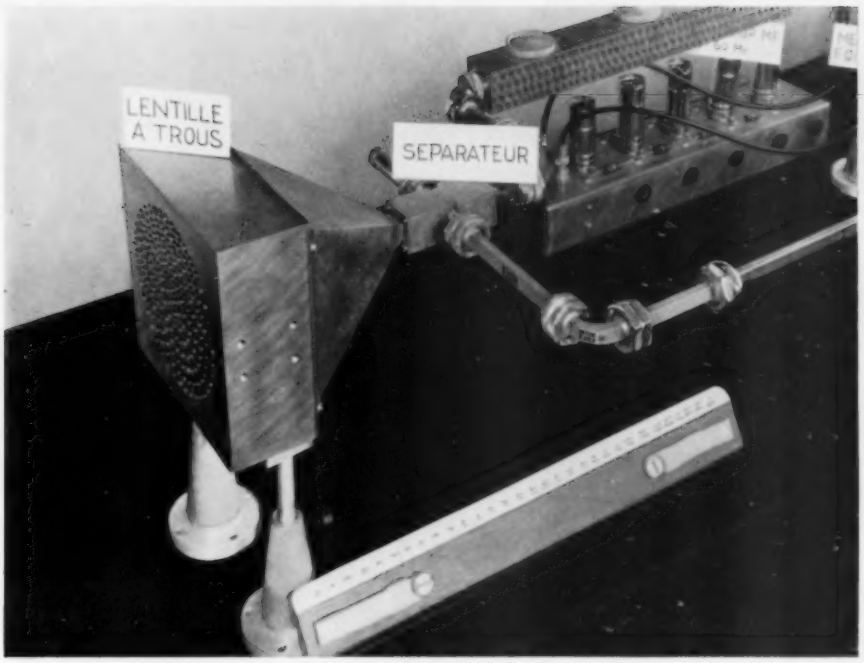
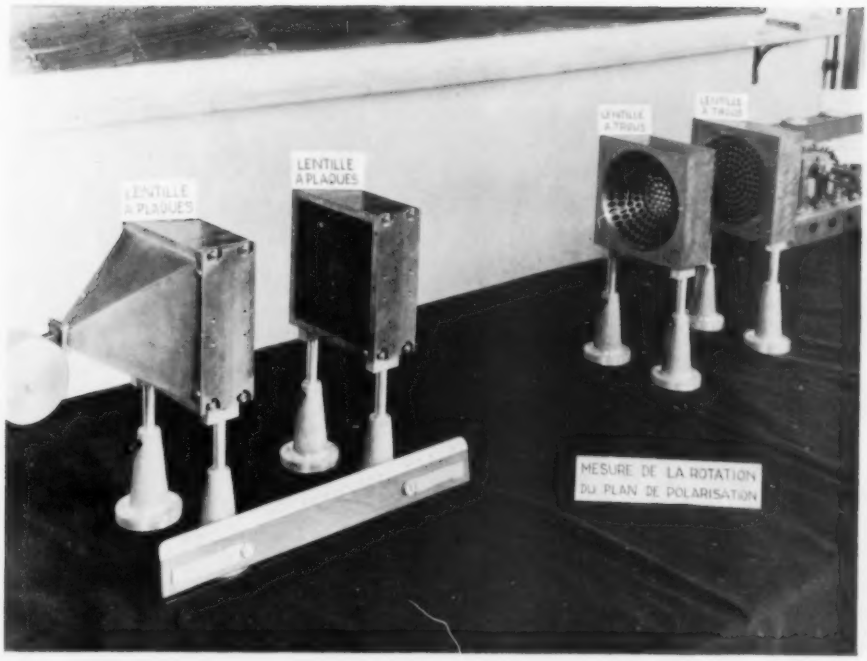
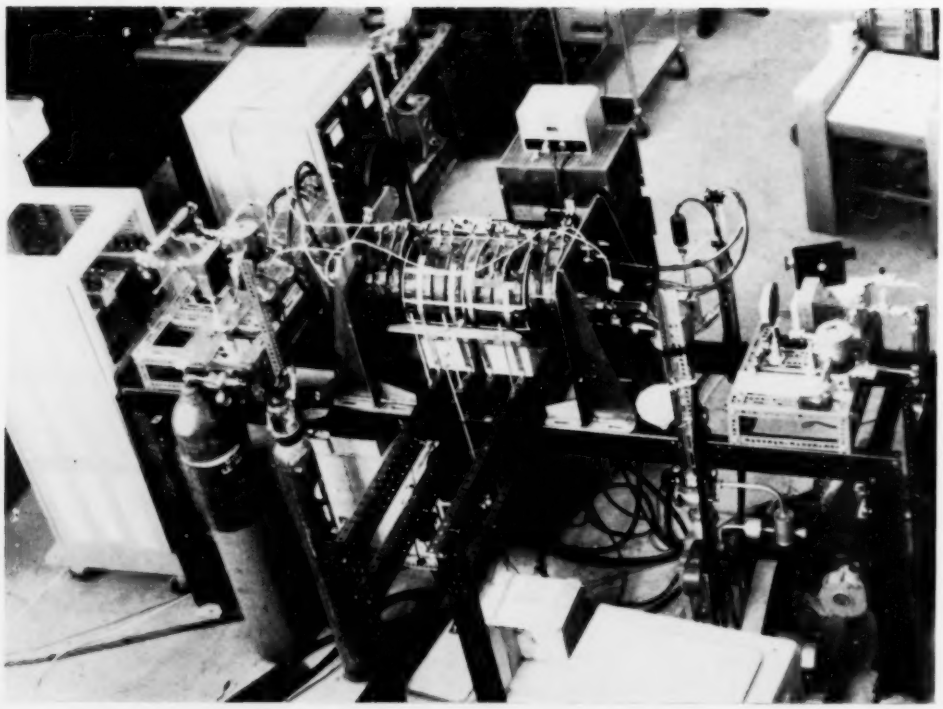


PHOT. 8.



PHOT. 9.

PHOT. 10.



PHOT. 11.—Vue d'ensemble de la source et du dispositif U.H.F.

J. Nucl. Energy, Part C: Plasma Physics, 1961, Vol. 3, pp. 129 to 134. Pergamon Press Ltd. Printed in Northern Ireland

CALCULATION OF RESONANT FREQUENCIES OF RE-ENTRANT CYLINDRICAL ELECTROMAGNETIC CAVITIES

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Rutherford High Energy Laboratory, Harwell, Berks.

(*Received* 10 *October* 1960)

Abstract—A method is described of calculating the resonant frequencies of cylindrical re-entrant electromagnetic cavities, which provides upper and lower bounds to the required frequency. These bounds converge towards each other as the number of terms in certain harmonic expansions is increased. The lower bound is increased by subtracting from the total field a singular part caused by the presence of sharp corners. Application to cavities of interest in the design of proton linear accelerators shows that accuracies of better than 1 per cent can be obtained without excessive computing time.

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1. INTRODUCTION

IN carrying out design studies of proton linear accelerators of the Alvarez type it is useful to be able to calculate the electromagnetic eigenfrequencies of resonant cavities of the shape shown in Fig. 1. It is hoped ultimately to be able to compute such eigenfrequencies by purely numerical methods based on finite-difference approximations, but it is helpful to be able to test computing programmes designed for this purpose on cavities of simpler geometry such as that shown in Fig. 2, where the axial hole has been omitted and the rounded corners replaced by sharp ones. The methods described below can be extended to include the axial hole (see Appendix) but for test purposes the geometry of Fig. 2 is adequate. It has been found that about one hour's computing time (on a Ferranti 'Mercury') suffices to determine frequencies to within 1 per cent, and in favourable and not unlikely cases, to within 0·25 per cent.

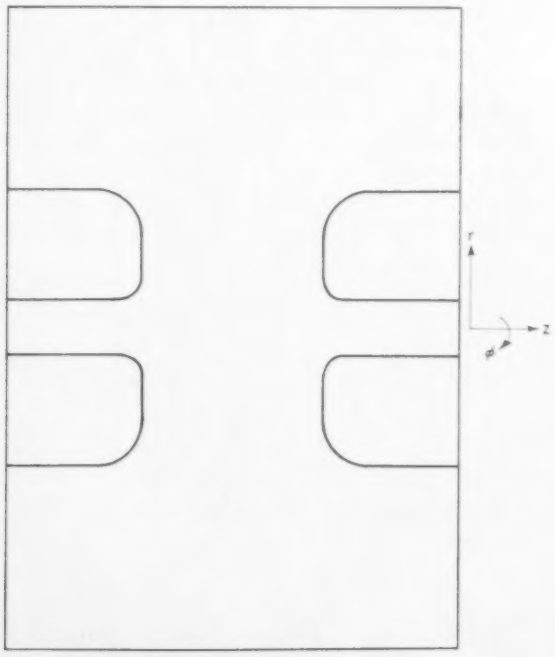

FIG. 1.—Geometry of practical cavity.

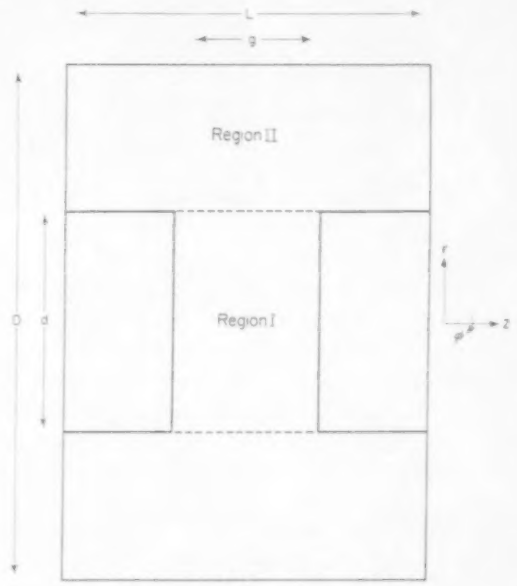

FIG. 2.—Geometry of cavity discussed in present paper.

The method is an adaptation to resonant cavities of one given by WALKINSHAW (1949) for progressive waves in corrugated wave-guides. It has been used by COMBE and FEIX (1960) who however obtained only an upper bound to the desired frequency whereas the present report finds upper and lower bounds, according to a theorem given by BELL (1951).

2. FIELD EXPANSIONS

The resonant mode of interest of the cavity shown in Fig. 2 is the lowest *E*-mode. The appropriate solution of the Maxwell equations may be derived

from a single-component Hertz vector U which satisfies

$$\nabla^2 U + (\omega^2/c^2)U = 0 \tag{1}$$

where ω is the desired (angular) frequency. The fields are related to U by

$$E_z = \frac{\partial^2 U}{\partial z^2} + \frac{\omega^2}{c^2} U \tag{2a}$$

$$E_R = \frac{\partial^2 U}{\partial r \partial z} \tag{2b}$$

$$H\phi = \frac{i\omega\varepsilon}{c} \frac{\partial U}{\partial r} . \tag{2c}$$

With the assumptions of infinite conductivity on the metal walls, the boundary conditions are simply

$$E_R = 0 \text{ on } z = \pm g/2,\ r < d/2 \tag{3a}$$

$$E_R = 0 \text{ on } z = \pm L/2,\ d/2 < r < D/2 \tag{3b}$$

$$E_z = 0 \text{ on } r = d/2,\ |z| > g/2 \tag{3c}$$

$$E_z = 0 \text{ on } r = D/2. \tag{3d}$$

We now construct expressions for U in the two regions, (I), $r < d/2$, and (II), $r > d/2$. The necessity that the two expressions satisfy the conditions (3) and yield the same values of E_z and H_ϕ on the common boundary $r = d/2$ is found to yield a double set of equations of which the condition of consistency gives a relationship involving ω, L, g, D and d which can in principle be solved to any desired degree of accuracy.

The general solution of (1) in region (I), which includes the axis $r = 0$, can be written in the form

$$U_{\mathrm{I}} = \sum A_n R_n(r) \cos 2n\pi z/g \tag{4}$$

where $R_n(r) = J_0(k_n r)$ for $k_n^2 > 0$

or $I_0(|k_n| r)$ for $k_n^2 < 0$

and $k_n^2 = \omega^2/c^2 - 4n^2\pi^2/g^2$.

The A_n are a so far undetermined set of coefficients. In region (II) the expression is

$$U_{\mathrm{II}} = \sum C_n S_n(r) \cos 2n\pi z/L \tag{4a}$$

where

$$S_n(r) = J_0(K_n r) Y_0(K_n D/2) - Y_0(K_n r) J_0(K_n D/2) \quad \text{for } K_n^2 > 0$$

or

$$I_0(|K_n| r) K_0(|K_n| D/2) - K_0(|K_n| r) I_0(|K_n| D/2) \quad \text{for } K_n^2 < 0$$

and $K_n^2 = \omega^2/c^2 - 4n^2\pi^2/L^2$.

The quantities C_n are also undetermined as yet.

These forms for U satisfy the boundary conditions (3a), (3b), and (3d).

3. MATCHING THE FIELDS IN THE TWO REGIONS

Sufficient relations between the coefficients A_n and C_n can be obtained by requiring (i), that (3c) be satisfied,

(ii), that E_z and H_ϕ should be continuous at $r = d/2$, $|z| < g/2$.

Using relation (2a) we find that

$$E_z^{\mathrm{II}}(d/2) = \sum c_m \cos 2m\pi z/L \tag{5}$$

where $c_m = C_m K_m^2 S_m(d/2)$.

The fourier expansion (5) must vanish for $|z| > g/2$ and must agree for $|z| < g/2$ with the expression

$$E_z^{\mathrm{I}}(d/2) = \sum a_n \cos 2n\pi z/g$$

where $a_n = A_n k_n^2 R_n(d/2)$.

Hence $$Lc_0 = ga_0 \tag{6a}$$

$$c_m = \frac{2g^2}{\pi L^2} m \sin m\pi g/L \sum_n \frac{(-1)^n a_n}{m^2 \dfrac{g^2}{L^2} - n^2} . \tag{6b}$$

Similarly from (2c) we find

$$H_\phi^{\mathrm{II}}(d/2) = \sum d_m \cos 2m\pi z/L \tag{7}$$

where $$d_m = \frac{i\omega\varepsilon}{c} C_m S_m'(d/2),$$

the prime denoting differentiation, and

$$H_\varphi^{\mathrm{I}}(d/2) = \sum b_m \cos 2m\pi z/g \tag{8}$$

where $$b_m = \frac{i\omega\varepsilon}{c} A_m R_m'(d/2).$$

The expression (7) is valid for $0 < |z| < L/2$, (8) for $0 < |z| < g/2$. For the range $g/2 < |z| < L/2$ the value of $H_\phi(d/2)$ given by (7) will be sustained by surface currents, but for $0 < |z| < g/2$ the Fourier expansion (8) must equal the function (7), the latter being a Fourier expansion valid over a greater range. The Fourier analysis results in the equations

$$A_0 R_0' = C_0 S_0' + \sum_{m=1}^{\infty} C_m S_m' \frac{\sin (m\pi g/L)}{m\pi g/L} \tag{9a}$$

$$A_n R_n' = \frac{2g}{\pi L} (-1)^n \sum_{m=1}^{\infty} C_m S_m' \frac{m \sin (m\pi g/L)}{m^2 \dfrac{g^2}{L^2} - n^2} \tag{9b}$$

where the argument $d/2$ has been omitted from all functions R and S.

4. THE ALTERNATIVE ELIMINATIONS

Our procedure is now to use equations (6) and (9) to eliminate either the set of coefficients A_n or the set C_n.

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This will yield an infinite set of simultaneous homogeneous linear equations in the non-eliminated coefficients; for consistency, the determinant of the matrix of this set of equations must vanish and its vanishing constitutes a relationship between the desired frequency ω and the dimensions L, g, D and d. In practice, to find the lowest zero of this infinite determinant, regarded as a function of ω, one examines the trend of the lowest zero of a sequence of determinants of finite order N, trying to establish the limit of this zero as $N \to \infty$. This means that only a finite number of the non-eliminated coefficients are used in the expansion (4) or (4a), so that the boundary conditions on the surface $r = d/2$ cannot be exactly satisfied.

If the inner coefficients A_n are eliminated, the condition (3c) cannot be satisfied with the now finite set C_n, and so there is a mismatch in E_z on $r = d/2$. If the outer set C_n are eliminated, the fields in the outer region will produce an H_ϕ on $r = d/2$ which cannot be matched by the now finite set of A_n for the inner region, and so a mismatch in H_ϕ will result. In either case the truncated function U can be regarded as a variational function, the parameters being the non-eliminated expansion coefficients A_n or C_n, which is an approximation to the true function U, satisfying the wave-equation (1) but not the boundary conditions on $r = d/2$. Use can now be made of the theorem given by BELL (1951) to the effect that a variational function giving a mismatch in E_z will give an upper bound for ω, which decreases as the number of variation parameters increases, whilst a function causing a mismatch in H_ϕ gives an increasing lower bound. This has been verified by the computations to be described below.

Let V_{lm}, U_{lm} $(l, m = 0, 1, \ldots, N)$ be the elements of the determinants obtained respectively by eliminating the sets A_n, C_n. Then it is found that both are symmetric determinants with elements as follows:

$$V_{00} = \frac{\pi^2 L^3}{2g^3}\frac{\omega^2}{c^2}\frac{S_0}{S_0'}\left(1 - \frac{LR_0'S_0}{gR_0S_0'}\right) \tag{10a}$$

$$V_{0m} = \frac{\pi^2}{2}\frac{L^3}{g^3}\frac{\omega^2}{c^2}\frac{S_0}{S_0'}\frac{\sin(m\pi g/L)}{m^2\pi g/L} \quad (m \neq 0) \tag{10b}$$

$$V_{lm} = A_{lm} - \frac{\pi^2 L^3}{4g^3}\frac{K_l^2 S_l}{l^2 S_l'}\delta_{lm} \quad (l, m \neq 0) \tag{10c}$$

where $$A_{lm} = \sin(l\pi g/L)\sin(m\pi g/L) \times \sum_{n=1}^{\infty} \frac{k_n^2 R_n/R_n'}{\left(\frac{l^2g^2}{L^2} - n^2\right)\left(\frac{m^2g^2}{L^2} - n^2\right)} \tag{11}$$

and $$\delta_{lm} = 1(l = m) \quad \text{or} \quad 0(l \neq m).$$

$$U_{00} = B_{00} - \frac{\pi^2 L^3}{2g^3}\frac{c^2}{\omega^2}\left(\frac{R_0'}{R_0} - \frac{g}{L}\frac{S_0'}{S_0}\right) \tag{12a}$$

$$U_{lm} = B_{lm} - \frac{\pi^2 L^3}{4g^3}\frac{c^2}{\omega^2}\frac{R_l'}{k_l^2 R_l}\delta_{lm} \tag{12b}$$

where

$$B_{lm} = \sum_{n=1}^{\infty} \frac{n^2 \sin^2(n\pi g/L)}{\left(\frac{n^2g^2}{L^2} - l^2\right)\left(\frac{n^2g^2}{L^2} - m^2\right)}\frac{S_n'}{K_n^2 S_n} \quad (l, m \neq 0) \tag{13}$$

Care must be taken when programming the summations (11) and (13), since every number in the store of a digital computer is rational and thus if N, the order of determinant, is large enough values of l and m will occur which cause the expression (11) to become, and (13) to include, indeterminate forms of the type 0/0. If this occurs in (11) then either $\sin(m\pi g/L)$ or $\sin(l\pi g/L)$ vanishes, or both vanish. The appropriate forms for the matrix elements of row l if $l\pi g/L = M\pi$ are

$$A_{lm} = \frac{\pi(-1)^{M+1} k_M^2 R_M}{2MR_M'}\frac{\sin m\pi g/L}{M^2 - m^2\frac{g^2}{L^2}} \quad (l \neq m)$$

$$A_{ll} = \frac{\pi^2 k_M^2 R_M}{4M^2 R_M'}.$$

These forms obviously cover the case $\sin m\pi g/L = 0$ also. On the other hand if in (13) $n\pi g/L = M\pi$ for some n then the corresponding term in the summation vanishes unless $l = m$, where it becomes

$$\frac{\pi^2 L^2 S_n'}{4g^2 K_n^2 S_n}.$$

5. SUBTRACTING THE SINGULARITY ON THE COMMON BOUNDARY

It is found in practice that the upper and lower bounds of the lowest eigenfrequency do not, for a wide variety of geometries, approach each other to within 1 per cent for determinants of order up to about 20. The computation is therefore very time-consuming. This indicates that the Fourier expansion of the fields at $r = d/2$ is only slowly convergent. By analogy with static two-dimensional problems we may expect that near the points $r = d/2$, $z = \pm g/2$, the electric field has a singularity of the form $(g/2 \pm z)^{-1/3}$. In the following analysis we assume a form for E_z on $r = d/2$ in which the part representing the singularity is separated from the Fourier expansion.

Thus, on the surface $r = d/2$, let

$$E_z = ef(z) + \sum_{n=1}^{\infty} e_n \cos 2n\pi z/g \quad (|z| < g/2) \tag{14}$$

where e is a parameter and f a dimensionless function. In the inner region we expand U in the form

$$U_I = \sum u_n R_n(r) \cos 2n\pi z/g.$$

The assumption (14) then requires

$$u_n k_n^2 R_n(d/2) = ef_n + e_n \tag{15}$$

where the quantities f_n are defined by

$$f(z) = \sum f_n \cos 2n\pi z/g \text{ in } (-g/2, g/2). \tag{16}$$

In the outer region the expansion for U takes the form

$$U_{II} = \sum V_m S_m(r) \cos 2m\pi z/L. \tag{17}$$

Matching E_z at $r = d/2$, we find

$$LV_0 K_0^2 S_0 = egf_0$$

$$\tfrac{1}{2}LV_n K_n^2 S_n = egF_n + \frac{n}{\pi L} \sin(n\pi g/L) \times \sum_m \frac{(-1)^m e_m}{\dfrac{n^2}{L^2} - \dfrac{m^2}{g^2}} \quad (n \neq 0) \tag{18}$$

where $$gF_n = \int_{-g/2}^{g/2} f(z) \cos(2n\pi z/L)\, dz.$$

Matching H_ϕ we find

$$\frac{gef_0 R_0'}{k_0^2 R_0} = gV_0 S_0' + g \sum V_m S_m' \frac{\sin(m\pi g/L)}{m\pi g/L}$$

$$\frac{g}{2} \frac{ef_n + e_n}{k_n^2} \frac{R_n'}{R_n} = \frac{(-1)^n}{\pi L} \times \sum_m \frac{mV_m S_m' \sin(m\pi g/L)}{\dfrac{m^2}{L^2} - \dfrac{n^2}{g^2}} \quad (n \neq 0) \tag{19}$$

Here as before all functions R and S are evaluated at $r = d/2$.

Elimination of the V_m between (19) and (18) now leads to the vanishing of a determinant W_{lm} which differs from the determinant U_{lm} of Section 4 only in the following respects:

(i) The first element in the first column is

$$W_{00} = \pi L^3/g^3 \left\{ G_0 - \tfrac{1}{2}\pi f_0 \left(\frac{R_0'}{k_0^2 R_0} - \frac{gS_0'}{LK_0^2 S_0} \right) \right\} \tag{21}$$

(ii) The S'th element in the first column is $(S \neq 0)$

$$W_{s0} = \pi L^3/g^3 \left\{ G_s - \tfrac{1}{2}\pi(-1)^s \frac{f_s R_s'}{k_s^2 R_s} \right\} \tag{22}$$

where

$$G_s = \sum_n \frac{n \sin(n\pi g/L)}{n^2 - \dfrac{s^2 L^2}{g^2}} \frac{F_n S_n'}{K_n^2 S_n} \tag{23}$$

In (23) the occurrence of forms 0/0 must be anticipated.

Elimination of the e_m between (18) and (19), in which e is treated as the zeroth member of the set, leads to the vanishing of exactly the same determinant V_{lm} which was described in Section 4. Thus introducing the function $f(z)$ can only increase the lower bound on ω, not decrease the upper bound.

A suitable form for $f(z)$ is given by

$$f(z) = (1 - (2z/g)^2)^{-\mu} \quad (0 < \mu < 1) \tag{24}$$

One then finds

$$f_0 = \tfrac{1}{2}\sqrt{\pi}\, \frac{\Gamma(1-\mu)}{\Gamma(\frac{3}{2} - \mu)}$$

$$f_n = \sqrt{\pi}\, \Gamma(1-\mu)\, g\,(n\pi)$$

$$F_n = \tfrac{1}{2}\sqrt{\pi}\, \Gamma(1-\mu)\, g\,(n\pi g/L)$$

where $$g(x) = J_{\frac{1}{2}-\mu}(x)/(\tfrac{1}{2}x)^{\frac{1}{2}-\mu}.$$

Computations have been carried out with $\mu = 1/3$.

6. RESULTS OF COMPUTATIONS

These are shown in Figs. 3, 4. 5, and 6 for a variety of geometries, the lowest eigenvalues of ω^2/c^2 being plotted against N (order of determinant), the upper curves being obtained by eliminating the A_n, the lower by eliminating the C_n, and the intermediate by eliminating the C_n and using the function (24). No particular units of length need be used. The geometry of the cavity of Fig. 3 represents one point on the curve given by COMBE and FEIX (1960), whose cavities, for the purpose of computing the lowest E-mode, can be taken as the right half $(z > 0)$ of the cavities treated here. It is seen that with $N = 20$ an accuracy (i.e. a relative discrepancy between upper and improved lower bounds) of about $\frac{1}{2}$ per cent can be obtained in

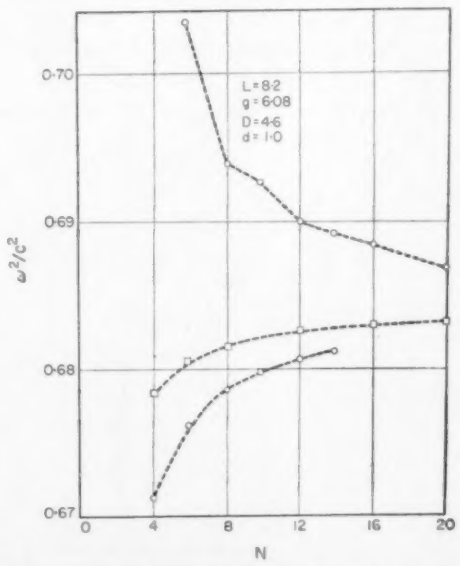

FIG. 3.—Trend of approximations as functions of order of determinant.

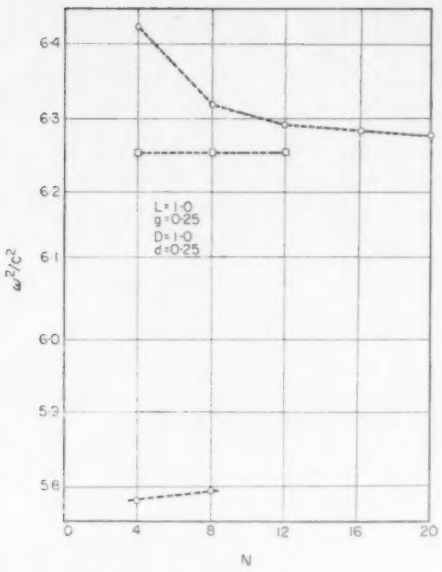

FIG. 4.—Trend of approximations as functions of order of determinant.

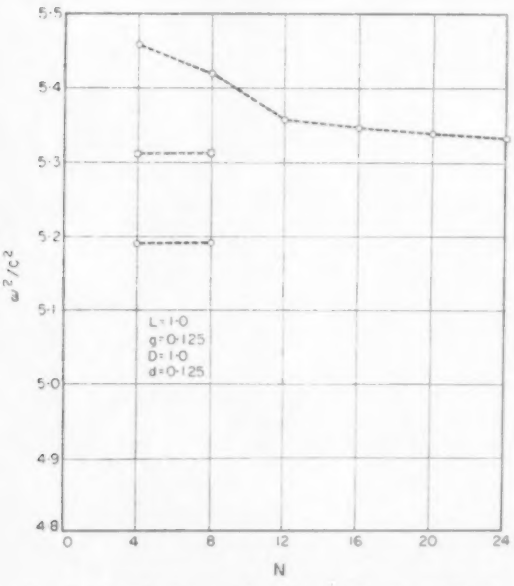

FIG. 5.—Trend of approximations as functions of order of determinant.

this case. Even better accuracies can be obtained for the geometries used in Figs. 4, 5 and 6. The utility of the function $f(z)$ is evident. In general the improved lower curve rises very slowly with N, whereas the upper curve descends rapidly.

This is fortunate, as the computation on the general case takes about twice as long for a given N on either lower curve, as for the same N on the upper, because in the lower curve each term in the summation in each matrix element requires Bessel functions of the second kind as well as of the first. On a Ferranti Mercury computer, an upper curve up to $N = 20$ takes about half an hour.

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APPENDIX: EXTENSION TO CAVITIES HAVING AN AXIAL HOLE

Here we summarize the analysis dealing with the geometry of Fig. 7. We change the notation used for radial functions.

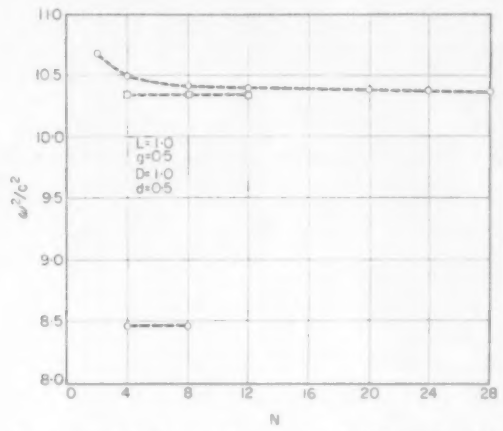

FIG. 6.—Trend of approximations as functions of order of determinant.

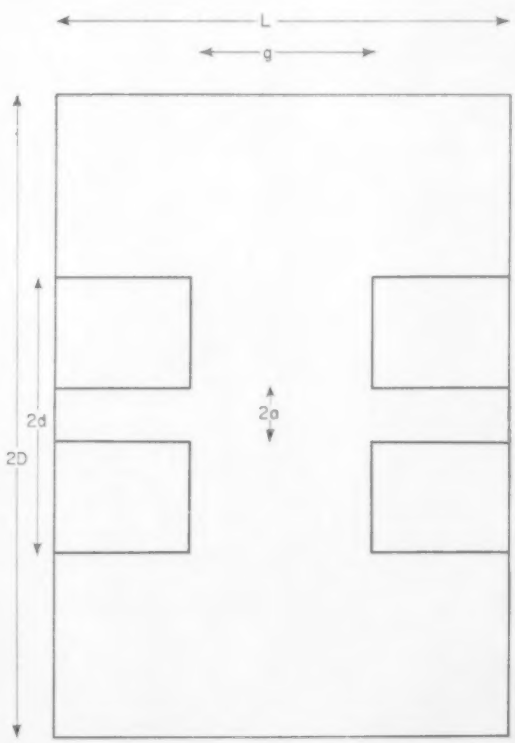

FIG. 7.—Geometry of cavity discussed in Appendix.

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The function U is expanded as follows in the three regions (I) $0 < r < a$, $|z| < L/2$; (II) $a < r < d$, $|z| < g/2$; (III) $d < r < D$, $|z| < L/2$.

$$U_{\mathrm{I}} = \sum A_n R_n(r) \cos 2n\pi z/L$$

where $R_n(r) = J_0(K_n r)$ or $I_0(|K_n|r)$;

$$U_{\mathrm{II}} = \sum (B_n S_n(r) + C_n T_n(r)) \cos 2m\pi z/g$$

where $S_n(r) = J_0(k_n r)$ or $I_0(|k_n|r)$

$T_n(r) = Y_0(k_n r)$ or $K_0(|k_n|r)$;

$$U_{\mathrm{III}} = \sum D_n U_n(r) \cos 2n\pi z/L$$

where $U_n(r) = J_0(K_n r)\ Y_0(K_n D) - Y_0(K_n r) J_0(K_n D)$

or $I_0(|K_n|r)\ K_0(|K_n|D) - K_0(|K_n|r)\ I_0(|K_n|D)$.

Matching E_z at $r = a$ and $r = d$ leads to equations which can be written in matrix form

$$A = Z(S_a B + T_a C)$$
$$D = X(S_d B + T_d C).$$

Here, A, B, C, D are vectors of coefficients and X, Z, S_a, T_a, S_d, T_d are matrices, in particular S_a and T_a are diagonal matrices whose elements are $S_n(a)$ and $T_n(a)$. Matching H_ϕ at $r = a$ and $r = d$ leads to matrix equations

$$S_a' B + T_a' C = YA$$
$$S_d' B + T_d' C = UD$$

where U, Y, S_a', etc., are further matrices. Elimination of the A_n and D_n, for example. leads to the matrix equations

$$(S_a' - YZS_a)B = (YZT_a - T_a')C$$
$$(S_d' - UXS_d)B = (UXT_d - T_d')C$$

from which we deduce the determinantal equation

$$|(S_a' - YZS_a)^{-1}(YZT_a - T_a')(UXT_d - T_d')^{-1} \times (S_d' - UXS_d) - I| = 0$$

where I is the unit matrix of infinite order. Curtailing this determinant to one of finite order N implies the use of a finite number of expansion coefficients in region (II), and thus a mismatch in H_ϕ. Clearly the method of subtracting singularities at $r = a$ and $r = d$ could be applied, and would lead to very tedious analysis and programming.

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THE SHOCK MODEL OF THE DYNAMIC PINCH

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Abstract—Various models of the dynamic z-pinch are discussed and photographic evidence from experiments using hydrogen and deuterium discharges in a straight tube is given in support of a shock model which allows for energy spent in ionization and dissociation. Effectively, the ionization and dissociation reduce γ from 5/3 to the region of 1·2. The temperature of the shocked gas is calculated from the measured shock velocity, and a curve of temperature against initial gas pressure is given. This temperature varies between 0·5 and 25 eV for pressures of 10,000 μ to 100 μ which corresponds to 10 to 30 per cent of the directed radial energy of the ions. Thus the pinch, during the implosion phase, consists of a rapidly imploding skin of low energy plasma.

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1. PINCH MODELS

ONE method of producing a hot plasma is by magnetic compression in a cylindrical pinch discharge. The pinch was first studied theoretically by NORTHRUP (1907) and experimentally by WARE (1951) and recently investigations have been intensified in the thermonuclear programmes of various countries. In many experiments the pinch velocity is much greater than the sound speed so that the compression is irreversible. This method of heating, known as the dynamic pinch, is desirable because energy is put in at a greater rate compared with the reversible case in which the pinch is subsonic.

In order to develop a theory of the dynamic pinch the hydrodynamic equations for the gas are used with the electromagnetic circuit equations. These equations are coupled by the pressure at the pinch surface and their simultaneous solution gives the pinch velocity as a function of the circuit parameters. Three different hydromagnetic models are commonly employed: the snowplough, free particle and shock. In the snowplough model, originally developed by ROSENBLUTH (1954) and also used later by COLGATE (1958), it is assumed that a particle hit by the magnetic piston is not reflected. Thus, in the reference frame of the piston, all the energy of the particle is dissipated when it is brought to rest. This implies that the particle has an infinite number of degrees of freedom and the ratio of specific heats ($\gamma = 1 + 2$/number of degrees of freedom) is 1. It follows that the density in the pinch surface on the snowplough model is infinite and the temperature zero. In both the free particle and shock models energetic particles move ahead of the piston and a disturbance front forms within the pinch. In the free particle model considered by ROSENBLUTH (1954), the mean free path of particles incident on the front is much greater than the distance between the piston and front; thus the particles move freely in a radial direction, the energy is contained in one degree of freedom and $\gamma = 3$. In the shock model the mean free path of the incident particles is much smaller than the distance between the piston and the front and, for a simple strong shock in a monatomic gas with three degrees of freedom, $\gamma = 5/3$. Shock model calculations were made by ALLEN (1957) and WRIGHT (1959). Photographic investigations of the dynamic pinch have been made by KOMEL'KOV (1959) and BORZUNOV *et al.* (1958).

The models can be distinguished by comparing the velocity of the front of energetic particles propagated ahead of the piston with the piston velocity. It may be shown that the density behind a plane shock of Mach number M is

$$\rho = \rho_0 \frac{\gamma + 1}{\gamma - 1 + \frac{2}{M}}$$

where ρ_0 is the initial density. If the piston velocity is v and the shock velocity u, the continuity equation is

$$\rho(u - v) = \rho_0 u.$$

Eliminating ρ/ρ_0 from these equations, the ratio of shock front velocity to piston velocity is

$$\frac{u}{v} = \frac{\rho}{\rho - \rho_0} = \frac{\gamma + 1}{2\left(1 - \frac{1}{M}\right)}.$$

Generally, in dynamic pinch experiments, $M \gg 1$ and the velocity ratio is $\frac{1}{2}(\gamma + 1)$. For a simple strong shock with three degrees of freedom $\gamma = 5/3$ and the velocity ratio is 4/3. When the particles have an infinite number of degrees of freedom, $\gamma = 1$ and the

velocity ratio is 1; this is the snowplough case. At the other extreme, when the particles have only one degree of freedom, $\gamma = 3$ and the velocity ratio is 2. In this limit there are no collisions, the motion is reversible, and the shock model degenerates to the free particle model.

In these models, calculations are usually made for a totally-ionized, infinitely conducting gas, whereas in actual experiments the gas is often initially unionized or poorly pre-ionized. For a simple strong shock with $\gamma = 5/3$, the energy from the piston is equally divided between thermal motion and the streaming motion of the particles between the piston and shock front. However, for the piston speed observed in these experiments, (1 to 10 cm/μsec), it is not permissible to neglect the energy spent in dissociating and ionizing the neutral gas. This energy comes from the thermal motion so the temperature of the shocked gas and the shock velocity are reduced and the particle density increased compared with the simple case. The ionization and dissociation reduce the γ since they provide a method for the particles hit by the piston to lose their energy and so one would expect the modified shock theory to tend towards the snowplough model.

The hydrodynamic curves for a plane shock in hydrogen allowing for a dissociation potential of 2·24 eV per atom and an ionization potential of 13·60 eV and assuming thermal equilibrium behind the shock front, have been calculated by NIBLETT (1960). Fig. 1 shows a graph of piston speed against the shock front speed for this case together with the graphs for the other three models. At the lower velocities the modified shock theory tends to the snowplough model, but for higher velocities, when the particle energy becomes greater than the ionization and dissociation energy, the curve tends towards the simple strong shock model. Over the experimental pressure range, (100 μ to 10,000 μ) the curves are practically independent of pressure. From the shock front speed the temperature of the gas between the piston and front may be calculated. Fig. 3 gives this temperature plotted against shock velocity in deuterium for various gas pressures (for hydrogen the shock velocity should be multiplied by 2).

2. EXPERIMENTAL MEASUREMENTS

This section describes experiments to test these models using dynamic pinches in hydrogen and deuterium. The magnetic piston was produced by a straight z-mode discharge (i.e. current flowing parallel to the tube axis) driven by a low inductance condenser bank.

The discharge tube was of quartz, 15 cm diameter and 30 cm long. The current generator was a 200 μF, 15 kV capacitor bank of 5·5 mμH switched by 100 parallel spark gaps; it was developed by FITCH and MCCORMICK (1959). Initiating electrons were provided by a low energy pre-ionization capacitor discharged about 3 μsec before the main bank. The initial tube inductance was 8·1 mμH and initial rate of current rise about 1000 kA/μsec. The peak current was in the region of 500 kA. The pinch was observed radially by using a streak camera which had a writing speed of 25 mm/μsec.

Measurements were taken in the pressure range 100 μ to 10,000 μ; the lower limit is set by the light sensitivity of the camera and the upper by non-uniform breakdown. Even at the lowest pressure the mean free

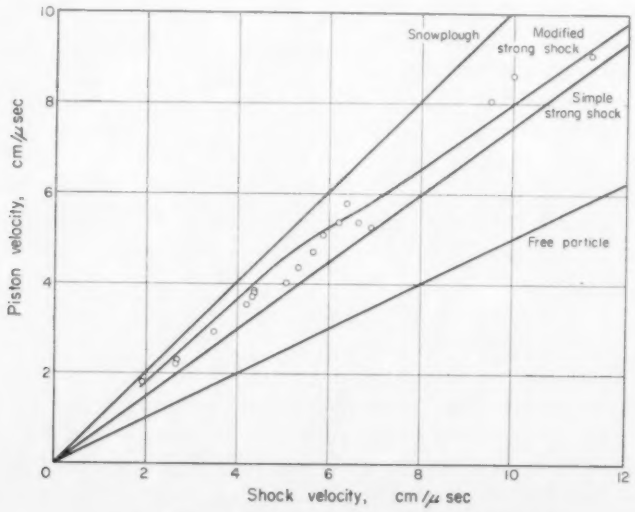

FIG. 1.—Comparison of pinch models for deuterium.

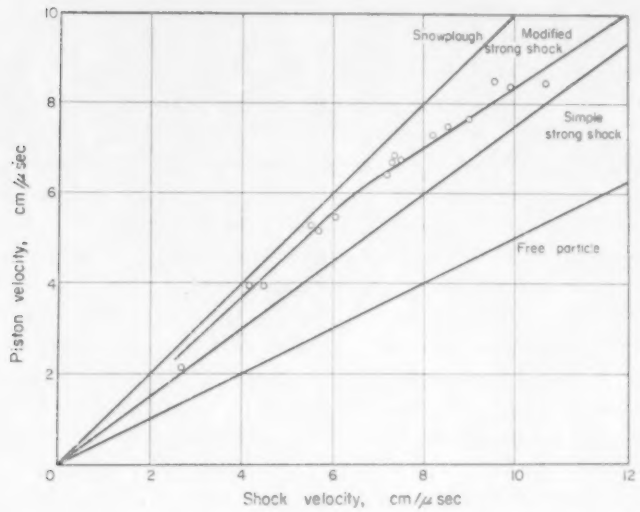

FIG. 2.—Comparison of pinch models for hydrogen.

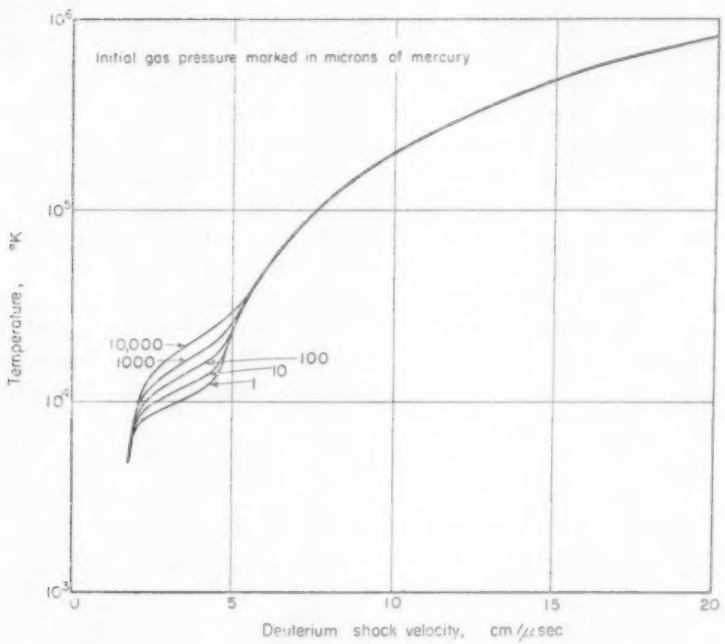

FIG. 3.—Temperature behind shock front on modified shock theory.

path for collisions between neutral atoms and between ions and neutral atoms is 0·1 cm and, for all the experiments, one would expect a shock front to develop. PHILLIPS (1960) has shown theoretically that when the initial gas density is sufficiently large for particle trapping to occur, the mean free path of ions in the current sheet in the incoming neutral gas is much less than their mean cyclotron radius and a shock can develop.

Fig. 4 shows a typical picture taken with an initial pressure of 640 μ (the black lines are due to the return conductor) together with its interpretation. The general discharge characteristics were as follows:

(i) In about the first 0·3 μsec no light was seen because current flowed near the tube surface where bars of the coaxial return conductor obscured the view.

(ii) From 0·3 μsec to the time of the first peak in

contraction the pinch moved at practically constant velocity. A cylindrical shell of luminosity was observed which was interpreted as defining the region occupied by shocked gas with the inner edge marking the shock front and outer edge the magnetic piston.

(iii) An intense burst of light was produced when the shock front reached the axis. This was due to energy from the radial motion of the ions rapidly heating the electrons which lost energy by radiation. A shock then travelled outwards from the tube axis and the pinch expanded. The expansion was limited by the magnetic pressure which caused a rapid deceleration and again the pinch contracted. After the start of the deceleration the discharge was not reproducible. The second implosion did not trap all the gas because light could be seen travelling outwards after the start of the second implosion. Sometimes this gas later formed third and fourth implosions (not seen in Fig. 4).

Measurements of the shock front velocity and piston velocity as a function of pressure are shown in Figs. 1 and 2. The results show, within experimental error, agreement with the modified shock theory. The readings were made near the beginning of the discharge where the separation of the shock front from the piston was small compared with the radius and the approximation of plane geometry used in the theory should be valid. The effect of the cylindrical geometry was observed at high compressions as an increase in the shock front velocity and this agrees with the theoretical conclusions of JUKES (1958) who showed that the speed of a cylindrically imploding shock front should be greater than the plane shock approximation near the axis.

The temperature of the shocked gas was calculated from the shock front velocity by using the theoretical curve in Fig. 3. During the implosion, the velocity and hence the temperature were practically constant. Graphs of the temperature against initial gas pressure for hydrogen and deuterium are shown in Fig. 5. The temperature ranged between 0·5 and 25 eV for pressures of 10,000 μ to 100 μ; the results for hydrogen are in general agreement with photoelectric measurements of the total light output made by BORZUNOV *et al.* (1959) (see Fig. 5). The form of the curves may be explained by considering the energy in the shocked gas. Behind the shock front the energy is equally divided between the directed radial energy of the gas moving with the piston velocity and the internal energy of the shocked gas. In a simple strong shock all the internal energy appears as heat, but in the modified shock model some of this energy is used in ionization and dissociation. The degree of ionization depends on the energy available. At high pressures the pinch velocity is low and there is insufficient energy in the shocked gas to produce complete ionization. As the pressure is reduced the velocity and energy in the shocked gas increase but the temperature remains practically constant because the increase in energy goes into increasing the fraction of gas ionized. In effect, the constant temperature may be considered as being due to a latent heat of ionization and dissociation. Thus, for hydrogen and deuterium (which have the same ionization and dissociation potentials), the curves coincide at high pressure. As the pressure is reduced to below about 1500 μ (where the temperature is about 2 eV) the ionization is complete and the excess energy

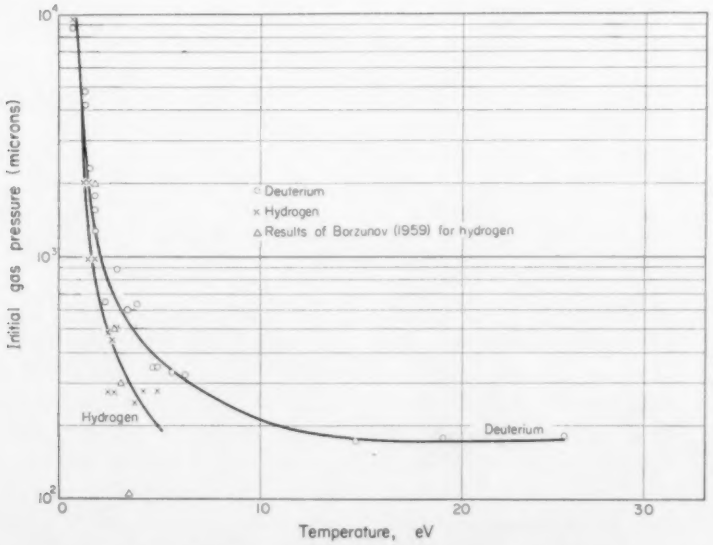

FIG. 5.—Temperature between piston and shock front.

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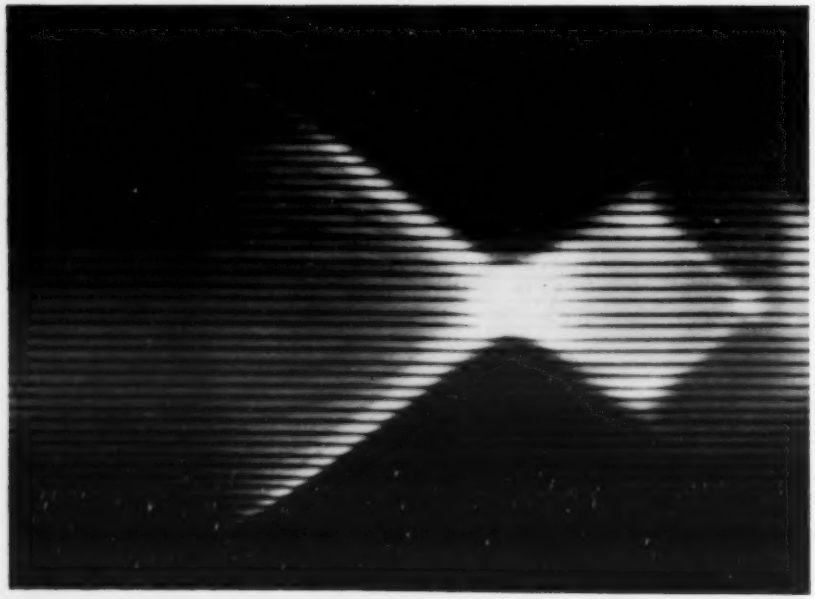

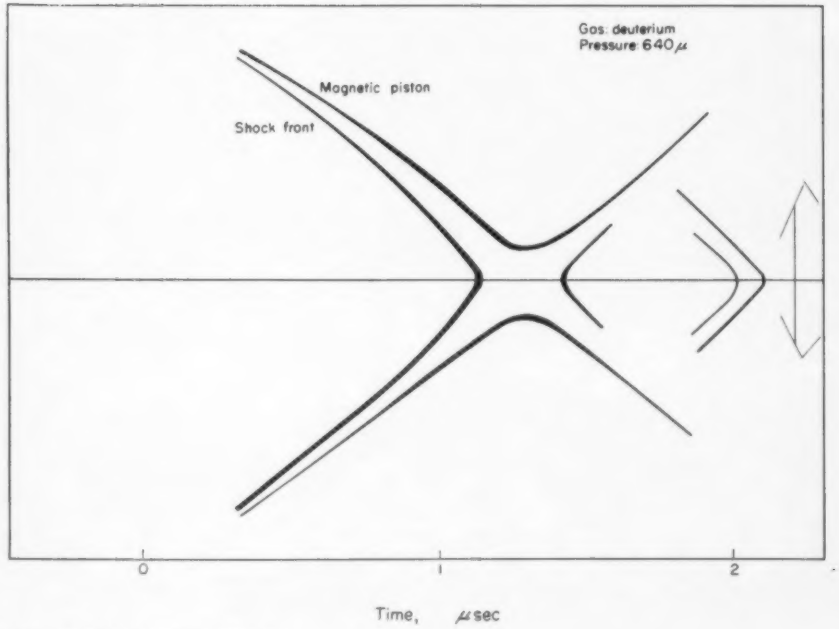

(a)

FIG. 4(a, b).—Radial streak photograph and its interpretation.

(b)

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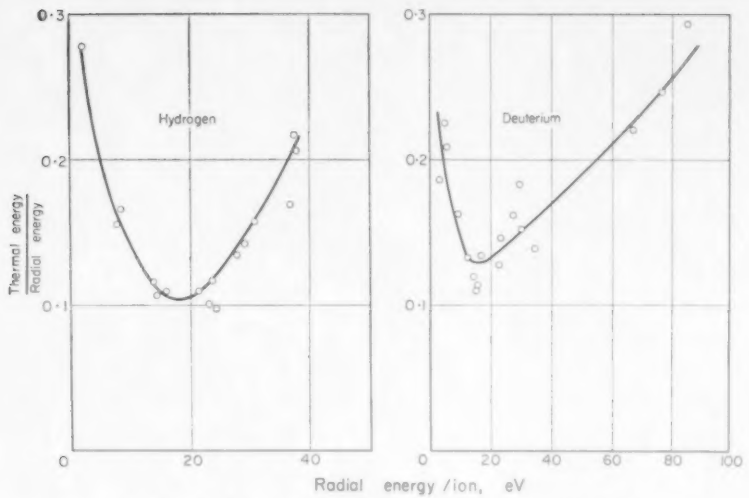

FIG. 6.—Ratio of thermal to radial energy in a pinching discharge.

in the shocked gas produces an increase in temperature. The curves diverge with deuterium having the higher temperature because of its greater mass.

The ratio of thermal energy to radially directed energy is plotted against the radially directed energy in Fig. 6. This has a minimum at about 0·1 when the radially directed energy is about 18 eV and initial pressure about 1500 μ. The internal energy in the shocked gas (which equals the radially direction energy) is also 18 eV, so, with the ratio of 0·1, the thermal energy is 2 eV. Thus the energy that was used in ionization and dissociation is about 16 eV per atom, which corresponds to complete ionization below 1500 μ and is in agreement with the conclusions from the temperature/pressure curve.

3. CONCLUSIONS

The shock and piston velocity have been measured in a linear z-pinch and shown to agree with a shock model modified to allow for ionization and dissociation. The ionization and dissociation provide an energy sink in the pinch surface and so, for the piston speed observed experimentally, one would expect the modified shock model to approximate to the snow-plough model. The shock model was used to calculate the temperature in the shocked gas from the shock velocity for various initial gas pressures; this was found to be in the region of 0·5 to 25 eV and, for hydrogen, is in the general agreement with the measurements of BORZUNOV *et al.* (1959).

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ELECTROSTATIC INSTABILITIES IN OHMICALLY HEATED PLASMAS

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Abstract—The dispersion equation for electrostatic oscillations in a current-bearing plasma is solved for marginal stability when the electron and ion temperatures are unequal. The result is applied to a plasma ohmically heated by a current which is constant with time. A criterion for instability is compared with a rather similar one for the formation of a runaway beam of electrons. Comparison is made with some experiments in which plasma has been ohmically heated (ZETA and Stellarator B1) and which has developed some instability.

INTRODUCTION

DETERMINATION of the maximum current density (amperes cm^{-2}) which may be carried by a fully-ionized gas in *quasi* equilibrium, or what may also be called a 'quiescent' state, is required for the evaluation of many parameters vital to plasma physics experiments. These include the pinched discharges (BUTT *et al.*, 1958) and the Stellarators (SPITZER, 1958). There is also the question whether Ohmic heating alone by thermal dissipation is a practicable way of achieving very high plasma temperatures (COOR *et al.*, 1958).

There are at least two known phenomena which may impose an upper limit on the current density and the maximum temperature attainable by simple Ohmic heating. These are:

(1) The formation of a runaway beam of electrons through the plasma. (GIBSON, 1957; DREICER, 1960.) As the proportion of total current carried by runaways increases, so the heating effect of the remaining Ohmic current decreases.

(2) The development of electrostatic instabilities between two or more sets of particles having a streaming motion relative to one another. (AUER, 1958). These can be subdivided into:

(a) the bulk of current carrying thermal electrons and the background ions, which we shall call current instability (BUNEMAN, 1958; PENROSE, 1960).

(b) runaway electrons and the remaining thermal particles, ions and electrons, which we shall call runaway instabilities. (AKHIEZER and FAINBERG, 1949; STRINGER, to be published.)

The purpose here is to consider in some detail a criterion for the onset of current instability (2a) and to compare it with a similar criterion for runaways (1). The phenomena associated with (2b) is being considered separately (STRINGER, to be published), although it will not of course be relevant if runaways themselves (1) are sufficiently suppressed in any given situation.

Current instability is due to resonance between the Doppler shifted plasma frequency of electrons coinciding with the natural ion plasma frequency in their mean frame of rest. (BUNEMAN, 1959). Competing with this instability mechanism is the phase damping predicted by LANDAU (1946). The first part of the paper discusses the unstable solutions of the dispersion equation for the problem, as formulated by the VLASOV-BOLTZMANN equation (1938). As such, it is a slight elaboration of the work of BUNEMAN (1959) JACKSON (1960) and PENROSE (1960), but it is given here for completeness. Coulomb collision between particles are neglected during the growth of instability. They are important, however, in determining the initial conditions from which an instability may grow, and they are considered in this role.

The second part of the paper is concerned with the stability problem of a plasma heated by a current of uniform density, which remains constant with time as the temperature rises due to dissipation. We suppose the current to be switched on with the plasma at a given 'preheat' temperature. The current requires an electric field to be present through the plasma and related to the current through Ohm's law. There is then the possibility of runaway electrons being drawn into a beam through the plasma by this field. The question of whether runaways or current instability will dominate in any given situation is discussed.

In experiments on plasma of high conductivity and low inductance it is more appropriate to consider conditions at *constant current* rather than at constant applied electric field (contrast DREICER, 1960). In a high current discharge it is the current which is limited and maintained relatively constant by the total electrical energy available. The presence of an associated magnetic field is neglected in the model we

consider here. If the plasma pressure is small and dynamical effects are negligible, the plasma would be almost force-free, that is, current would be constrained to flow almost along the magnetic field lines.

There are two points of view on the further significance of electrostatic instabilities for plasma heating and containment. One has been put forward by GABOR and BUNEMANN that instabilities may lead to electrodynamic turbulence and more rapid effective heating than would be achieved by Coulomb inter-particle collisions. On the other hand, the point which has been expressed by SPITZER and others, is that instabilities may produce random electric fields which enable plasma to escape rapidly across a confining magnetic field. We shall not embark on these further questions here, but confine ourselves to discussion of under what circumstances instabilities may be anticipated. In any case our analysis is confined to the usual linearized approximations and can say nothing of the further behaviour of an instability whose amplitude has grown excessive.

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THE DISPERSION EQUATION AND ITS SOLUTION

Consider two Maxwellian distribution of electrons and ions at temperatures T_e, T_i in which there is a drift velocity u between the mean centres of the distributions. This is the first and a crude approximation to describe a current bearing plasma in a steady electric field. We choose the frame in which the electron distribution is at rest and the ion distribution has a drift velocity u along Ox. The stability of the system will be investigated with respect to small electrostatic perturbations of the form $\exp i\,(kx\text{-}\omega t)$ where k is real, but ω may be complex. The electric field is assumed to have an x-component only, and collisions are neglected. The distribution functions are

$$f_e(v_x) = \pi^{-(1/2)} n_e \beta_e \exp -\beta_e^2 v_x^2 \qquad (1a)$$

$$f_i(v_x) = \pi^{-(1/2)} n_i \beta_i \exp -\beta_i^2 (v_x - u)^2 \qquad (1b)$$

where $$\beta_e = (m/2kT_e)^{1/2}\,,\ \beta_i = (M/2kT_i)^{1/2} \qquad (2)$$

and $$n_j = \int_{-\infty}^{+\infty} f_j(v_x)\, dv_x \ (j = i, e). \qquad (3)$$

If the ions of mass M carry a charge Ze and the plasma as a whole is electrically neutral, then

$$n_e = Zn_i.$$

It is now convenient to define two plasma frequencies for electrons of mass m and ions

$$\omega_{p_e}^2 = \frac{4\pi n_e e^2}{m}\,,\ \omega_{p_i}^2 = \frac{4\pi n_i Z^2 e^2}{M} = \frac{4\pi n_e Z e^2}{M} \qquad (4)$$

and we shall drop the suffix e from n_e calling n the electron particle density.

The dispersion equation has been derived from the VLASOV-BOLTZMANN equation and may be written (BUNEMAN, 1959):

$$k^2/2 = \omega_{p_e}^2 \beta_e^2 K(\omega\beta_e/k) + \omega_{p_i}^2 \beta_i^2 K([\omega - ku]\beta_i/k) \qquad (5)$$

where

$$K(z) = 2ze^{-z^2} \int_0^z \exp t^2\, dt - 1 - i\pi^{1/2} z e^{-z^2} \qquad (6)$$

Defining $p = \omega\beta_e/k$ and $q = -(\omega - ku)\beta_i/k$

the dispersion equation (5) becomes

$$k^2/2\omega_{p_e}^2 \beta_e^2 = g(p) + \frac{ZT_e}{T_i}\, g(q) - i\pi^{1/2}\left(pe^{-p^2} - \frac{ZT_e}{T_i}\, qe^{-q^2} \right) \qquad (7)$$

with $$u\beta_e = p + q\sqrt{\left[\frac{Zm}{M} \cdot \frac{T_i}{ZT_e}\right]^{1/2}} \qquad (8)$$

and $$g(z) = 2ze^{-z^2} \int_0^z \exp t^2\, dt - 1. \qquad (9)$$

One can investigate the dispersion equation (7, 8) for marginal stability ($\mathrm{Im}(\omega) = 0$) by the relatively simple Nyquist method (see, for example, PENROSE, 1960). k^2 is plotted in the complex plane as a function of ω/k, as ω/k traverses the real axis from $-\infty$ to $+\infty$. If this curve encircles any part of the real and positive k^2-axis then there will be instabilities for that band of real k.

We shall find those areas on a graph of $u\beta_e$ as a function of ZT_e/T_i and also the parameter R (defined as $(M/Zm)^{1/2}$) for which k is real.

Putting $\mathrm{Im}\,(k^2) = 0$ gives

$$ZT_e/T_i = pe^{-p^2}/qe^{-q^2}. \qquad (10)$$

No generality is lost in assuming $p > 0$, but (10) then shows $q > 0$ also. In general there are three roots to (8) and (10) which we shall call A, B, C, see Fig. 1.

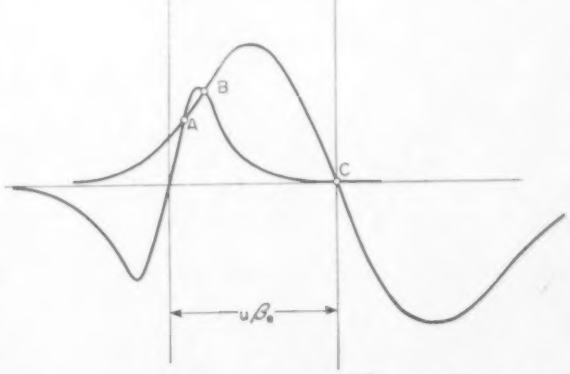

FIG. 1.—Sketches of curves pe^{-p^2}, $\frac{ZT_e}{T_i} qe^{-q^2}$ showing the three typical roots, A, B and C.

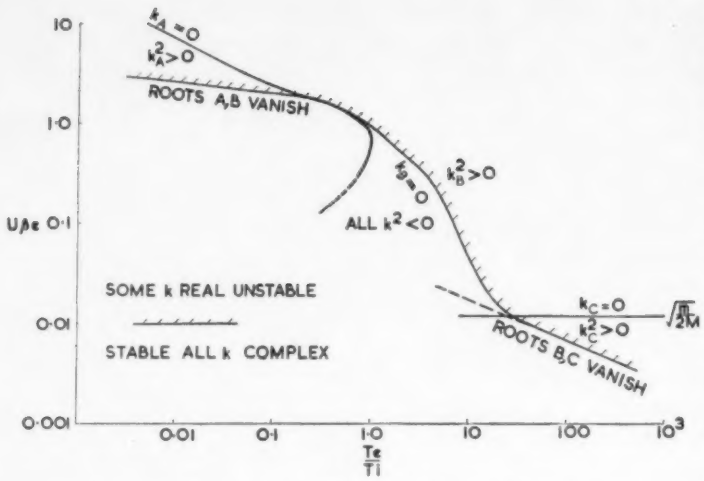

FIG. 2.—Plot of wave number k for ω/k real, $\beta_e^2 \equiv m/2kT_e$ and $Z = 1$.

Thus $p_A q_A$ etc. may be substituted in turn into (7) to determine the curves k_A etc. $= 0$, Fig. 2, and on which side k is real ($k^2 > 0$). It is easy to do this numerically using tabulated values of $g(x)$ (see, for example, HARRIS, 1948). For illustration we shall consider two extreme cases analytically, using the approximations for $g(x)$:

$$g(x) \to 1/2x^2 \quad \text{for} \quad x \gg 1 \quad (\gtrsim 3 \text{ roughly})$$

$$\approx -1 \quad \text{for} \quad 0 < x \ll 1.$$

Case 1 $\quad (ZT_e/T_i \ll 1, \quad u\beta_e \gtrsim 1).$

Root A is given by $p \gg 1$, $q \ll 1$, hence $k_A{}^2 \gtrless 0$ as $p \gtrless \left(\frac{T_i}{2ZT_e}\right)^{1/2}$. For

$T_i/ZT_e \gtrsim 20$, $p \simeq u\beta_e$ hence $k_A{}^2 \gtrless 0$

as $$u\beta_e \lesseqgtr \left(\frac{T_i}{2ZT_e}\right)^{1/2}.$$

Root B is given by $p \gg 1$, $q \ll 1$, hence $k_B{}^2$ is always positive.

Root C is given by $p \ll 1$, $q \gg 1$, hence $k_C{}^2$ is always negative. The lower limit of $u\beta_e$ to the area in which $k_A{}^2 > 0$ is given by the vanishing of roots A and B that is when

$$pe^{-p^2} \geqslant (\text{max. of } qe^{-q^2}).\ ZT_e/T_i. \tag{11}$$

Hence $$p \simeq u\beta_e \quad \text{if} \quad u\beta_e \gtrsim \sqrt{\frac{m}{M}\frac{T_i}{T_e}}.$$

Case 2 $\quad (ZT_e/T_i \gg 1, \quad u\beta_i \ll 1).$

Root A is given by $p \ll 1$, $q \ll 1$ and hence $k_A{}^2$ is always negative.

Root B is given by $p \ll 1$, $q \gg 1$ and $k_B{}^2 \gtrless 0$ as $2q^2 \lesseqgtr \frac{ZT_e}{T_i}$, while

$$p \simeq \frac{ZT_e}{T_i} qe^{-q^2} \tag{12}$$

where

$$u\beta_e = p + q[mT_i/MT_e]^{1/2}. \tag{8}$$

Root C is given by

$$u\beta_e \simeq q[mT_i/MT_e]^{1/2}$$

hence $$q \simeq u\beta_i \tag{13}$$

and $$k_C{}^2 \gtrless 0 \quad \text{as} \quad u\beta_e \lesseqgtr [mZ/2M]^{1/2}. \tag{14}$$

The region of $k^2 > 0$ is bounded below by the vanishing of roots B and C, leaving only root A. This limit can be found by making $p = u\beta_e - q\beta_e/\beta_i$ a tangent to the curve $p = \frac{ZT_e}{T_i} qe^{-q^2}$.

One finds, equating dp/dq,

$$u\beta_e = q\beta_e/\beta_i + (ZT_e/T_i)qe^{-q^2} \tag{15}$$

where $$(2q^2 - 1)e^{-q^2} = \frac{T_i}{ZT_e}\left(\frac{m}{M}\frac{T_i}{T_e}\right)^{1/2} \tag{16}$$

or approximately,

$$u\beta_e \cong q\beta_e/\beta_i = q(mT_i/MT_e)^{1/2} \tag{17}$$

where

$$q^2 \approx \frac{3}{2}\ln(ZT_e/T_i) + \frac{1}{2}\ln(M/mZ) + 0(2\ln q) \tag{18}$$

One concludes that the system is electrostatically stable for $u\beta_e$ less than the limit which is bounded by

(i) roots A, B vanishing for $ZT_e/T_i \ll 1$, $u\beta_e > 1$
(ii) wave number $k_B = 0$ for $ZT_e/T_i \gtrsim 1$, $u\beta \lesssim 1$
(iii) roots B, C vanishing for $ZT_e/T_i \gg 1$, $u\beta_e \lesssim (mZ/2M).^{1/2}$ (19)

ELECTROSTATIC INSTABILITIES IN OHMICALLY HEATED PLASMA

The foregoing criteria can now be applied to determine the onset of electrostatic current instability in an initially quiescent plasma heated by a constant electric current. Ohmic dissipation heats electrons which then convey energy to the ions through Coulomb collisions. If the heating current is small compared to the density, so that the heating rate is slow compared to the collision rate for the transfer of momentum between particles, it is reasonable to suppose that ions and electrons achieve separate Maxwellian distributions, but with $T_e \neq T_i$.

On the other hand, the characteristic growth rates

of electrostatic instabilities, once conditions are ripe for them, have been shown (STRINGER, unpublished) to be $\sim\omega_{p_i}$. This may be so rapid that further effects of collisions *during* the growth of an instability are negligible. For example the quantity $\omega_{p_i}/2\pi\nu$ where ν is the electron momentum transfer rate, is $\approx 3 \times 10^8 \times kT_e^{3/2}\,(nMZ/m)^{-1/2}$ where kT_e is in electron volts. If $kT_e \approx 10$ eV and $n \approx 10^{14}$ cm^{-3} then $\omega_{p_i} \approx 10 \times 2\pi\nu$ for deuterium.

We shall assume a fully-ionized plasma under the initial ($t = 0$) conditions $T_i = T_{i0}$, $T_e = T_{e0}$ heated by a uniform current density j (e.s.u.) $= n|e|u$ such that j/n remains constant for $t > 0$. Under quiescent conditions the progress of such heating is given by well-known equations (SPITZER, 1956)

$$\frac{3}{2}\frac{d}{dt}(nkT_e + nkT_i/Z) = \eta j^2 = \frac{(neu)^2\pi^{3/2}m^{1/2}Ze^2\ln\Lambda}{\gamma_E 2^{5/2}(kT_e)^{3/2}} \tag{20}$$

$$\frac{3}{2}\frac{d}{dt}nkT_i = \frac{k(T_e - T_i)2^{5/2}\pi^{1/2}n^2Z^2e^4\ln\Lambda}{mM(kT_i/M + kT_e/m)^{3/2}} \tag{21}$$

where η is the plasma resistivity, and $\ln\Lambda$ and γ_E are slowly varying functions of n, T_e, and Z respectively. We shall asume that $T_e/T_i \gg m/M$ and also that any expansion of the plasma during heating is negligible.

We divide (21) by (20) and confine attention to $dT_i/dT_e \ll 1$. This is always a permissible approximation in the early phase of heating providing the plasma is initially constrained by, let us say, a sufficiently strong magnetic field. Then

$$dT_i/dT_e = 32\gamma_E Z\pi^{-1}k(T_e - T_i))/Mu^2. \tag{22}$$

This has the approximate solution, for $dT_i/dT_e \ll 1$ (23)

that

$$T_i - T_{i0} \cong 16\gamma_E Z\pi^{-1}k\Big((T_e - T_{e0})^2 + 2(T_{e0} - T_{e0})\Big) \times (T_{e0} - T_{i0})/Mu^2$$

or, if $T_{e0} = T_{i0} = T_0$,

$$T_i - T_{i0} \cong 16\gamma_E Z\pi^{-1}k(T_e - T_0)^2/Mu^2. \tag{24}$$

This equation represents the initial progress of heating and is *independent of the rate of heating in time*. Curve (24) can be plotted on the T_e/T_i plane and compared with the curve of marginal stability, Fig. 3. For convenience let us define

$$x \equiv 2ZkT_e/Mu^2 \tag{25a}$$

$$y \equiv 2kT_i/Mu^2. \tag{25b}$$

The quantities $R = (M/mZ)^{1/2}$ and Z appear as

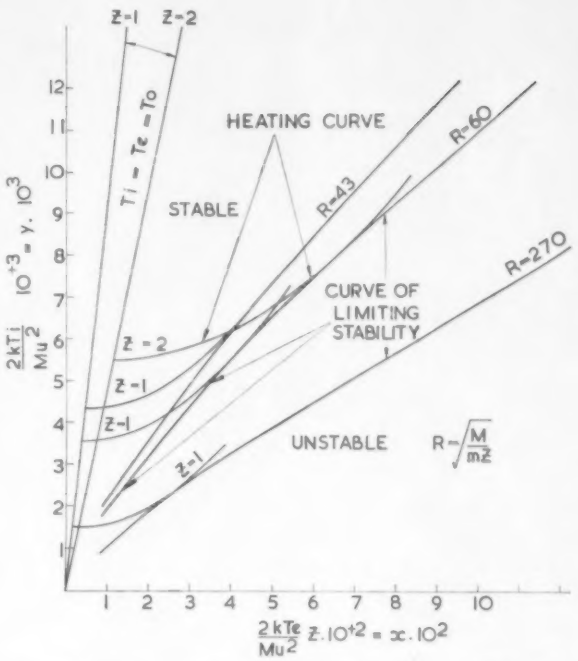

FIG. 3.—T_e–T_i diagram. Curves of marginal stability for $R = \sqrt{M/mZ} = 43$, 60 and 270 and heating at constant current for $Z = 1$ and 2. The area beneath the curve is unstable.

independent parameters. The approximate heating curve (24) is parabolic, with its axis vertical, and vertex lying on the line $x = Zy$ at a point x_0 representing T_0. The stability curve is almost linear, the area beneath it representing instability. The heating curve will lie entirely within the stable region for all initial values of $x = x_0$ greater than a certain critical value x_0^*, which can readily be found. For $x_0 < x_0^*$ the heating curve enters the unstable region at some point x_1y_1. The maximum values of x_1y_1, corresponding to $x_0^*y_0^*$, are x_1^* y_1^* and likewise may be found graphically.

For $x > x_0^*$ *Ohmic heating does not lead to current instability*, so the corresponding T_0^* is a *critical preheat temperature*.

RUNAWAY ELECTRONS

By writing $E_0/\eta = j = neu$ it is easily shown from GIBSON's formula (1957) that the fraction f of electrons with sufficient speed in the direction of the steady field E_0 to run away is

$$f = \exp -4\gamma_E\pi^{-1/2}(M/mZ)^{1/2}(2ZkT_e/Mu^2)^{1/2} \tag{26}$$

$$= \exp -4\gamma_E\pi^{-1/2}(Rx)^{1/2}. \tag{27}$$

Hence we can also plot the runaway fraction as a function of x, R, Z (Fig. 4). The *lower the temperature* (x), *the greater the potential runaway fraction* f. Keeping f very small requires $u \ll (2kT_e/m)^{1/2}$, which

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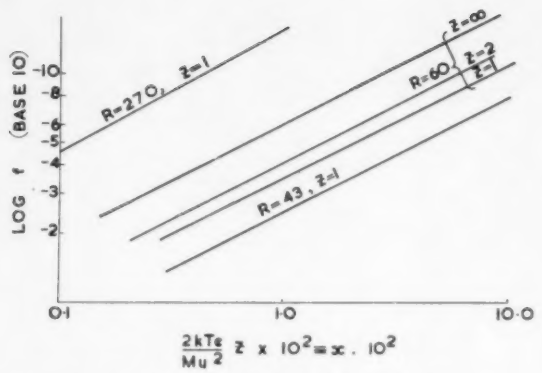

FIG. 4.—Plot of runaway fraction f against electron temperature.

is also a qualitative condition for the validity of the transport theory implicit in (20 and (21). Generally in plasma experiments it is required that f should be $\leqslant 10^{-4}$, a somewhat arbitrary figure based on estimates of the energy carried away by runaways and hence the runaway current.

It will be convenient to write x and y more practically in terms of the current to density ratio j/n, or, what is equivalent, if j/n is constant across the section of a gas discharge, I/N, where I is total current and N is the electron line density.

$$x \text{ (or } y) = ZkT_e(\text{or } kT_i) \times 5 \times 10^{-20} \cdot \frac{M_H}{M}\left(\frac{n}{j}\right)^2 \quad (28)$$

where kT is in electron volts, j/n in amps cm and M_H is the mass of the hydrogen ion.

CONCLUSION TO THEORY

If the initial, or preheat temperature T_0 of a plasma is below a certain critical value T_0^* then Ohmic heating at constant current inevitably induces an electrostatic instability of the current at upper temperatures less than T_{e1}^* T_{i1}^* related to T_0^*. Precise relationships can be found in a simple, graphical way from Fig. 3 using equation (28).

In certain circumstances one expects this effect may be masked by the formation of a beam of runaway electrons, (which itself may be unstable, although this is not considered here) carrying most of the plasma current. The fraction of potential runaways f can be found from Fig. 4. Assuming that $f > 10^{-4}$ represents a catastrophic runaway one concludes from Table 1 that plasmas of H and D are so affected when $T_0 > T_0^*$ and current instabilities are precluded. On the other hand, where the plasma is one of multi-charged, heavier ions ($Z > 1$, $R > 60$, with perhaps doubly-ionized He a marginal case) current instability can take place in the absence of appreciable runaways for preheat temperatures below T_0^*.

EXPERIMENTAL OBSERVATIONS

Application of existing experimental data to this simplified theory is imprecise. This is because of experimental uncertainty over the initial electron and ion preheat temperatures and omission from the theory of energy losses due to ionization and radiation and the effect of collisions. There is also the complication of the drain of plasma electrons due to 'pump out' and the gain of material from the walls as well as possible hydromagnetic instabilities. However, in both Stellarator B1 and ZETA pinch experiments it is possible to estimate from observations those quantities I/N and T_e relevant to the theory. One can then calculate the quantity $u\beta_e$ and the critical upper value of T_e/T_i for current instability. The potential runaway fraction f can also be calculated. These quantities are tabulated in Table 2 for both the early phase of the current pulse and just prior to its extinction (upper and lower figures respectively).

The B1 results appear compatible with the previous theoretical discussion. Here COOR *et al.* and BERNSTEIN *et al.* have found peaks in X-ray emission near the beginning and end of the almost constant current heating pulse (see COOR, 1958 Fig. 10 and 12). The latter peaks coincide with sharp fluctuations in the heating plasma current before its sometimes final step-like extinction. We would associate the first X-ray peak with runaway electrons, when the electron temperature is lowest, whereas the latter peak, or peaks, are compatible with current instabilities precipitated if the electron temperature rises so that T_e

TABLE 1.—THE FRACTION f OF POTENTIAL RUNAWAYS AT THE CRITICAL TEMPERATURE $(2ZkT_0^*/Mu^2) = x_0^*$ REQUIRED TO AVOID CURRENT INSTABILITY IN VARIOUS PLASMAS

$R = (M/mZ)^{1/2}$	43	60		270
Z	1	1	2	1
Critical $x_0^* \times 10^2$ to avoid instability	0·45	0·35	1·10	0·15
Maximum runaway f at x_0^*	2×10^{-2}	8×10^{-3}	5×10^{-5}	3×10^{-6}

TABLE 2.—EXPERIMENTAL AND CALCULATED VALUES FOR B1 AND ZETA

Experiment	Stellarator B1	ZETA
Gas	Helium $Z = 2, R = 60$	Deuterium $Z = 1, R = 60$
Gas current I (amperes)	1200* 1000	$3–4 \times 10^5$* $\sim 2{\cdot}5 \times 10^5$
Line density N (electrons cm^{-1})	$>2 \times 10^{14}$* 5×10^{13}	$>2 \times 10^{17}$* 6×10^{16}
I/N (amp cm)	$<6 \times 10^{-12}$* 2×10^{-11}	$<2 \times 10^{-12}$* 4×10^{-12}
kT_e (eV) observed chiefly by resistance	10* ~ 90	<10* ~ 10
$u\beta_e = 10^{11}\, I/N\, kT_e^{1/2}$	0·2 0·2	0·06 0·13
Critical calculated T_e/T_i for current instability	2·5 2·5	9 6
Containment nt (sec cm^{-3}) i.e. approximate mean towards end of pulse	6×10^9	$\sim 10^{10}$
Calculated runaway fraction f	$<4 \times 10^{-4}$* 10^{-4}	$>10^{-8}$ (?)* 10^{-6}

(* Upper tabulated quantities depend on the usually chosen starting conditions. Below them are those observed just prior to the current chop near, or at the end of the current pulse. B1 data from COOR *et al.*, ZETA from WILSON, GIBSON *et al.*)

appreciably exceeds T_i. One may apply a simple test of whether T_e could in fact exceed T_i sufficiently towards the end of the pulse. From (21), assuming $T_e \gg T_i$, we have

$$\frac{T_i}{T_e} \approx 6 \times 10^{-8} Z^2 \frac{M_H}{M} \overline{nt} (kT_e \text{ eV})^{-3/2}.$$

In B1 it seems likely that $T_e/T_i \sim 2$ or 3, but in ZETA it is more probable that $T_e \approx T_i$.

Qualitatively similar behaviour has been observed to terminate the current pulse in ZETA, but in this case (WILSON, GIBSON and co-workers, 1960) the phenomena appears to be entirely consistent with a runaway electron process.

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NON-ADIABATIC MAGNETIC TRAPS

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Abstract—A novel method of injection into a mirror machine, reported recently by SINELNIKOV *et al.*, is analysed in detail by numerical and perturbation solutions of the equations of motion of a single particle. The principle of the method is to inject particles through one of the mirrors, along the lines of force. The magnetic field in the centre of the machine has a small, spatially periodic radial component which, for certain initial conditions, causes a resonant interchange of energy from the longitudinal to the transverse component of the particle's velocity. This results in reflection of the particles by the end mirror, and by suitable adjustment of the mirrors the particles may be contained inside the machine for an appreciable number of transits.

The possibility of increasing the accumulation of particles by combining this method of injection with molecular-ion dissociation is also discussed.

1. INTRODUCTION

THE well-known phenomenon that a charged particle may be reflected from regions of strong magnetic field forms the basis of many experimental devices for the magnetic confinement of hot plasma. (POST, 1958; BARNETT *et al.*, 1958; KURCHATOV, 1958).

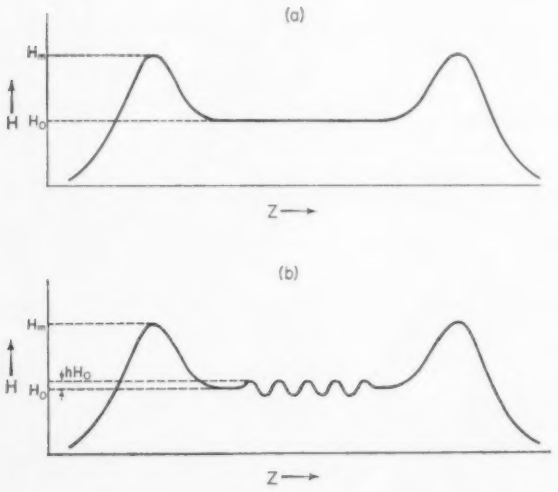

FIG. 1.—(a) Simple mirror machine configuration.
(b) Modulated mirror machine.

A typical mirror machine, or adiabatic trap (see Fig. 1a) consists of a central region of magnetic field H_0 bounded at each end by regions of stronger field H_m (the magnetic mirrors). Particles satisfying certain conditions may be confined in such a trap by repeated reflections from the mirrors; with the exception of a somewhat restricted group of 'absolutely contained' particles, confinement follows from the assumption that the magnetic moment of the particle in the field is an adiabatic invariant.

The magnetic moment μ of a particle is given by

$$\mu = \tfrac{1}{2}mv_\perp^2/H \tag{1.1}$$

where $v_\perp$ is the component of velocity perpendicular to the magnetic field H. It can be shown (SPITZER, 1956) that particles will be reflected from the mirrors if their magnetic moment at the centre of the trap satisfies the relation

$$\mu > W/H_m \tag{1.2}$$

where W is the total energy of the particle. An equivalent criterion is that the velocity vectors of the particles should be inclined to the central field at an angle greater than $\sin^{-1}(R^{-1/2})$, where R is the mirror ratio, H_m/H_0.

If μ is conserved during each reflection from a mirror, and in the absence of collisions or other effects which alter μ, particles satisfying (1.2) should be contained indefinitely.

The assumption of invariance of the magnetic moment is generally considered to be justified if

$$\frac{1}{\omega H}\frac{\partial H}{\partial t} \ll 1 \tag{1.3}$$

$$\frac{r_L}{H}|\nabla H| \ll 1 \tag{1.4}$$

where $\omega = eH/mc$ is the Larmor frequency and $r_L = v_\perp/\omega$ is the Larmor radius. That is to say, the magnetic field must vary slowly in a Larmor period and little over a Larmor radius.

Using numerical methods, GARREN *et al.* (1958)

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have shown that, for a typical mirror machine configuration, the maximum fractional change in magnetic moment at each reflection may be represented by an expression of the form

$$\frac{\Delta\mu}{\mu} = Ae^{-BL/r_L} \tag{1.5}$$

where A and B are constants and L is the distance in which the field goes from H_0 to H_m. It can be seen that the fluctuations in μ can be made as small as we please by increasing the ratio L/r_L. In practice, it is generally considered only necessary to ensure that fluctuations of μ due to non-adiabatic effects are small compared with the inevitable fluctuations due to collisions between particles, which of course set the ultimate limit on containment.

A consequence of the adiabatic invariance of μ is that particles cannot be accumulated in a stationary magnetic trap by injection through one of the mirrors. Experiments designed to create plasma by injection of energetic particles have, up to the present, relied on either (a) increasing the magnetic field continuously while injecting through the mirrors (POST, 1958) or (b) dissociating molecular ions (BARNETT *et al.*, 1958; KURCHATOV, 1958) or ionizing fast neutral particles (GIBSON *et al.*, 1958) inside the trap.

In method (a) the magnetic field is increased slowly enough for the adiabatic criterion (1.3) to be applicable, so that $v_\perp$ increases but $v_\parallel$ (the component of velocity parallel to the field) remains constant. This increases the angle between the velocity vector and the field, so that (1.2) is satisfied and the particles are trapped. In this system accumulation is only possible so long as the magnetic field is increasing. Method (b), in which either the mass or charge state of the particle is changed, creates particles inside the trap satisfying (1.2), and is, therefore, essentially a steady-state method.

There are, however, certain stationary systems for which the adiabatic assumption is not valid and for which it is possible to inject particles through one mirror and contain them for a period which, although necessarily limited, may be long enough to result in appreciable accumulation of particles inside the trap. Such systems we shall call non-adiabatic traps.

A particularly interesting non-adiabatic trap has recently been investigated by SINELNIKOV *et al.* (1960a, b). The familiar configuration of a mirror machine is modified by incorporating a small spatial modulation of the central field (Fig. 1b) which may be represented by

$$H = H_0\left\{1 + h\sin\left(\frac{2\pi z}{\lambda}\right)\right\} \tag{1.6}$$

where $h \ll 1$, and λ is the wavelength of the modulation.

Particles are injected through one mirror along the lines of force, so that initially their magnetic moment is zero. In the absence of the modulation, such particles would pass straight out of the other mirror. However, by arranging their velocity V to satisfy the relation

$$2\pi V \approx \omega\lambda \tag{1.7}$$

where $\omega = eH_0/mc$, a resonant interaction with the modulated field results in the particles acquiring a velocity component perpendicular to the field. The interaction is, essentially, between the axial component of velocity and the spatially-periodic radial component of the field, so that a particle experiences a transverse force which oscillates at the gyrofrequency. The particle therefore gains energy of gyration about the field lines by cyclotron resonance, and because the total energy is conserved, the velocity component in the direction of the field decreases. Clearly, in order to encounter the radial component of the field, the particles must be injected off axis.

Thus although the amplitude of the field variation may be small, and (1.4) be satisfied, the additional condition of resonance results in highly non-adiabatic behaviour of the particles. On their first transit of the trap they may acquire an appreciable magnetic moment, and may therefore be reflected from the end mirror. After the first reflection, further transits of the particles through the region of modulated field will also result in large perturbations of the magnetic moment. In general these will be smaller than the initial change from zero, because the decrease in $v_\parallel$ tends to shift the particles from resonance, and account must now be taken of the phase of the particle in its Larmor orbit as it re-enters the modulated field. The cumulative effect of these changes in μ will eventually result in the condition (1.2) being no longer satisfied and the particles will pass out of one of the mirrors and be lost. However, this may not occur before the particles have completed many transits of the system.

In this paper the motion of particles in this type of non-adiabatic trap will be analysed, with particular reference to the factors which influence the total containment time. The possibility of combining this injection method with dissociation of molecular ions as a means of accumulating a high temperature plasma will also be discussed.

2. EQUATIONS OF MOTION

The magnetic field in the central region of the mirror machine of Fig. 1(b) is taken as being described by a

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vector potential, in cylindrical polar co-ordinates (r, θ, z)

$$\boldsymbol{A} = (0, A_\theta, 0), \chi \text{ where } A_\theta = \tfrac{1}{2} r H_0 f(z).$$

The magnetic field is therefore

$$\boldsymbol{H} = (H_r, 0, H_z), \text{ where } H_r = -\tfrac{1}{2} r H_0 \cdot \frac{df}{zp}$$

and $H_z = H_0 f(z)$.

The Hamiltonian $\mathscr{H}$ for the motion of a particle of mass m and charge e, is

$$\mathscr{H} = \frac{1}{2m}\left\{ pr^2 + \left(\frac{p_0}{r} - \frac{e}{c} A_\theta\right)^2 + p_z^2 \right\} \tag{2.1}$$

from which the equations of motion may be written down immediately; thus

$$\frac{d^2 r}{dt^2} = \frac{p_\theta^2}{m^2 r^3} - \frac{e^2 H_0^2}{4m^2 c^2} r f^2 \tag{2.2}$$

$$\frac{d^2 z}{dt^2} = \left\{ \frac{p_\theta}{mr} - \frac{erH_0 f}{2mc} \right\} \frac{erH_0}{2mc} \cdot \frac{df}{dz}. \tag{2.3}$$

In (2.2) and (2.3), p_θ is a constant of the motion and is related to the motion of the particle about the axis by

$$p_\theta = mr^2 \frac{d\theta}{dt} + \frac{e}{c} r A_\theta$$

$$= mr^2 \frac{d\theta}{dt} + \frac{e}{c} r^2 H_0 f. \tag{2.4}$$

It is convenient to introduce the variable $\tau = \frac{1}{2}\omega t$, where $\omega = eH_0/mc$, and to replace p_θ by $\frac{1}{2}mC$. Then equations (2.2) to (2.4) become

$$\frac{d^2 r}{d\tau^2} = \frac{C^2}{r^3} - r f^2 \tag{2.5}$$

$$\frac{d^2 z}{d\tau^2} = (C - r^2 f) \frac{df}{dz} \tag{2.6}$$

$$C = r^2 \left(\frac{d\theta}{d\tau} + f \right). \tag{2.7}$$

These equations have a first integral

$$\left(\frac{dr}{d\tau}\right)^2 + \left(\frac{dz}{d\tau}\right)^2 + \left(\frac{C}{r} - rf\right)^2 = v^2 \tag{2.8}$$

where the parameter v is related to the total velocity V of the particle by

$$V = \tfrac{1}{2}\omega v \tag{2.9}$$

Using (2.8), (2.5) and (2.6) can be combined into the single equation

$$\frac{d^2 r}{dz^2} = \frac{\left(1 + \left(\frac{dr}{dz}\right)^2\right)\left(C - r^2 f\right)\left(\frac{C}{r^3} + \frac{f}{r} - \frac{dr}{dz} \cdot \frac{df}{dz}\right)}{v^2 - \left(\frac{C}{r} - rf\right)^2} \tag{2.10}$$

with z now the independent variable.

In most of this work the field function is taken as

$$f(z) = 1 + h \sin z \tag{2.11}$$

where $\lambda/2\pi$ is taken as the unit of length. The condition for resonance, therefore, becomes $v \approx 2$.

Consider a particle that enters the modulated field parallel to the axis, at a radius r_0. The boundary conditions for (2.10) are then

$$r = r_0, \frac{dr}{dz} = 0 \text{ at } z = 0$$

which are equivalent to the boundary conditions for (2.5) and (2.6)

$$r = r_0, \frac{dr}{d\tau} = 0, z = 0, \frac{dz}{d\tau} = v, \text{ at } \tau = 0.$$

For such initial conditions, $C = r_0^2$.

It is convenient to consider a normalized magnetic moment ξ defined by

$$\xi = \frac{v_\perp^2}{v^2} = 1 - \frac{\left(\frac{dz}{d\tau}\right)^2}{v^2}. \tag{2.12}$$

ξ can have values between 0 and 1, and the condition for the reflection of a particle from a magnetic mirror of ratio R is

$$\xi > \frac{1}{R}. \tag{2.13}$$

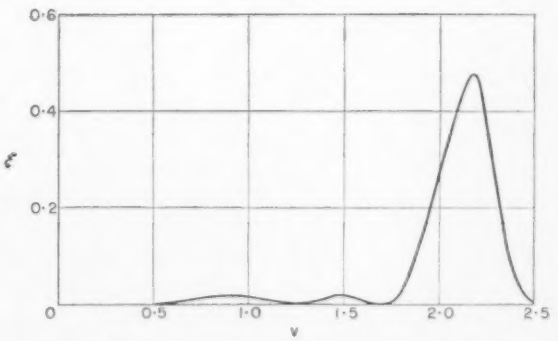
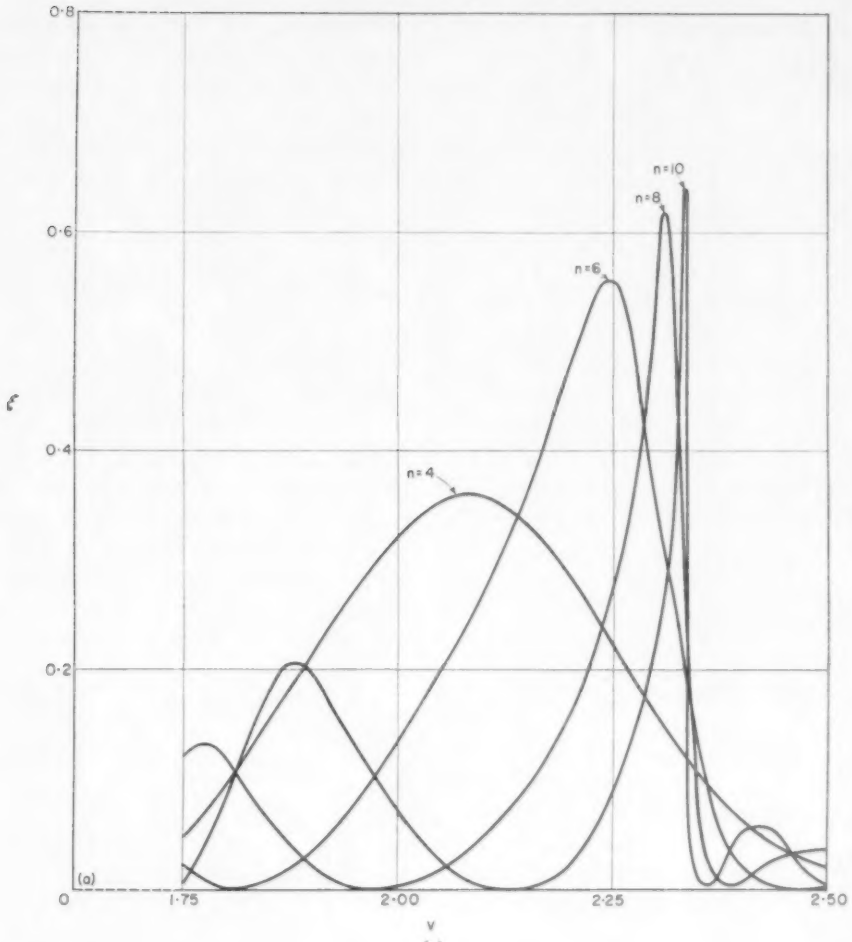
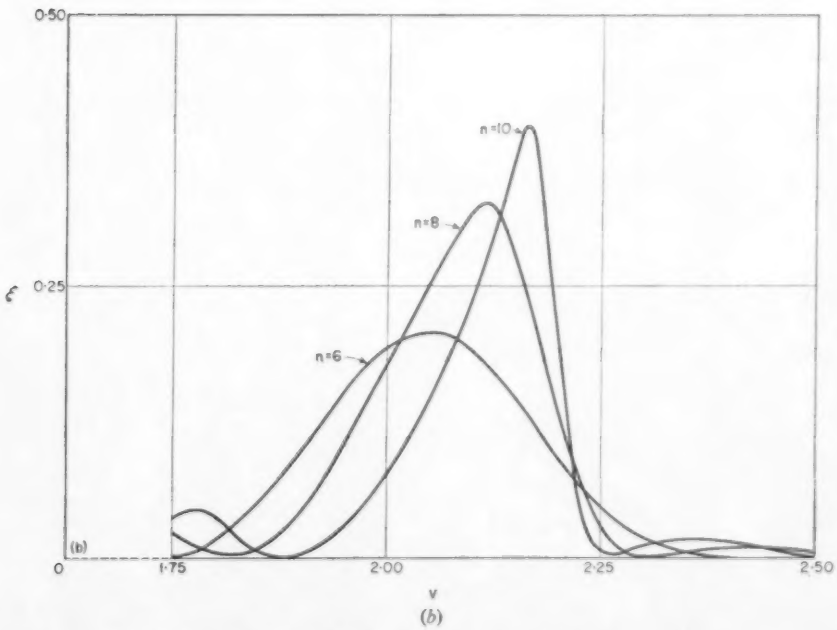


FIG. 2.—ξ as a function of v, first transit. $n = 5$, $h = 0{\cdot}05$.

Numerical solutions of (2.10) have been obtained which give ξ after the particle has passed through an integral number n of wavelengths of the modulation. Figure 2 shows ξ as a function of v, for $n = 5$ and $h = 0{\cdot}05$. Figure 3 shows details of the resonance



(a)



(b)

FIG. 3.—ξ as a function of v, first transit. (*a*) $h = 0{\cdot}05$; (*b*) $h = 0{\cdot}025$.

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for values of n up to 10, and $h = 0{\cdot}05$ and $0{\cdot}025$ respectively. For all these cases, $r_0 = 2$.

These results show clearly the main features of the interaction. There is a pronounced resonance for $v \approx 2$. As n is increased, the height of the resonance increases at first rapidly (approximately as n^2) and then more slowly; the effect of increasing n further is to increase the sharpness of the resonance and to shift the peak to a value of v somewhat greater than 2.

The shift in the resonance arises from the coupling between the transverse and axial velocity components through the conservation of energy. The axial velocity decreases as the particle moves through the modulated field; by taking v slightly greater than 2 this tends to draw the particle into resonance, and to increase the total effect of the n periods. At the same time a second, smaller peak appears for v somewhat less than 2.

3. VARIATION OF THE WAVELENGTH OF THE MODULATION

In a system of constant wavelength, it is clearly not possible to satisfy the resonance condition (1.7) for every period of the modulation, since the axial velocity is continuously decreasing. If however, the wavelength of the modulation were also progressively decreased, it should be possible to satisfy the resonance condition more closely for each period.

As an example, ξ has been calculated as a function of v for a system of progressively decreasing wavelength given by

$$f(z) = 1 + h \sin (z + \alpha z^2) \tag{3.1}$$

where α is positive.

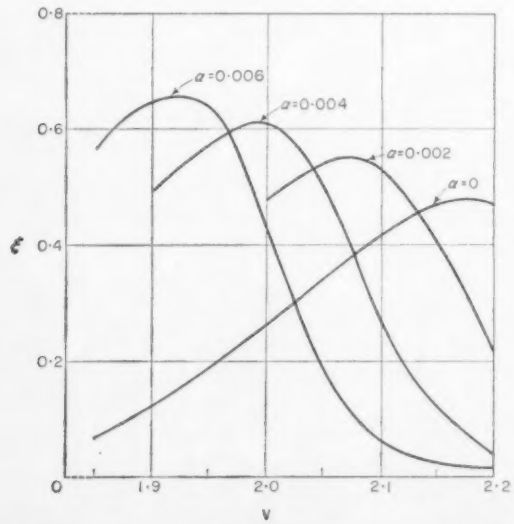

FIG. 4.—Alteration of the resonance by different staggering factors, α.

The results, for $h = 0{\cdot}05$ and $n = 5$, and different values of α are given in Fig. 4. As can be seen, the effect of increasing α is to increase the height of the resonance, and to a less marked extent, to decrease its width, while shifting the peak to lower v.

The expression (3.1) was chosen for convenience of computation. To carry this treatment to its logical conclusion, a system could be devised for which the points at which $\dfrac{d\theta}{dz} = 0$ on the particle trajectory would coincide with points of the field for which $H_r = 0$. This would ensure that the force on the particle was always acting to increase its transverse velocity, and for a given h and n, would produce the maximum ξ. However, without going to such lengths, it can be seen that in a system with uniform, or slightly staggered modulation, with $n = 5$ and $h = 0{\cdot}05$ it is possible to achieve $\xi = 0{\cdot}5$ to $0{\cdot}65$ on the first transit. Thus a particle injected into the mirror machine of Fig. 1(b) along the lines of force would be reflected from the end mirror if $R > 2$.

4. PERTURBATION THEORY OF THE FIRST TRANSIT

Let us expand r as a power series in h, thus

$$r = r_0 + hr_1 + \cdots \tag{4.1}$$

and use equation (2.10) with the assumption (2.11) for f. The equation for r_1 is

$$v^2 \frac{d^2 r_1}{dz^2} = -4r_1 - 2r_0 \sin (z) \tag{4.2}$$

with the boundary conditions $r_1 = \dfrac{dr_1}{dz} = 0$ at $z = 0$.

The solution of (4.2) is then

$$r_1 = \frac{r_0}{v^2 - 4}\left(2 \sin (z) - v \sin \left(\frac{2z}{v}\right)\right) \tag{4.3}$$

and for values of $z = 2n\pi$,

$$r_1 = -\frac{r_0 v}{v^2 - 4} \sin \left(\frac{4n\pi}{v}\right) \tag{4.4}$$

$$\frac{dr_1}{dz} = \frac{2r_0}{v^2 - 4}\left\{1 - \cos \left(\frac{4n\pi}{v}\right)\right\} \tag{4.5}$$

inserting (4.4) and (4.5) in (2.8) and (2.12) immediately yields

$$\xi = \frac{8r_0^2 h^2}{v^2 - 4}\left\{1 - \cos \left(\frac{4n\pi}{v}\right)\right\}. \tag{4.6}$$

In Fig. 5 the results of (4.6) are compared with the direct numerical solution of (2.10), taking $h = 0{\cdot}05$. Agreement is excellent for $n = 1$, but it can be seen

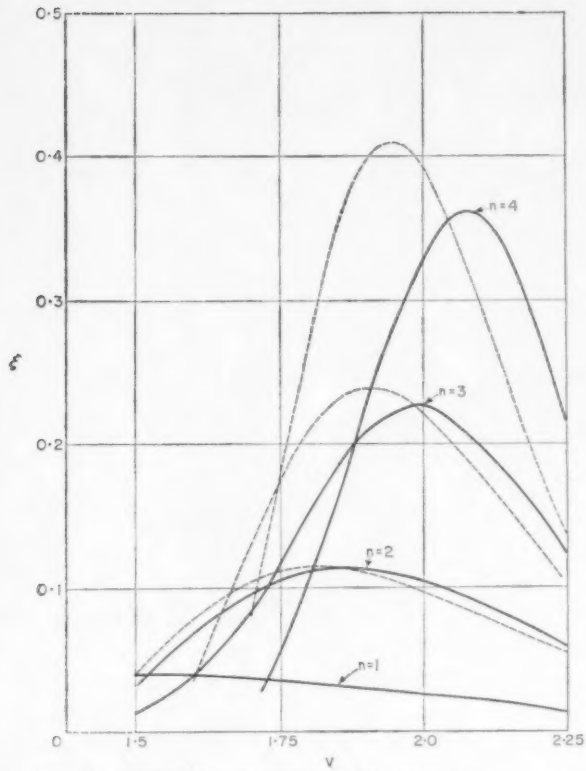

FIG. 5.—Comparison of numerical integration (————) and perturbation theory (– – – – –), first transit.

that to obtain reasonable agreement beyond $n = 3$ it is necessary to go beyond second order effects.

The effects of higher order terms can be obtained by an extension of the methods outlined in this section, but will not be considered here. However, the simple perturbation theory is valuable in that it shows that, for small ξ, $\xi \propto h^2 r_0^2$ and $\xi_{max} \propto n^2 h^2 r_0^2$.

5. EFFECT OF MAGNETIC MIRRORS

Let us now consider the behaviour of a particle which has gained sufficient magnetic moment on its first transit to be reflected by the end magnetic mirror, and now moves back through the modulated field.

To avoid using computing time in evaluating the exact path of the particle in the mirror, it is assumed that the mirror is perfectly adiabatic and reflects the particle with the same velocity components along and transverse to the field, and with its guiding centre at the same radius. The only variable introduced by the mirror will then be a difference in the phase of the particle in its Larmor orbit, given by

$$\phi = \phi(u, L) \tag{5.1}$$

where u is the axial velocity component of the particle as it leaves the modulation, and L is a characteristic length associated with the mirror.

In general $\phi(u, L)$ will be a complicated function, but for simplicity we assume that the mirror can be represented by a length $L/2$ of uniform magnetic field H_0 terminating in a plane P at which u is reversed (Fig. 6a). The phase change of a particle reflected by this system is then given by

$$\phi\,(u, L) = \frac{\omega L}{u} = \frac{2L}{u} \tag{5.2}$$

since $\omega = 2$ in the units used here.

FIG. 6.—(a) Representation of magnetic mirror.
(b) Representation of a trap with two mirrors.

A derivation of the phase change at a more realistic mirror is given in the Appendix, but in what follows (5.2) will be used. This simplification does not affect the validity of the general description of the particle behaviour.

If now the particle leaves the modulation at the point $r = a_1$, $(dr/d\tau) = b_1$ in phase space and suffers a phase change ϕ in the mirror, it will return to the modulation at the point $r = a_2$, $(dr/d\tau) = b_2$ given by

$$a_2^2 = \frac{1}{2}\left(a_1^2 + b_1^2 + \frac{C^2}{a_1^2}\right) + a_1 b_1 \sin\phi + \frac{1}{2}\left(a_1^2 - b_1^2 - \frac{C^2}{a_1^2}\right)\cos\phi \tag{5.3}$$

$$b_2 = \frac{1}{a_2}\left\{a_1 b_1 \cos\phi - \frac{1}{2}\left(a_1^2 - b_1^2 - \frac{C^2}{a_1^2}\right)\sin\phi\right\} \tag{5.4}$$

where C has the same meaning as in (2.7).

With these initial conditions, the return path of the particle through the modulation can be computed. As an example, we take a system with $h = 0{\cdot}05$ and $n = 5$ and choose a particle injected at $r_0 = 2$ with $v = 2{\cdot}15$. This particle will receive $\xi = 0{\cdot}475$ on its first transit. The value of ξ after the second transit as a function of the mirror parameter L is shown in Fig. 7. It can be seen that ξ may be reduced to zero (which corresponds to the particle retracing its path in (ξ, z) space back to the point of injection), or still further increased by suitable choice of L.

Consider now a magnetic trap with two mirrors, having characteristic lengths L_1, L_2 (see Fig. 6b).

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L_1 is chosen from Fig. 7 to maximize ξ on the second transit. The value of ξ after the third transit as a function of L_2 is then given in Fig. 8.

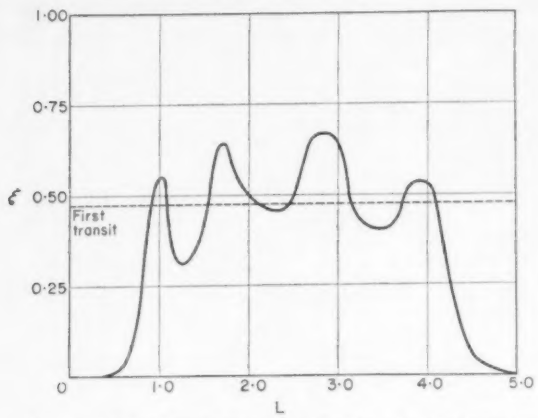

FIG. 7.—ξ after second transit as a function of mirror length L.

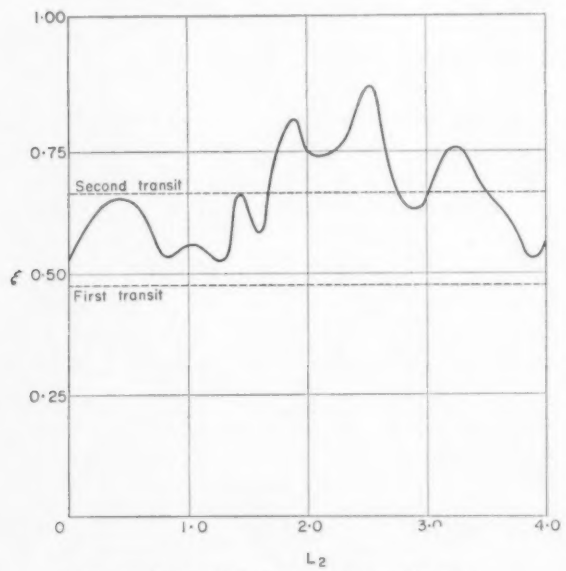

FIG. 8.—ξ after third transit, as a function of L_2.

It can be shown that for a wide range of (L_1, L_2) the particle will still be well trapped ($\xi > 0{\cdot}5$) after three transits. Having now set L_1 and L_2, the system is fixed, and the particle will oscillate between the mirrors until (2.13) is violated, when it will pass out of one of the mirrors.

The variation of ξ over many traverses has been computed for a typical pair of values L_1, L_2 and the results given in Fig. 9. In this case ξ fell to 0·05 after the 27th transit, and the particle would be lost unless the mirror ratio were greater than 20. However, up to the 27th transit, it would have been contained by a mirror ratio of 2·5.

The choice of L_1, L_2 in this case was to a certain extent arbitrary, and there is no reason to suppose

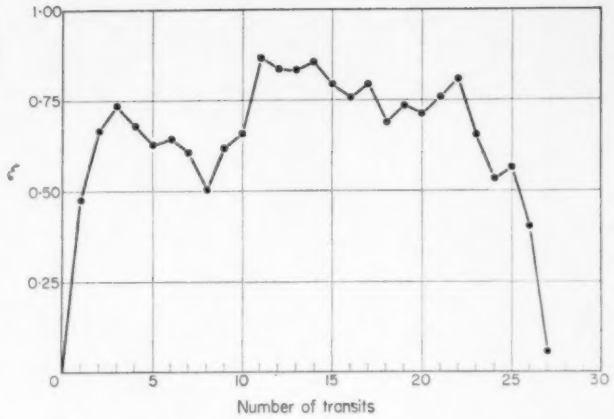

FIG. 9.—Variation of ξ over many transits. $L_1 = 2{\cdot}854$, $L_2 = 3{\cdot}122$.

that this case represents the upper limit of containment time. To try and obtain the conditions for maximum containment by the computational methods used here would obviously be a marathon task. However, the foregoing treatment suggests that it should be possible to adjust L_1 and L_2 experimentally by altering, either physically or electrically, the distances between the mirrors and the modulated field region, and so to 'tune' the trap for maximum containment.

SINELNIKOV *et al.*, (1960b) have reported experimental results for the trapping of electrons in a system of this kind. Their experiments were designed to inject large currents and to accumulate a space charge of electrons which could be used as a potential well for ions. The behaviour of the particles was affected by the space charge, and there was also considerable ionization of the residual gas. No measurements were made of the containment time of single particles, but from the injected current and the accumulated density one can infer that the electrons were probably contained for several hundred transits of the system.

6. REGION ACCESSIBLE TO TRAPPED PARTICLES

One simple result follows from the constants of the motion. Consider a particle in the region of constant field H_0 having magnetic moment $\xi > 0$. Let r_G be the radial position of its guiding centre, and r_L be its Larmor radius (see Fig. 10). From equation (2.4) we have

$$p_0/m = r^2 \frac{d\theta}{dt} + \tfrac{1}{2}\omega r^2, \tag{6.1}$$

and since initially $(d\theta/dt) = 0$ and $r = r_0$, and p_0 is a constant of motion

$$r^2 \frac{d\theta}{dt} = \tfrac{1}{2}\omega (r_0^2 - r^2). \tag{6.2}$$

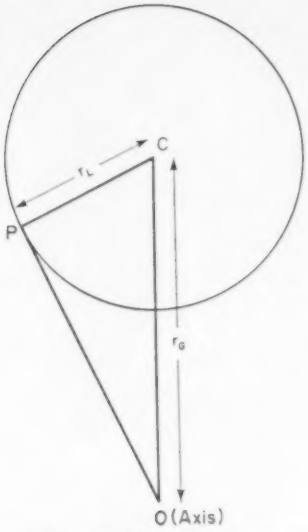

FIG. 10.—Representation of Larmor orbit. $CP = r_L$, $OC = r_0$.

So if we choose on the orbit in Fig. 10 the point P at which $(d\theta/dt = 0)$, clearly $OP = r_0$, from which it follows that

$$r_G^2 = r_L^2 + r_0^2. \tag{6.3}$$

The maximum value of r_L is $V/\omega = v/2$, in the units of the previous sections. Therefore, the particle must lie between the limits

$$r = \tfrac{1}{2}\{(4r_0^2 + v^2)^{1/2} \pm v\} \tag{6.4}$$

which for the examples computed in Section 5, where $r_0 = 2$ and $v = 2{\cdot}15$ are $r_{\max} = 3{\cdot}345$ and $r_{\min} = 1{\cdot}195$ respectively.

Owing to the precession of the guiding centre when a particle is reflected by a magnetic mirror, the particles are in fact confined to the annular volume bounded by these radii, a result that is independent of any assumption of adiabaticity of the mirrors.

7. POSSIBILITY OF MOLECULAR DISSOCIATION

An obvious extension of this method of injection is to make the injected particles molecular ions, and to arrange that, during their limited containment some are dissociated into atomic ions of larger e/m (for example, $H_2^+ \rightarrow H^+ + H^0$ by collision with neutral gas molecules). The atomic ions, having the same velocity but half the gyrofrequency of their parents, are further from resonance with the modulated field and might therefore be expected to have significantly longer containment times. To confirm this it is necessary to consider in detail the effect on their magnetic moment of repeated passages through the modulated field.

The dissociation of a particle of charge e and mass m_1 into a particle of the same charge and mass m_2 at any point in the orbit calculations of Section 5 may be simulated by changing the constants C and v to C' and v' given by

$$C' - r^2 = \gamma(C - r^2) \tag{7.1}$$

$$v' = \gamma v \tag{7.2}$$

where γ is the ratio m_2/m_1. Continuing the calculation then gives the orbit of the dissociated particle.

Consider first the passage of a dissociated particle through a region of continuously modulated field. As an example, we take a particle with $\xi = 0{\cdot}565$, $r = 1{\cdot}84$, $\gamma = \frac{1}{2}$, $\dfrac{dr}{dz} = 0$ at $z = 0$, and follow the variation of ξ over 25 periods of the modulation.

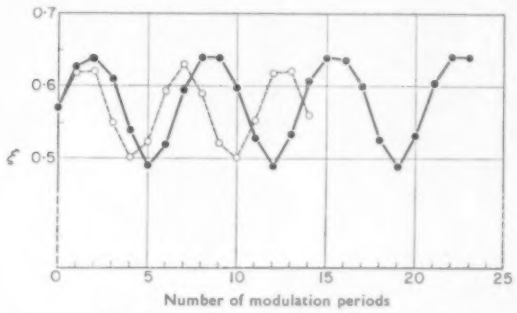

FIG. 11.—Variation of ξ for dissociated particle in continuous modulation. Numerical computation (———○———) and perturbation theory (– – –○– – –).

Figure 11 shows the results; it can be seen at once that ξ varies about a mean value in a periodic manner and that the amplitude of the variation is small. This suggests that a perturbation theory might be used to describe the particle's behaviour. As in Section 4, therefore, we take

$$r = r_0(\tau) + hr_1(\tau) \tag{7.3}$$

$$z = z_0(\tau) + hz_1(\tau) \tag{7.4}$$

and use equations (2.5) and (2.6) with the assumption (2.11) for f.

To zero order (2.5) and (2.6) become

$$\frac{d^2r_0}{d\tau^2} = \frac{C^2}{r_0^3} - r_0 \tag{7.5}$$

$$\frac{d^2z_0}{d\tau^2} = 0 \tag{7.6}$$

the solutions are

$$r_0^2 = r_L^2 + r_G^2 + 2r_L r_G \cos(\phi_0 + 2\tau) \tag{7.7}$$

$$z_0 = u\tau \tag{7.8}$$

where u is the axial velocity component, r_L and r_G are

defined in Fig. 10 and ϕ_0 is the phase angle of the particle in its Larmor orbit at $\tau = 0$.

From (2.6), to first order

$$\frac{d^2z_1}{d\tau^2} = (C - r_0^2)\cos(z_0). \tag{7.9}$$

Combining (7.7) to (7.9) and integrating, with $\tau = 2n\pi/u$ we have

$$\frac{dz_1}{d\tau} = \frac{4r_L r_G}{4-u^2}\left[\sin(\phi_0) - \sin\left(\phi_0 + \frac{4n\pi}{u}\right)\right] \tag{7.10}$$

and since, to first order

$$\xi = \xi_0 - \frac{2hu}{v^2}\frac{dz_1}{d\tau} \tag{7.11}$$

the variation of ξ is given by

$$\xi = \xi_0 - \frac{8r_L r_G hu}{v^2(4-u^2)}\left[\sin(\phi_0) - \sin\left(\phi_0 + \frac{4n\pi}{u}\right)\right] \tag{7.12}$$

The results of (7.12) are compared with the direct numerical integration of the equation of motion in Fig. 11. It can be seen that whereas (7.12) gives a satisfactory description of the maximum amplitude fluctuations in ξ there is appreciable discrepancy in where they occur. This is because u is considered as constant and the second sine term in (7.12) is sensitive to small changes in u. To obtain an accurate description of the particle orbit would require higher order terms to be considered, but from (7.12) we can at least infer that the maximum change in ξ per transit is

$$(\Delta\xi)_{\max} \approx \frac{16r_L r_G hu}{v^2(4-u^2)} \tag{7.13}$$

which, for the initial parameters used previously gives

$$(\Delta\xi)_{\max} \approx 0{\cdot}1. \tag{7.14}$$

The effect of these fluctuations in ξ on the containment of the dissociated particles depends on whether their cumulative effect can at some stage reduce ξ to a very low value, or whether it merely causes ξ to fluctuate between fixed limits. We shall not attempt to give a rigorous answer to this question; however, taking the particle of Fig. 9 and simulating dissociation ($\gamma = \frac{1}{2}$) at several different points on its orbit, we integrated (2.10) to follow the dissociated particle for 30 transits. In every case, the ξ of the new particle showed appreciable fluctuations (typically, $|\Delta\xi| \sim 0{\cdot}05$ per transit) but in no case did the value of ξ fall below 0·4. That is to say, the dissociated particle would certainly have been contained for longer than its parent ion.

The computational methods used restricted the number of transits that could be followed to about 30, so to obtain some further idea of the particle's behaviour over many transits we used (7.10) to give the change in u per transit and (5.2) to give the change in phase at each reflection. The constants of the motion were used to adjust r_L and r_G each time and the variation of ξ was 'followed' for 250 transits. The general behaviour closely resembled the results of the numerical solutions, and in the five cases chosen the lowest value reached by ξ was 0·2.

A result which follows from (7.13) is that dissociation into still lighter particles, for example, $\gamma = \frac{1}{3}, \frac{1}{4}$, has very little effect on the magnitude of the fluctuations in ξ. This has been confirmed by direct computation.

8. DISCUSSION

To make full use of a system with magnetic mirrors as a device for containing a hot plasma, the mean free path for containment of a single particle should be longer than the mean free path for scattering by other particles. For the densities and temperatures of interest in thermonuclear research, the scattering mean free path generally corresponds to 10^6 or more transits between the mirrors, and it has been confirmed experimentally that the containment of a single particle can be made of this order in an adiabatic trap (RODIONOV, 1959).

It is difficult to imagine that a practical non-adiabatic trap could be devised for which the containment of externally injected particles would be as long as this. However, the results of this paper at least indicate how the containment in such a trap might be optimized. In the first place, it appears that for the first transit $\xi \propto n^2h^2$ and for subsequent transits $\Delta\xi \propto h$. This suggests that n should be made large, and h as small as consistent with trapping on the first transit. h could be further reduced if the modulations were staggered in the manner described in Section 3. Secondly, for any given modulation system, the trap could be 'tuned' for maximum containment by suitable adjustment of the mirrors, as described in Section 6.

Combining this method with molecular-ion dissociation may lead to much longer containment. Although the mechanism which traps the injected particles also causes them to be lost, it does not follow that the dissociated particles must also escape, since these are created inside the trap. The fluctuations in their magnetic moment do not necessarily indicate poor containment, since similar fluctuations have been found by GARREN *et al.* for particles contained in an unmodulated trap. CHIRIKOV (1959) has suggested

that resonances between the higher harmonics of the field and the Larmor rotation of the particle may result in large changes in ξ for certain particles in a simple mirror machine, and this effect may very well be more significant in the modulated case.

Finally, two practical advantages of this system may be mentioned. Firstly, it should be possible to achieve longer path lengths, L, for the injected beam that in systems with an internal injector, such as OGRA. This in turn reduces the critical current for 'burnout' which varies as $1/L^2$ (see KURCHATOV). Secondly, the undissociated fraction of the beam passes out of one of the mirrors and may be led away and disposed of, whereas in OGRA it strikes the internal injector and becomes a source of unwanted neutral gas inside the machine. Both these features could lead to a reduction in critical current and physical size of apparatus required to obtain burn-out.

9. CONCLUSIONS

It seems that by combining SINELNIKOV's method of trapping externally injected particles for a limited time in a non-adiabatic trap with molecular ion dissociation, the accumulation of particles in the trap might be significantly increased, particularly if large mirror ratios are used.

Whether the containment of the dissociated ions can be made so long that it is limited only by collisional scattering between particles is a matter for more sophisticated theory, or for experiment.

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APPENDIX

Derivation of the Phase Change upon Reflection by a Magnetic Mirror

Let us consider a mirror with parabolic shape, given by

$$H = H_0(1 + z^2/L^2). \tag{A 1}$$

Since we are concerned only with reflected particles, the mirror ratio is immaterial. The characteristic length L is assumed to be long enough for the assumption of invariance of the magnetic moment to be justified.

The phase change of a particle of magnetic moment μ and total energy W is

$$\phi = 2\int_0^{t_p} \omega \, dt \tag{A 2}$$

where $\omega = \dfrac{eH(z)}{mc}$. Changing the variable we have

$$\phi = \frac{e}{c}\sqrt{\frac{2}{m}}\int_{H_0}^{H_p} \frac{HdH}{\dfrac{dH}{dz}\sqrt{W-\mu H}} \tag{A 3}$$

where H_p is the maximum field reached by the particle and t_p the time it takes to get there. H_p is given by

$$H_p = W/\mu \tag{A 4}$$

and by combining (A 1) (A 3) (A 4) we obtain

$$\phi = \pi\left(\frac{L}{r_L}\right)(1 + \tfrac{1}{2}\eta) \tag{A 5}$$

where $r_L = v_\perp/\omega_0$ is the Larmor radius in the field H_0 and

$$\eta = \left(\frac{W}{\mu H_0} - 1\right).$$

Similar expressions can be derived for fields of different shape.

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SELF-CONSISTENT FIELD THEORY OF PLASMA BETATRONS

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Abstract—A quasi-stationary, self-consistent field theory is developed for the space-charge neutralized plasma betatron. It is found that the equilibrium orbit shrinks in the first phase of the acceleration process, and this sets a limit to the number of particles which can be accelerated. Exceeding this limit the beam collapses. During the acceleration process, the beam passes through a betatron damping dominated and a self-pinching phase, and finally enters the relativistic regime. Limits where these phases set in, as well as the beam behaviour in different phases is given.

1. INTRODUCTION

IN recent years several schemes have been proposed for containment of positive ions with the electromagnetic field of a relativistic electron beam (BENNETT 1934, 1955; BUDKER 1956; FINKELSTEIN 1958; DE PACKH, 1958; CHRISTOFILOS, 1958). The principles of these schemes are fundamentally similar, the intended applications, however, fall into two distinctly separate groups: high current strong focussing particle accelerators, like BUDKER's machine, and thermonuclear machines as the Astron (CHRISTOFILOS, 1958). In the formulation of the operation of these machines both groups face similar difficulties. On one hand beams of particles are employed as in accelerators, on the other hand particle densities and self-fields are high enough so that the machines can be considered to be plasma devices. The plasma behaviour of the systems call for a self-consistent field treatment, while their beam character prevents us from using the powerful magnetohydrodynamic plasma models, based on the assumption of small Larmor radii and the guiding centre approximation. In these devices the Larmor radii far exceed some of the beam dimensions.

Self-consistent behaviour of electron-ion beams in their own electromagnetic field-neglecting external guiding fields—has been treated in various papers. The stationary state of a relativistic electron sheath in the homogeneous magnetic field of Astron has been investigated by (TONKS 1959, 1960). We attempt here to carry out a similar calculation for the quasi-stationary behaviour of a particle beam in the field of a plasma betatron (FINKELSTEIN, 1958; ROGERS *et al.*, 1959).

The plasma betatron is essentially an ordinary betatron accelerating electrons and ions on the same equilibrium orbit in opposite directions. The presence of electrons and ions of the same density in the beam leads to charge neutralization as well as pinching of the beam due to the self-magnetic fields. We will assume that the beam minor diameter is small compared to the characteristic dimensions of the guiding fields. It is assumed furthermore that interparticle collisions and radiation play no important role in the process. The self-consistent field treatment means that each particle moves under the influence of the electromagnetic field composed of the guiding field and the field produced by the particles themselves. This latter field can be deduced from the density and velocity distribution of the particles through Maxwell's equations.

Since in the first phase of the acceleration process the electrons are non-relativistic and ions (in present machines) always remain so we start with the treatment of the non-relativistic case and proceed later with the relativistic limit. We restrict ourselves to "quasi-stationary" processes (the term will be explained later), hence fast processes, like instabilities, fall outside the scope of the present treatment.

2. NON-RELATIVISTIC CASE

(a) *The equilibrium orbit*

We choose a cylindrical co-ordinate system, with the symmetry axis of the betatron as z axis. The magnetic field is characterized by the vector potential:

$$\mathbf{A} = \mathbf{A}^b + \mathbf{A}^s \tag{1}$$

where $\mathbf{A}^b$ is the contribution of the betatron field and $\mathbf{A}^s$ comes from the self-field of the beam. Both fields are independent of the azimuthal co-ordinate φ, and magnetic field lines lie in $r = z$ planes. The vector potential can therefore be chosen in the simple form:

$$A_r = A_z = 0; \qquad A_\varphi = A_\varphi(r,z). \tag{2}$$

The non-relativistic motion of a particle in this field can be described by the Hamiltonian:

$$\mathscr{H} = \frac{P_r^2}{2m} + \frac{P_z^2}{2m} + \frac{1}{2m}\left(\frac{P_\varphi - qrA_\varphi}{r}\right)^2 + qV \tag{3}$$

where the scalar potential V is also independent of φ. Therefore P_φ is a constant of motion. At $t = 0$ (injection time) the particle energy as well as A_φ is zero. Consequently from equation (3)

$$P_\varphi = 0. \tag{4}$$

Equation (3) reduces therefore to:

$$\mathscr{H} = \frac{P_r^2}{2m} + \frac{P_z^2}{2m} + \psi \tag{5}$$

where

$$\psi = \frac{q^2 A_\varphi^2}{2m} + qV. \tag{6}$$

This describes a particle moving in the 2-dimensional $r - z$ plane under the influence of the effective potential $\psi(r,z)$ which is a function of both impressed and self-field.

Our programme is the following: We use the requirements for self-consistency in order to express every quantity in terms of the impressed betatron field quantities, and some beam parameters. The φ component of the particle velocity becomes, using the Hamiltonian equation (3)

$$v_\varphi = r\dot{\varphi} = -\frac{qA_\varphi}{m}. \tag{7}$$

The beam will be accommodated at a radius $r = R$ and plane of symmetry $z = 0$, where the effective potential ψ has a minimum, and produces a current:

$$I = \sum_{e,i} Nqv_\varphi \approx N^e q^e v_\varphi{}^e = -\nu A_\varphi(R,0) \tag{8}$$

where e and i stand for electrons and ions respectively. N is the number of particles per unit beam length, and

$$\nu = \left(\frac{Nq^2}{m}\right)^e. \tag{9}$$

The contribution of the beam to the vector potential is proportional to the beam current:

$$A_\varphi{}^s(R,0) = \lambda I \tag{10}$$

where λ is a geometrical factor related to the self inductance of the beam L through:

$$\lambda = \frac{L}{2\pi R}. \tag{11}$$

The resulting vector potential at the beam can be expressed in terms of the betatron field, by combining equations (1), (8), and (10):

$$A(R,0) = \frac{A^b(R,0)}{1 + \nu\lambda} \tag{12}$$

dropping index φ.

Even for a thin beam particles will oscillate in the potential well ψ, around the $r = R$, $z = 0$ circle as the beam axis. Since this is a common axis for both electron and ion component beams the space charge field vanishes here:

$$\left(\frac{\partial V}{\partial r}\right)_{R,0} = 0; \quad \left(\frac{\partial V}{\partial z}\right)_{R,0} = 0. \tag{13a}$$

Furthermore the definition of R is:

$$\left(\frac{\partial \psi}{\partial r}\right)_{R,0} = 0; \quad \left(\frac{\partial \psi}{\partial z}\right)_{R,0} = 0 \tag{13b}$$

while from equations (6), (13a) and (13b):

$$\left(\frac{\partial A}{\partial r}\right)_{R,0} = 0; \quad \left(\frac{\partial A}{\partial z}\right)_{R,0} = 0. \tag{13c}$$

In a cylindrical geometry, the flux ϕ and vector potential are related simply through:

$$A = \frac{\phi}{2\pi r} \tag{14}$$

or differentiating:

$$\frac{\partial A}{\partial r} = \frac{1}{2\pi r}\frac{\partial \phi}{\partial r} - \frac{\phi}{2\pi r^2}. \tag{15}$$

At the beam axis the self-magnetic field is zero, hence

$$\frac{1}{2\pi R}\left(\frac{\partial \phi^s}{\partial r}\right)_{R,0} = B^s = 0. \tag{16}$$

Combining equations (13c), (15), (16) and (1) leads to:

$$\left(\frac{\partial \phi^b}{\partial r}\right)_{R,0} = \frac{\phi(R,0)}{R}. \tag{17}$$

This can be rewritten, with help of equations (14) and (12);

$$B^b(R,0) = \frac{A^b(R,0)}{(1 + \nu\lambda)}\frac{1}{R} \tag{18}$$

where B^b is the betatron magnetic field. After some manipulation we obtain from equation (18)

$$\nu\lambda = -\left(\frac{\partial A^b}{\partial r}\Big/ B^b\right)_{R,0.} \tag{19}$$

Notice that N and ν are functions of R. Relating ν_0 to the 'betatron' orbit radius $R_0\left[\left(\frac{\partial A^b}{\partial r}\right)_{R,0} = 0\right]$:

$$\nu = \nu_0 \frac{R_0}{R} \tag{20}$$

equation (19) becomes:

$$\nu_0\lambda = -\frac{R}{R_0}\left(\frac{\partial A^b}{\partial r}\Big/ B^b\right)_{R,0}. \tag{21}$$

For a given betatron field distribution, the left side of equation (21) can be plotted as a function of R. As

$\nu_0\lambda > 0, \frac{\partial A^b}{\partial r} < 0$ for any equilibrium orbit. The physical meaning of equation (21) can be seen in Fig. 1. The equilibrium orbit of any particle will be located at such an r, where the betatron condition, equation (13c) with the *resultant* field is satisfied. As the slope of the vector potential for the ring current is necessarily positive at the beam axis the beam shifts automatically to a *smaller* orbit where the *negative* slope of the guiding field vector potential compensates for this. The larger the number of injected beam electrons (ν_0) the larger negative $\frac{\partial A^b}{\partial r}$ is necessary for this compensation. A shrinkage of the beam diameter has a similar effect, through increasing λ.

Moving inward from the 'betatron orbit' R_0, the right side of equation (21) reaches a maximum, and then falls off to zero. As an important consequence of (21) it should be noticed that *the number of beam particles at a given λ is limited* in the machine. Similarly *if the beam pinching exceeds a certain limit* (with a given ν_0) equation (21) can not be satisfied any more and *the beam collapses.**

The effect of beam pinching is also visible on Fig. 1. The beam 'entrenches' itself into the betatron field. This results however, in an undesirable side effect: the stronger the pinching, the deeper the potential well, and the less the energy acquired by the particles during the acceleration process. Combining equations (7) and (12):

$$v_\varphi = -\frac{qA(R,0)}{m(1+\nu\lambda)}. \tag{22}$$

Using equation (19) and rearranging (22):

$$v_\varphi = -\frac{q}{m} RB^b(R,0). \tag{23}$$

The beam current can also be expressed in terms of the betatron quantities. From equations (8), (12), (19) and (20):

$$I = -\nu_0 R_0 B^b(R,0). \tag{24}$$

One purpose of plasma betatrons is to create an extremely strong magnetic field around the beam, with a relatively weak guiding field. It is comforting to find that this is still possible despite the restriction posed by (21). Although the current is limited by equations (24) and (21) it can be shown that the quantity B^s/B^b is not limited.

The unknown beam parameter λ still appears in our equations. In order to find the variation of λ with the betatron field quantities, we have to analyse the dynamics of the beam.

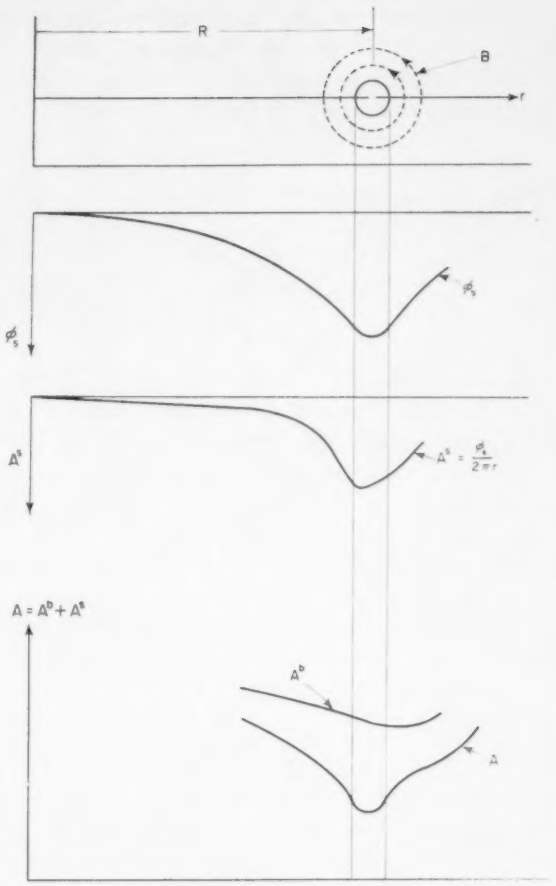

FIG. 1.—Construction of the total self-consistent vector potential resulting from the superposition of external and beam fields.

(b) *Motion in the r—z plane*

Besides circulating on their equilibrium orbit, the particles perform an oscillatory motion in the r and z direction in the effective potential well ψ. In the course of time (as the acceleration process goes on) this potential well is changing. This is due partly to the change in the guiding field, partly to changes in the self-consistent potential part. These changes effecting the oscillatory particle motion, effect (self-consistently) the shape of the potential well.†

In order to calculate particle oscillation in the $r - z$ plane and its effects to the field, we make the following assumptions:

* Such a beam behaviour was first suggested by MAISONNIER and FINKELSTEIN (1959) CERN 59-11. They do not apply a self-consistent field model and arrive therefore to different results.

† Somewhat similar investigations were carried out by LAWSON J. D. *J. Electr. Contr.* **3**, 587 (1957); and EHRMAN J. B. (1960) *Phys. Fluids* **3**, 121.

(i) The beam is so thin that the toroidal geometry can be replaced by a cylindrical geometry (Fig. 2).
(ii) The vector potential depends on ρ but not on θ. This means the convenient choice of a betatron with $n = \frac{1}{2}$.
(iii) Interparticle collisions, as well as radiation, can be neglected.
(iv) Each particle of a certain species is mono-energetic, and passes through the centre $\rho = 0$ (zero angular momentum about the beam axis).
(v) The velocity distribution of the particles is isotropic, their phases are random.
(vi) The oscillation frequency in the potential well is large compared to the characteristic frequencies describing any kind of change in the potential well.
(vii) The potential well can be approximated with a paraboloid.
(viii) The electron and ion beams are space charge neutralized ($V = 0$).

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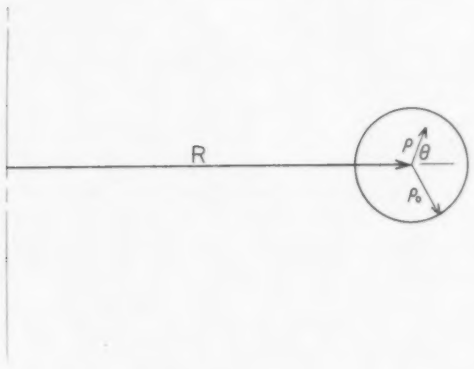

FIG. 2.—Beam geometry in the r–z plane.

The model set up by those assumptions is reasonably close to a real case to yield at least qualitatively applicable results.

The equation of motion of a particle in the potential well is:

$$m\frac{d^2\rho}{dt^2} + \frac{\partial\psi}{\partial\rho} = 0. \tag{25}$$

Due to condition (vii):

$$\frac{\partial\psi}{\partial\rho} = \alpha\rho \tag{26}$$

hence equation (25) leads to harmonic oscillations with the frequency

$$\omega = \sqrt{\frac{\alpha}{m}}. \tag{27}$$

We use condition (vi) to introduce the adiabatic invariant:

$$\frac{E}{\omega} = \frac{1}{2}\sqrt{\alpha m}\,\rho_0^2 \tag{28}$$

where E is the transverse particle energy and ρ_0 the oscillation amplitude, or in other words, the beam radius. We choose to express α from equation (26) as:

$$\alpha = \frac{1}{\rho_0}\left(\frac{\partial\psi}{\partial\rho}\right)_{\rho_0} \tag{29}$$

which leads to

$$\left(\frac{\partial\psi}{\partial\rho}\right)_{\rho_0}^{1/2}\rho_0^{3/4} = \text{ad. inv.} \tag{30}$$

In the course of the acceleration process $\frac{\partial\psi}{\partial\rho}$ increases, hence the beam radius shrinks. With help of equations (1) and (6) and considering condition (viii) we write:

$$\left(\frac{\partial\psi}{\partial\rho}\right)_{\rho_0} = \frac{q^2}{m}(A^b + A^s)_{\rho_0}\left(\frac{\partial A^b}{\partial\rho} + \frac{\partial A^s}{\partial\rho}\right)_{\rho_0}. \tag{31}$$

We distinguish between two basically different regimes. If

$$\left(\frac{\partial A^s}{\partial\rho}\right)_{\rho_0} = (B^s)_{\rho_0} \ll \left(\frac{\partial A^b}{\partial\rho}\right)_{\rho_0} \tag{32}$$

then automatically $|A^b| \gg |A^s|$, and self-fields play no important role in the process, equation (31) becomes:

$$\left(\frac{\partial\psi}{\partial\rho}\right)_{\rho_0} = \frac{q^2}{m}A_{\rho_0}{}^b\left(\frac{\partial A^b}{\partial\rho}\right)_{\rho_0} \approx \frac{q^2}{m}A_{\rho_0}{}^b\,\rho_0\left(\frac{\partial^2 A^b}{\partial\rho^2}\right)_{\rho_0} \tag{33}$$

considering that $\left(\frac{\partial A^b}{\partial\rho}\right)_0 = 0$. Using equation (33) in (30) we find:

$$A^b\left(\frac{\partial^2 A^b}{\partial\rho^2}\right)_{\rho_0}\rho_0^4 = \text{ad. inv.} \tag{34}$$

We introduce for convenience a parameter κ describing the momentary status of the betatron field. Every betatron field quantity F (some function of the spatial co-ordinates of course) is a linear function of the time dependent κ, $F(\mathbf{r},t) = \kappa F_0(\mathbf{r})$. This leads to:

$$\rho_0^2 \propto \frac{1}{\kappa}. \tag{35}$$

This is the usual formula for betatron damping as

expected. The condition for this 'betatron' regime, equation (32) can be re-expressed as:

$$\left|\rho_0^2\left(\frac{\partial^2 A^b}{\partial \rho^2}\right)\right| \geqslant \left|\frac{\mu_0 I}{2\pi}\right|. \tag{36}$$

The left side of inequality (36) is independent of κ in the betatron regime, while the right side is linear in it. We conclude therefore that *every plasma betatron operates in the first phase of the acceleration process as an ordinary betatron.* As κ increases equation (36) is not satisfied any more and self-pinching gradually takes over. This leads to the pinch dominated regime where:

$$(B^s)_{\rho_0} \geqslant \left(\frac{\partial A^b}{\partial \rho}\right)_{\rho_0}. \tag{37}$$

The quantity $\nu\lambda$ is limited by the condition imposed by equation (19). If $\nu\lambda \ll 1$ it follows from (12) that

$$A^b \geqslant A^s. \tag{38}$$

Using conditions (37) and (38) in equation (31) we arrive to the adiabatic invariant:

$$\rho_0 A^b = \text{ad. inv.} \tag{39}$$

or for the shrinking of the beam radius in the pinch dominated regime:

$$\rho_0 \propto \frac{1}{\kappa}. \tag{40}$$

3. RELATIVISTIC CASE

The calculation for the electrons after reaching the relativistic domain goes on similar lines as the previous non-relativistic calculations. The Hamiltonian is now

$$\mathscr{H} = c\sqrt{m_0^2c^2 + P_r^2 + P_z^2 + \left(\frac{P_\varphi - qrA_\varphi}{r}\right)^2} + qV \tag{41}$$

Setting again $P_\varphi = 0$ we obtain:

$$\mathscr{H} = c\sqrt{m^2c^2 + P_r^2 + P_z^2} + qV \tag{42}$$

where

$$m^2c^2 = m_0c^2 + q^2A^2. \tag{43}$$

As the orbital particle energy largely exceeds the transversal energy:

$$m^2c^2 \geqslant P_r^2 + P_z^2. \tag{44}$$

This leads to the approximate expression for the Hamiltonian:

$$\mathscr{H} = \frac{P_r^2}{2m} + \frac{P_z^2}{2m} + \psi \tag{45}$$

where

$$\psi = m_0c^2\sqrt{1 + \frac{q^2A^2}{m_0^2c^2}} + qV = m_0c^2\gamma + qV \tag{46}$$

is again the effective potential. γ is the usual relativistic factor:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v_\varphi^2}{c^2}}} = \sqrt{1 + \frac{q^2A^2}{m_0^2c^2}} \tag{47}$$

Assuming again charge neutrality, γ plays the role of the potential. The equilibrium position of the beam is where:

$$\left(\frac{\partial \gamma}{\partial r}\right)_{R,0} = 0 \quad \text{or} \quad \left(\frac{\partial A}{\partial r}\right)_{R,0} = 0. \tag{48}$$

Self-consistency yields the relationship:

$$\nu_0\lambda = -\gamma\frac{R}{R_0}\left(\frac{\partial A^b}{\partial r}\Big/ B^b\right)_{R,0} \tag{49}$$

and the current becomes now:

$$I = -\frac{\nu_0 R_0 B^b(R,0)}{\gamma} \tag{50}$$

Note that in the extreme relativistic case where $\gamma \gg 1$, γ is proportional to κ, hence the current is approximately constant. This is obvious since saturation current is reached when all particles move with the velocity of light.

The adiabatic invariant becomes now:

$$\rho_0^{3/2}(A^{1/2})_{\rho_0}\left(\frac{\partial A}{\partial \rho}\right)_{\rho_0} = \text{ad. inv.} \tag{51}$$

It is easy to see that this agrees with equations (30), (31). The condition for the betatron regime is again described by equation (36). In this regime the betatron damping yields:

$$\rho_0^2 \propto \frac{1}{\kappa} \tag{52}$$

the same as in the non-relativistic case. Notice that in the $\gamma \gg 1$ limit the left and right hand side of the inequality (36) are independent of κ. Hence if the electron beam did not start self-pinching in the non-relativistic region, it will never pinch.

In the pinch dominated regime:

$$\frac{\rho_0 A^b}{\gamma^{1/2}} = \text{ad. inv.} \tag{53}$$

This leads in the $\gamma \gg 1$ limit to:

$$\rho_0^2 \propto \frac{1}{\kappa}. \tag{54}$$

This is the same law as for betatron damping. This, as well as the remark following equation (52) leads to the recognition that self-pinching becomes less important in the extreme relativistic limit, although if

the beam was pinch dominated in the non-relativistic region it always remains so.

It is instructive to point out that the ratio:

$$\frac{B^s}{B^b} \propto \frac{1}{\sqrt{\kappa}} \tag{55}$$

in the extreme relativistic regime. Therefore even if a sufficiently strong self-field was created, with continued acceleration the betatron field will sooner or later exceed the self-field. (In the radiation damping dominated BUDKER machine this is not the case).

For a toroidal beam λ depends logarithmically on ρ_0. Considering the shape of the betatron field and equation (49) the following statement can be made about the dynamics of the equilibrium orbit:

In the first phase of the acceleration process the beam orbit shrinks from the betatron orbit to some smaller $R_{\min}$ *or if* v_0 *exceeds a certain limit it collapses. The most dangerous phase for collapse is when the beam just reaches relativistic energies. In the later phases* (when γ increases considerably) *the orbit expands again and the danger of collapse is over.*

In all the previous considerations we dealt with the electron beam alone. This is justified since the small ion velocity makes the contribution of the ions to the magnetic field small compared to that of the electrons in the first part of the acceleration process. In later stages, however, ions might acquire high enough speeds to cause a second non-relativistic beam pinching, with a decrease in the beam radius again according to equation (49).

In order to find the actual variation of the beam radius in the course of the acceleration process, the numerical value of the adiabatic invariants must be known. This can be found only through analysing the initial conditions. If the variation of ρ is really established the variation of R can also be found using equation (21). Note that λ is a function of R and ρ only.

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RESEARCH NOTE

Experimental and theoretical observations on a fast linear pinch

(*Received* 8 *December* 1960)

AN experiment is being carried out at A.E.R.E. on a shock-heated linear z-pinch, with initial B_z field and strong pre-ionization (ASHBY and PAUL, 1959). In such a discharge, it ought to be possible to get detailed quantitative agreement between experiment and a one-dimensional magnetohydromagnetic calculation, provided that the right theoretical model is used. This is because Rayleigh-Taylor instability at the outside of the plasma should be inhibited by the shear between the B_z and B_θ fields, while magnetohydrodynamic surface instabilities should not have time to grow appreciably during the first one or two bounces. Within the plasma, $\partial B_z/\partial r$ and $\partial \rho/\partial r$ have the same sign, and this ought to ensure Rayleigh-Taylor stability. Electrode influences should not propagate along the tube faster than the magnetosonic speed, so that for several microseconds the central region of the plasma should behave as part of an infinite cylinder. Furthermore, in this experiment inertial effects predominate over transport effects.

To test this possibility, preliminary experimental data obtained at low bank voltage have been compared with calculations on the A.W.R.E. IBM 7090, using the computer programme developed by HAIN and HAIN (1959)* which solves the magnetohydrodynamic equations for the collapse of a *fully-ionized* cylindrical plasma. This initial comparison is intended to establish a broad agreement between theory and experiment, and then to bring out any improvements needed in the rather simple theoretical model. The results are encouraging, as shown below, and it would appear to be practicable to use the programme as a powerful tool to interpret experimental information, to interpolate details of plasma behaviour that are not yet brought out explicitly by the diagnostics, and to suggest new experiments. A subsequent step will be to repeat the calculations with a *partially-ionized* version of the programme†, which includes the effects of ionization, radiation loss and charge exchange.

Some experimental parameters are:

main bank capacity	37·7 μF
main bank voltage	11 kV
tube length	60 cm
tube radius	11·7 cm
maximum gas current	140 kA
initial rate of change of current	$3{\cdot}7 \times 10^{11}$ amp/sec
period of main discharge	12 μsec
filling pressure of hydrogen	14 μ
initial axial field	0·8 kG
preheat current	30 kA
preheat half-period	80 μsec

B_z and B_θ fields are measured with a magnetic probe, which can be moved within a quartz tube (8 mm in diameter) fixed diametrically through the centre of the discharge. Each point on a graph such as Fig. 1(a) represents a separate shot. There is considerable spread but in several cases departures from simple theory exist which are sufficiently marked to represent real physical effects. Measurement of B_z (wall) and the circuit variables, as functions of time, gives information about the velocity and density of plasma which leaves a thin ionizing region near the wall (COLGATE, FERGUSON and FURTH, 1958) and about the arrival of shocks or other pressure waves from the interior.

Before the main bank is fired, the gas is ionized by I_z current from a separate 'preheat' bank. In the discharge analysed here, sufficient energy is supplied to the gas to ionize it completely, (50 eV per atom), but one cannot expect the thermal pressure of a fully-ionized plasma to be supported by the B_z field at the end of the preheat phase, since the magnetic configuration is observed to be unstable. On the other hand pressure balance must certainly exist, since the velocity of sound for hydrogen plasma at 5 eV is 4 cm/μsec, while the ionization phase lasts for 80 μsec. The most plausible initial state is that a hot *partially-ionized* gas in the body of the tube is supported, not magnetically, but by the pressure of a cool, dense, almost neutral layer of gas near the wall. Diffusion of plasma through the neutral gas will be inhibited by charge exchange. (The observations give some support for such a model). B_z may then be assumed to have the same value in the plasma as in the insulating wall, which could not be true if the plasma were contained magnetically. Nevertheless, in the preliminary calculations the gas is simply assumed to be initially fully-ionized, of *uniform* density, with $T = 2$ eV.

When the main bank is fired, B_θ begins to rise linearly with time at the outside of the tube, and pushes the plasma in, thus compressing ρ and B_z. By changing the initial gas density and observing the change in collapse velocity it appears that nearly all the gas is swept up, despite the probable incomplete ionization; this is presumably due to charge exchange between ions and neutrals. Since the velocity of the B_θ 'piston' is comparable with the magnetosonic speed in the undisturbed plasma, a shock develops in the B_z field, which runs into the centre. No substantial disturbance is observed at the centre until about 1 μsec after the start; this is quite distinct from the discharge with no preheat.

Although the shock should be extremely thin, (say $\lesssim$ 0·1 cm, depending on the model), this cannot be observed because the probe tube has a finite size. The time variation of B_z seen by a single probe (e.g. Fig. 2) is not inconsistent with the existence of a thin shock. In the calculation, a thin shock requires a small timestep and is therefore expensive in machine time. In these preliminary runs the shock was therefore considerably widened by the Von Neumann technique (RICHTMYER, 1957) as is apparent in Fig. 1(a,b,c). The fully-ionized programme does include a variety of phenomena such as anisotropic resistive diffusion; separate temperatures, heating and heat conduction for ions and electrons; equipartition etc., but in this experiment only dynamical phenomena are really observable. In any case the ion and electron temperatures and the shock thickness should be determined largely by the neutral component, which is not yet included in the calculation.

* See also BRAGINSKII, GEL'FAND and FEDORENKO (1959).
† Written by A. J. Taylor and now being tested.

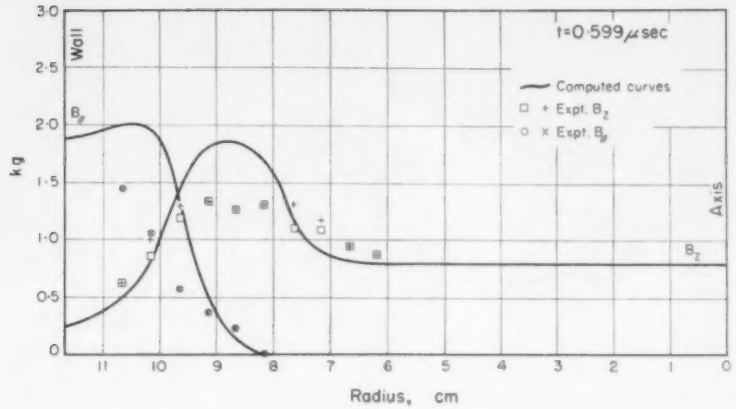

FIG. 1 (a)

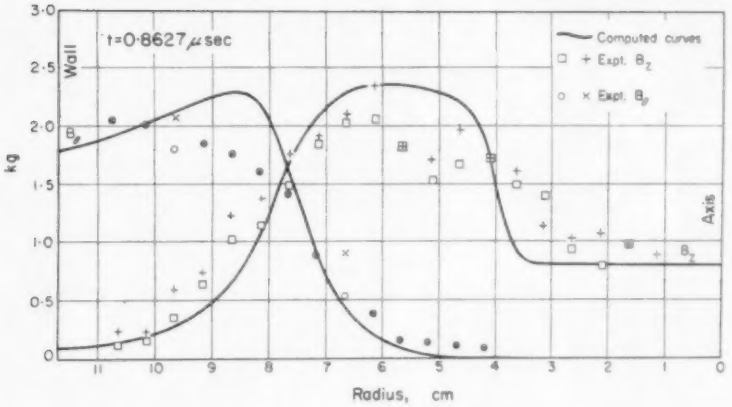

FIG. 1 (b)

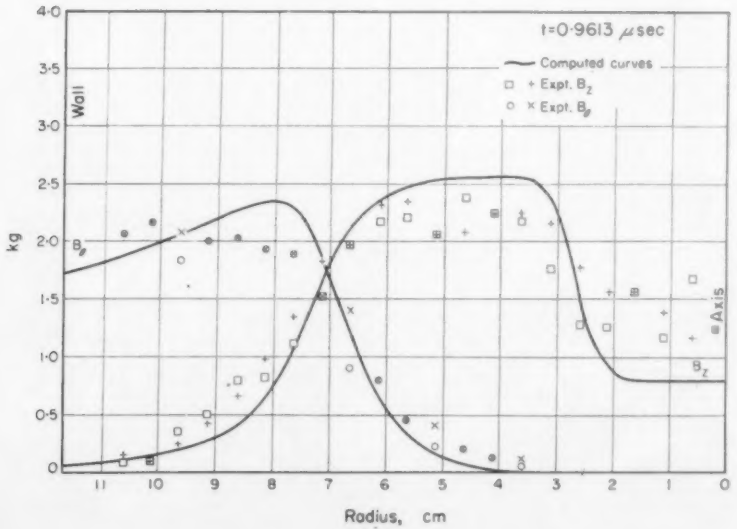

FIG. 1 (c)

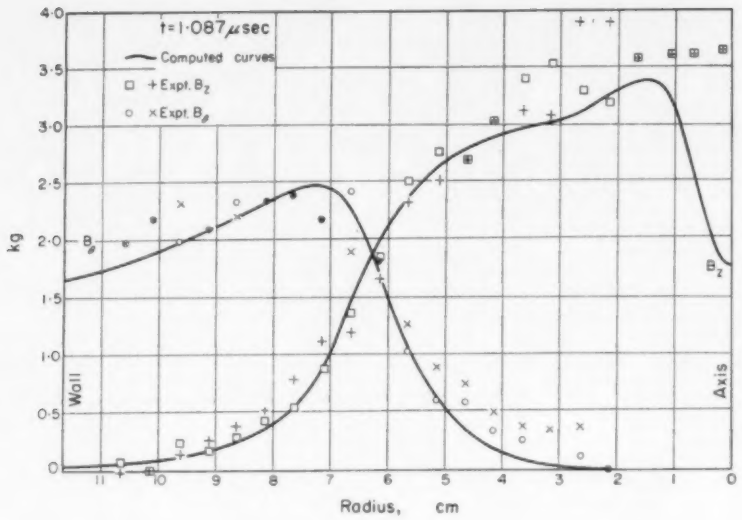
t=1·087 μsec
Computed curves
□ + Expt. B_z
○ × Expt. B_θ
kg
Wall
Axis
B_θ
B_z
Radius, cm

FIG. 1 (d)

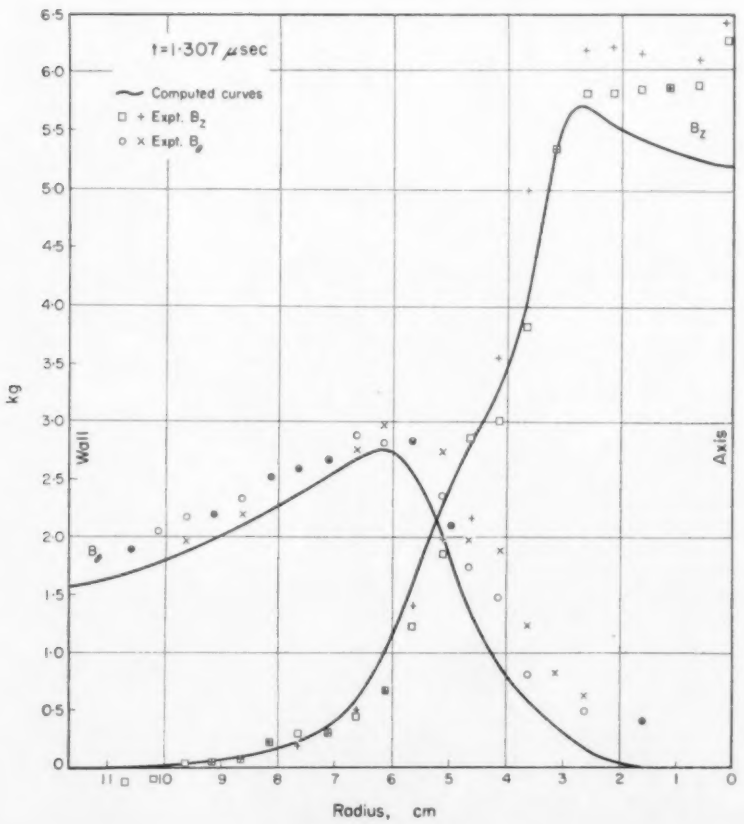
t=1·307 μsec
Computed curves
□ + Expt. B_z
○ × Expt. B_θ
kg
Wall
Axis
B_θ
B_z
Radius, cm

FIG. 1 (e)

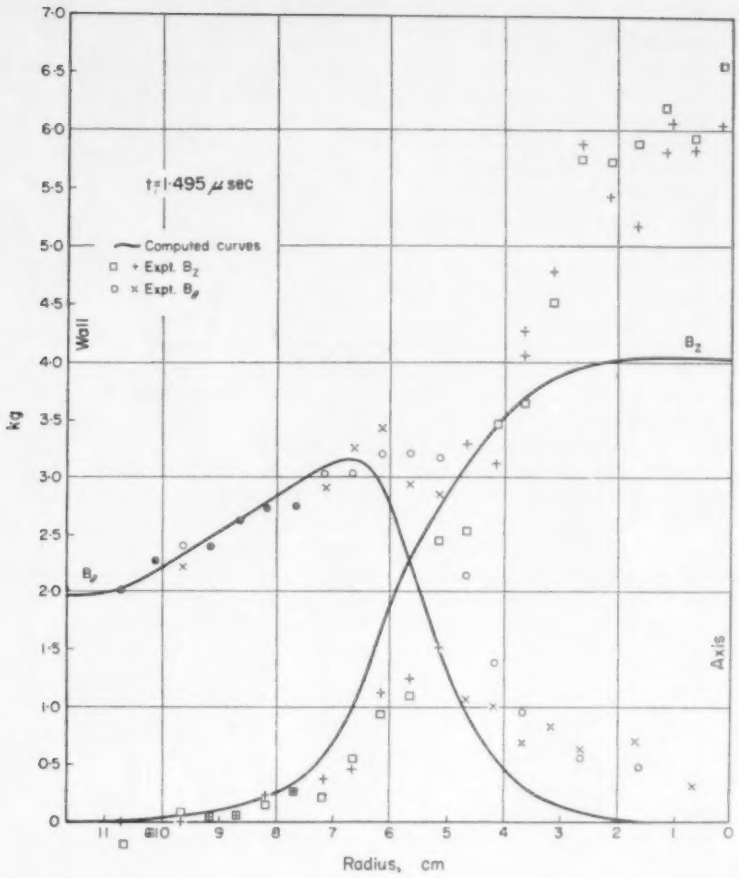

FIG. 1 (f)

Figures 1 and 2 show the good agreement between the experimental results and a preliminary theoretical computation in which no *ad hoc* assumptions are made at all. This allows one to use the remaining points of *disagreement* to obtain further information about the experimental conditions. Specifically, we study the mass of gas emitted by the wall and the initial density distribution.

(i) In the first run, reproduced here, conducting 'pressureless' plasma was assumed to emerge from the wall, after the initial gas had moved in, following the hypothesis of COLGATE *et al.* (1958). Its density was given the very low value of 10^{12} ion pairs/cm^3, i.e. about $\rho_0/100$, (where ρ_0 is the density corresponding to 14 μ). Thus in the region surrounding the main pinch $j \times B \simeq 0$, i.e. $j_z/j_\theta \simeq B_z/B_\theta$. This means that B_θ should be approximately a vacuum field while B_z should depart markedly from vacuum form, since residual B_z flux is pumped out of the insulating wall by inward-moving plasma.

It is clear from the experimental results that B_z is not a vacuum field, so that conducting plasma is indeed present in this outer region, but neither is B_θ a vacuum field here during the first microsecond (Figs. 1(b) and 1(c)). Thus a substantial $j \times B$ force is necessary to accelerate the plasma outside the main pinch, so that it must have considerable mass. In support of this, Figs. 1(a) and 1(b) show that at the beginning of the discharge the current sheath is moving inwards more slowly than simple theory predicts. These dynamical effects may be due to ionization of the layer of dense neutral gas at the wall suggested earlier. By Figs. 1(d) and 1(e) B_θ has become a vacuum field in the outer region, presumably because acceleration has ceased.

(ii) A consequence of (i) is that B_z flux is not pushed inwards so rapidly. One can see from Figs. 1(b) and 1(c) that there is a deficit of B_z in the hump, and an excess in the outer region, compared to theory.

(iii) The experimental current sheath is a little wider than theory predicts, as shown e.g. in Figs. 1(c) and 1(d); this is partly due to (ii) and partly also to cooling of the plasma by ionization and radiation, which will increase its resistivity. To simulate this latter effect, a run was done in which T_e and T_i

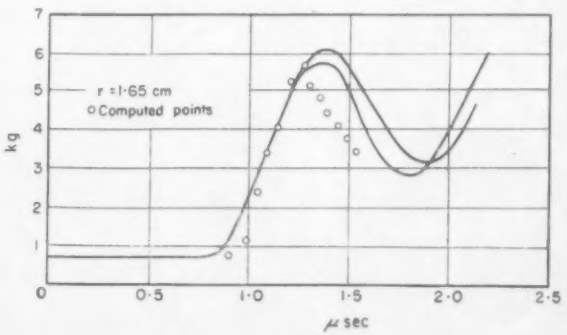

FIG. 2.

were not allowed to exceed 5 eV. This did slightly widen the theoretical current sheath, leaving a residual diffusion of B_θ which should presumably be attributed to finite probe width. But it is apparent that the width of the current sheath gives very little information about the electron temperature.

(iv) In general there is good agreement up to collapse time, and the right central compression of B_z; subsequently the theoretical plasma bounces back too quickly, as shown in Fig. 2 and Figs. 1(e) and 1(f). (Fig. 2 is not taken at the centre, because exact cylindrical convergence in the calculation produces a peak which would not be picked up by a finite probe.) The main cause is probably tamping due to the kinetic pressure of the inward-moving shell of dense plasma which is known from (i) to exist outside the current sheath (i.e. the point $B_\theta = B_z$).

To test this idea, a series of 'tamper runs' is being carried out, in which extra mass is allowed to emerge from the wall after the initial gas has moved in. In the first such run, ρ (wall) was maintained at ρ_0 until 0·5 μsec, and then allowed to decrease gradually. This considerably improved the agreement between experiment and theory in the outer region but slowed down the inward motion of the main plasma. To get an overall fit it will be necessary to reduce the density of the initial gas below ρ_0 and to increase the mass of the tamper further. It is planned by *adjusting the initial conditions and boundary conditions on* ρ in this way to reconstruct the detailed space-time behaviour of the discharge from the probe traces and circuit quantities up to about 3 μsec.

A more detailed account of this work will be submitted for publication shortly.

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BOOK REVIEW

Nuclear Fusion—The Second Geneva Series on the Peaceful Uses of Atomic Energy. Edited by WILLIAM P. ALLIS, Van Nostrand, 1960. viii + 488 pp., $12.50 (94*s*).

THE official publication of the papers on thermonuclear research presented at the Second International Conference on the Peaceful Uses of Atomic Energy, held at Geneva in September 1958, took the following course. Preprints of most of the papers, translated where necessary, were made available to specialists in the field before the conference took place. Early in 1959 appeared Volumes 31 and 32 of the *Proceedings*, which contained the 111 thermonuclear papers in full, efficiently edited and excellently printed.

The book under review comprises a large selection of these papers arranged in eleven chapters, each covering one aspect of thermonuclear research and each with a brief introduction. By judicious editing Dr. Allis has succeeded in compressing the bulk of the material presented at the conference into one volume. The significant omissions are concerned with reacting plasmas, and few will disagree with the editor's comment that such studies seem premature. A comprehensive subject index and a complete list of papers in numerical order are included.

The chief criticism of this book must be the protracted interval between the Conference and its publication. This will inevitably result in its being of little value or appeal to scientists active in the rapidly changing field of thermonuclear research, a situation aggravated by the prior publication of a book markedly similar in purpose and content (*Plasma Physics and Thermonuclear Research*, Pergamon Press, 1959).

For the general reader, the book provides a compact account of the status of thermonuclear research at the time of the Conference, and thus a useful background essential to the appreciation of subsequent published work in the field. It might have benefitted in this respect had the introduction given a fuller exposition of the fundamental physical principles involved.

P. A. DAVENPORT

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CURRENT PAPERS IN PLASMA PHYSICS, ACCELERATORS AND THERMONUCLEAR RESEARCH

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SOME MEASUREMENTS OF FLUCTUATIONS IN A PLASMA*

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Abstract—Measurements of the fluctuations in anode voltage, tube current, and number density have been made in a hot-cathode mercury-vapour discharge at a pressure of about 1μ, under both space-charge limited and temperature-limited emission conditions. Substantial low-frequency fluctuations occur from zero to several hundred kc/s with strong components at about 60 and 100 kc/s. Potential distribution curves show noise generation over the first 20 cm of the plasma column. This is poorly correlated from point to point but appears to propagate with a velocity of the same order as the electron thermal velocity. Over the remaining 60 cm of the experimental tube, the signal is well correlated and travels at more than 10^9 cm/s. Despite the non-uniform noise distribution, the column appears as a uniform impedance (to external signals), and an approximate expression has been derived for its impedance/frequency characteristic. Number density measurements, made by a microwave cavity perturbation technique, indicate fluctuations increasing in amplitude towards the cathode, and persisting even when the discharge is temperature-limited and other fluctuations have been substantially reduced.

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1. INTRODUCTION

EXAMINATION of quantities associated with d.c. plasmas, such as anode voltage (V_a), discharge current (I_a), light output, or number density (n_0), indicates the presence of fluctuations in their values which may be substantial. For example, it is not unusual to find potential fluctuations ranging from hundreds of millivolts to volts, and containing frequency components from zero to a few Mc/s. These instabilities and oscillations, which we shall term 'noise,' often impose considerable difficulties on obtaining clear results from experiments on plasma. At low frequencies, effects which it might be hoped to observe, such as the electrostatic sound waves postulated by TONKS and LANGMUIR (1929) are lost against the background fluctuations, while at high frequencies unwanted sidebands result from phase and amplitude modulation of applied signals.

It is clear that information is required on the characteristics of the noise, so that its production mechanisms may be elucidated. Further progress can then be made in the design of low-noise plasma tubes suitable both for laboratory use and for possible application in electron devices.

Some experimental work on hot cathode arc plasmas at pressures of a few microns has been carried out by previous investigators with the following general conclusions: The noise is not strongly dependent on tube volume or current (COBINE and GALLAGHER, 1947; MARTIN and WOODS, 1952). Probe measurements indicate propagation velocities above 10^7 cm/s, and suggest that a coherent random fluctuation of the whole plasma is involved (THONG SAW PAK, 1955). The low frequencies observed seem to imply ion-dominated generation mechanisms and several suggestions have been made:

(a) Ions trapped in the potential minimum existing in front of the cathode under space-charge-limited conditions are responsible (DALY and EMELEUS, 1955; 1956).

(b) There is an irregular movement of the boundary between parts of the cathode emitting temperature-limited and space-charge-limited currents (EMELEUS, 1956).

(c) There is an enhanced flicker effect, possibly associated with mode-jumping of standing space-charge waves in the gas, very close to the cathode (CLARKE, 1955).

There is one observation of the presence of striations moving at about 2×10^5 cm/s, in a low-pressure mercury vapour discharge (FOULDS, 1956). These disturbances could also be related to noise production.

Clearly, detailed experiments are required to describe the noise characteristics more accurately and to distinguish between the various generation mechanisms postulated. The work described herein is an attack on their problems utilizing the hot-cathode

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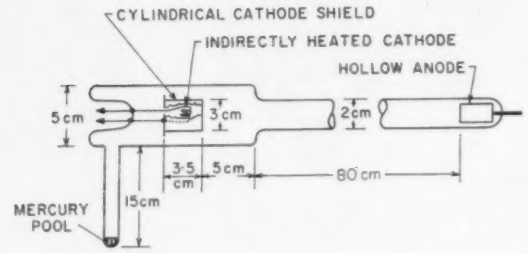
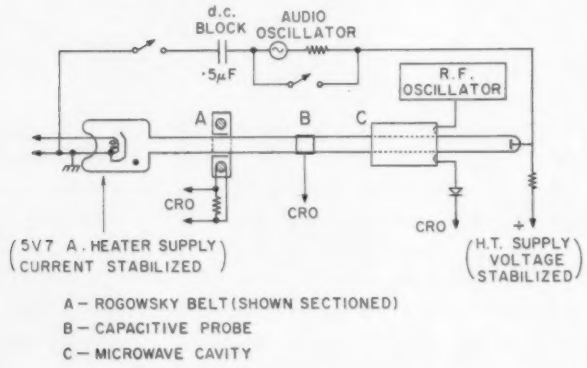
low-pressure mercury-vapour plasma, a common form of experimental discharge.

2. EXPERIMENTS

The experimental tube is shown in Fig. 1(a) and incorporates an indirectly-heated cathode from a commercial mercury-vapour rectifier tube. Over the



(a) EXPERIMENTAL DISCHARGE TUBE



(b) SCHEMATIC VIEW OF APPARATUS

FIG. 1.—(a, b).

20–200 mA range of I_a studied under space-charge-limited conditions, V_a remained sensibly constant between 45 and 50 volts, the actual value depending on the room temperature, which corresponded to a mercury vapour pressure of about 1 μ.

Figure 1(b) shows schematically the equipment used in the tests to be described. The elements capable of sliding along the tube consisted of a capacitive probe, made by winding a strip of copper around it, a microwave cavity fitted with coupling loops adjusted to excite the TM_{010} mode, and a Rogowsky belt. This last was simply a ferrite-cored current transformer using the discharge column as its 1-turn primary. The secondary was screened electrostatically from the tube, as indicated in the diagram.

A. *Two-terminal measurements*

Some observations on the noise characteristics can be made by examining the fluctuations in V_a. For example, to find out whether the tube was acting as a constant-current or constant-voltage noise generator, the external load and I_a were varied with the results shown in Fig. 2. It will be seen that when the load resistance is large compared with the d.c. resistance of the discharge, the noise voltage is not strongly dependent on discharge current or load impedance. Under these conditions, then, the noise source can be represented approximately by a constant-voltage generator in series with the discharge, or possibly by a number of such generators throughout the discharge. This agrees with the observations of COBINE and GALLAGHER (1947).

Some information on the internal impedance properties of the plasma was sought, particularly to see whether the effective impedance seen by the noise generator(s) would be the same as that offered to an externally-applied signal. Such an experiment was carried out by connecting a resistor across the discharge and measuring the ratio of voltage developed across the resistor to that across the discharge (see Fig. 1b). This measurement showed that the impedance to both noise and to an injected 300 kc/s signal was close to the d.c. resistance of the discharge. This was found to be true also when the discharge resistance was halved by doubling the anode current.

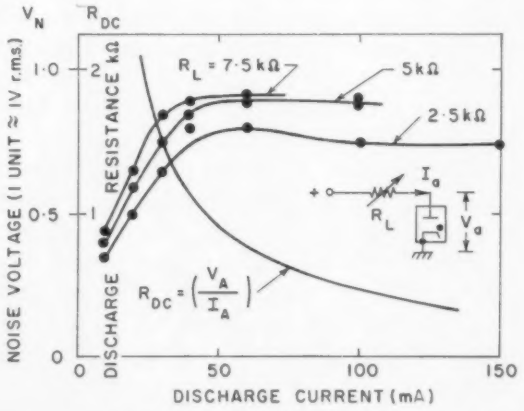


FIG. 2.—Noise voltage developed across a variable load resistance.

More detailed measurements of impedance as a function of the frequency of the injected signal suggested two basic assumptions for a phenomonological theory of this characteristic: first, that $I_a \propto G_0$, where G_0 is the conductance of the discharge (this simply expresses the constancy of V_a under steady state conditions), and second, that under transient conditions $(dG/dt) \propto (G_0 - G)$, where G is the instantaneous value of conductance. This theory is

developed in the Appendix to yield the expression

$$Z = R_{dc} \cdot \frac{j\omega\tau}{1 + j\omega\tau}, \qquad (1)$$

where τ is a time-constant of the system, which in the actual discharge will be associated with the relaxation time of the system,

$$Z \to j\omega\tau \,.\, R_{dc} \text{ as } \omega \to 0,$$

$$\to R_{dc} \text{ as } \omega \to \infty.$$

It should be emphasized that equation (1) is only a first approximation, but agreement with the impedance measurements was obtained to within 25 per cent, the accuracy of the measurement, and a value $\tau \sim 4\ \mu$s was deduced from them. The expression can be re-written as

$$Z = \frac{R_{DC} \cdot j\omega L}{R_{DC} + j\omega L} \qquad (2)$$

where

$$\tau = \frac{L}{R_{DC}};$$

which will be recognized as representing a parallel connexion of inductance and resistance.

Inserting a typical value of $R_{DC} = 500\ \Omega$, appropriate to $I_a \sim 100$ mA, gives $L = 2$ mH. It should be possible to resonate this with a capacitor connected directly across the tube. With $C = 0{\cdot}5\ \mu$F, resonance was observed as 5 kc/s as expected.

It was necessary to carry out impedance measurements at low signal levels because the current waveform was not sinusoidal for applied signals of more than a few volts across the discharge and condenser in series. Sharp kinks occurred, rather than flattening of the waveform, suggesting discontinuous jumps in the discharge characteristics.

The distribution of the frequency components of the voltage fluctuations observable at the discharge anode was determined with the aid of a spectrum analyser. The spectrum shown in Fig. 3 was obtained; strong components near 60 kc/s and 105 kc/s are evident.

All measurements so far apply to space-charge-limited conditions. Under temperature-limited conditions the impedance Z was found to be very high, as expected, (see D below).

B. *Site of noise generation*

It has already been conjectured that the noise source is in the cathode region. In order to establish whether this was so, or whether the anode, cathode, or column could be considered individually as major noise sources a further series of experiments was performed: In the first of these, a 2 cm wide capacitive probe was connected to the Y-plate of an oscilloscope. The anode of the tube was short-circuited (for a.c.) to the cathode, through a $0{\cdot}5\ \mu$F condenser, the cathode being grounded

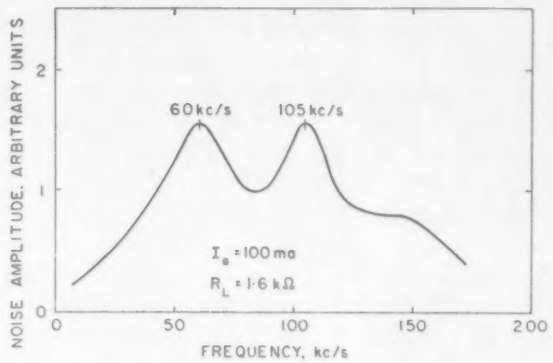


FIG. 3.—Noise frequency spectrum observed at anode.

to the oscilloscope. Rough estimates of the noise distribution could be made as the probe was moved along the column, and a distribution such as that shown in Fig. 4(a) was indicated. This experiment suggests that most of the noise is generated near the cathode, but does not rule out the possibility of continuous generation along the column. To resolve this, a second experiment was carried out employing two probes, one of which was fixed as a reference near the anode and connected to a differential amplifier, while the other probe was free to move and was connected to

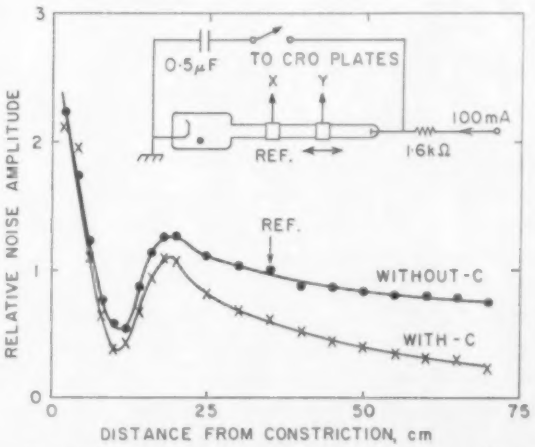


FIG. 4(a).—Comparison of noise distribution curves with and without condenser.

the other end of the differential amplifier. It was found that the difference between the two signals grew as the probes were separated, and maintained the same waveform as the signal obtained by adding the probe outputs, over all but the last 20 cm of the column towards

the cathode end. This suggests that over most of the column length there is little noise generation. It was further established from measurements made on photographs of the noise waveform that a negligible amount was generated at the anode.

To extend these initial results, a more refined technique was evolved employing a Tektronix oscilloscope having equal deflexion sensitivities on X- and Y-plates. If a single-frequency signal is applied to both inputs of such an oscilloscope, a sloping straight line will be observed on the screen if the waveforms are in phase, and an ellipse if they are out of phase. With asymmetrical waveforms containing many components, a more complicated shape can be expected and in fact, the noise trace is of 'aerofoil' appearance rather than elliptical, when derived from two points along the discharge column. If one probe is kept

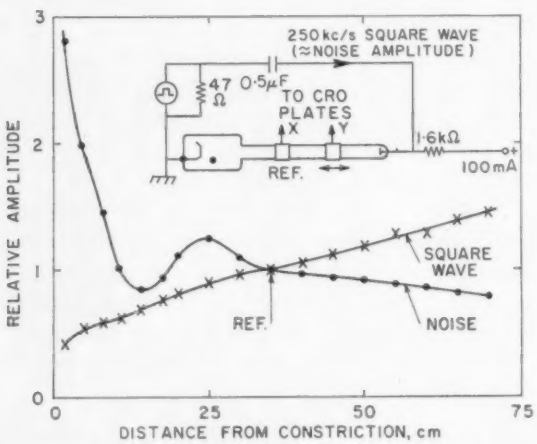


FIG. 4(b).—Comparison of distribution curves for noise and an externally-applied signal.

fixed while the other is moved, the slope of the trace on the oscilloscope screen gives a measure of the relative amplitudes of the two signals, while the thickening of the aerofoil shape gives an indication of the presence of phase shift.

Using this X/Y technique, careful measurements of the noise distribution along the column were made. The resolution was increased above the initial experiments by using 1 cm wide copper probes. These experiments served to confirm the shape of the distribution obtained in earlier tests and added the information that phase-shifting, or smearing, took place only over the first 20 cm of the column beyond the constriction at the cathode end. By taking an equivalent ellipse to circumscribe the trace observed on the oscilloscope, a rough estimate could be made of the phase-shift suffered by a 100 kc/s component over this region. From this figure between 2 and 5×10^7 cm/s was estimated for the velocity of propagation. This is in the range of electron thermal velocities. Over the remainder of the column, however, no further phase shift could be measured, even on the fastest sweeps of the oscilloscope, implying a velocity of propagation greater than 10^9 cm/s.

A series of noise distribution curves was obtained with various currents. The shape of the curves did not vary much, but the noise increased slightly as the current increased. The amount of phase-shift occurring in the smearing region appeared to decrease with increasing current.

C. *Voltage distribution due to externally-applied signals*

The probe technique was applied to determine the voltage distribution down the column of externally-applied signals of square and sinusoidal waveform, in the frequency range 350 kc/s to 1 Mc/s . It was found that there was in all cases a linear distribution of this external signal along the column. This applied to signals of from 2 to 3 times the magnitude of the noise, but it was also possible to determine the distribution of an externally-applied signal, in the special case of a square wave, for an amplitude below that of the noise. The oscilloscope trace obtained under these conditions is composed of two identical noise 'aerofoils' separated by a distance and angle which give a measure of the amplitude of the square wave. Such a test gave the experimental curves plotted in Fig. 4(b).

It is clear that the column appears uniform along its length to externally-applied signals, but not to the noise. Some powerful disturbances were applied along the column by means of a permanent magnet held outside the tube. The noise with the magnet in position increased by a considerable factor, 5 to 6 times, yet the distribution became linear within a few centimetres of the disturbance. This contrasts markedly with the 20 cm between the constriction and the uniform region of noise distribution in the undisturbed column. These results suggest that the current variations caused by the external generators, and by the magnets, are carried by the thermal group of electrons, but that in the first 20 cm beyond the cathode the noise is carried by some other mechanism.

D. *Temperature-limited operation*

If the cathode temperature is sufficiently reduced, the tube current falls to a value which is independent of the voltage across the tube. Under these conditions the appearance of the column is the same as for space-charge-limited operation at the same

current, but the cathode dark space is increased. Since the applied voltage does not affect the current, the impedance to an external signal approaches infinity—a fact which was verified experimentally.

It is relevant to examine whether noise signals should be observed on the capacitive probe under these conditions. If, for example, the noise could be represented by a single generator near the cathode in series with the column, then no noise should be observed on the column, since no noise current can flow. In fact, a noise voltage was observed all along the column (Fig. 5). This was less than when operating under space-charge-limited conditions, except near the cathode end.

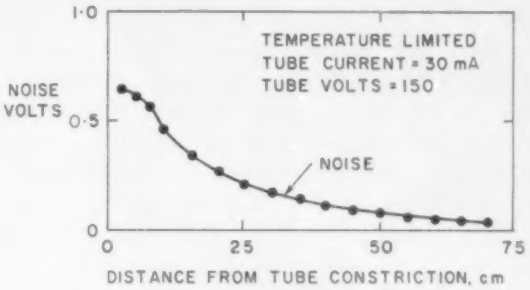


FIG. 5.—Potential distribution along the positive column.

E. *Number density measurement*

The number density of charged particles in the discharge is a function of current, a fact which was used in Section A in deducing the impedance. It is of interest, therefore, to measure the number density fluctuations due both to external signals and to the noise in the discharge. It is of particular interest to see whether there is any number density fluctuation under temperature-limited operation, when the current is constant. The number density of electrons in the discharge may be found by measuring the change in resonant frequency of a microwave cavity through which the discharge tube passes. The fluctuation in number density may then be found by exciting the cavity at a frequency slightly less than the resonant frequency and measuring the fluctuation in amplitude of the field in the cavity. (This converts f.m. to a.m. by working 'on the side' of the resonance curve). The first experiment was carried out under space-charge-limited conditions in the presence of an externally-applied signal. With the simple assumption that the number density in the positive column is proportional to the conductance of the discharge, it is shown in the Appendix that an externally-applied e.m.f. $\Delta V e^{j\omega}$ should produce a density variation

$$\frac{\Delta G}{G_0} = \frac{\Delta n}{n_0} = \frac{1}{j\omega\tau}\frac{\Delta V}{V_a}. \qquad (3)$$

Furthermore, this density variation should be independent of position along the column and should vanish under temperature-limited conditions. Accordingly, voltages at frequencies of 5 kc/s and 50 kc/s were applied. In the former case, no change of amplitude was found along the tube; in the latter case, however, there was no change over most of the tube length; though within 10 cm of the cathode the noise (see below) made quantitative measurement difficult. There was possibly a fall-off in signal over this length, but this is not certain. It was also verified that the signal disappeared entirely under temperature-limited conditions. Unfortunately, the amplitude of $\Delta n/n_0$ in terms of $\Delta V/V_a$ was not measured accurately, though it appeared to be of the expected order of magnitude.

Of greater interest was the measurement of noise along the tube length. As with the capacitive probe measurements, this did not correspond to the variation produced by an external signal. The results of this measurement are shown in Fig. 6. Curve *A* shows a measurement with a condenser across the discharge, so that the voltage across it is steady. The behaviour is much as with the external oscillator, except that at about 20 cm from the cathode the noise begins to increase rapidly. Unfortunately, the fluctuations in the region were sufficient to shift the frequency by an amount comparable to the half-width of the resonance

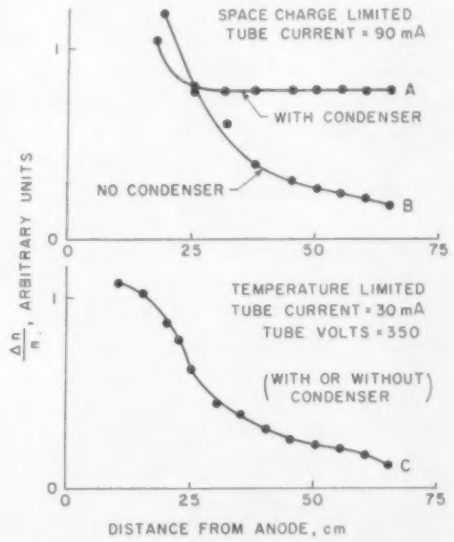


FIG. 6.—Variation of number density fluctuations along the positive column.

curve, so that the system was no longer linear. This represents a density fluctuation of ~10 per cent. Over most of the column the fluctuations were several per cent, but the measurements were not accurate. A more careful study is planned, and will be reported later.

When the condenser was removed, so that the external impedance was high, quite a different variation was found (curve *B*). In this case, the noise current was presumed to be small. Curve *C* shows a similar measurement under temperature-limited conditions. This is similar to curve *B*. Putting a condenser across the discharge made no difference at all, as would be expected, since the output impedance of the system was infinite. In all three cases, changing the discharge current by a factor of 5 or so made very little difference to the shapes of the curves or amplitudes of the fluctuations.

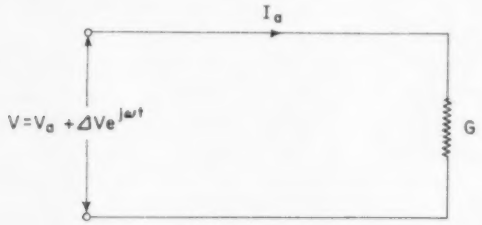


FIG. 7.—Simplified model of discharge.

F. *Measurement of current fluctuation*

The final experiment in this series was the measurement of current in the discharge column using the Rogowsky belt. No quantitative measurements were made, but the following two important observations were predicted and verified; first, there was no appreciable noise current under temperature-limited conditions; and second, under space-charge-limited conditions, the current was constant along the tube. This latter was determined by using two belts in opposition and noting that within the limits of measurement (a differential of about 5 per cent) there was no signal, whatever the belt separation.

3. DISCUSSION

The experiments described indicate the presence of a strong noise source located near the cathode of the discharge, in agreement with the findings of previous workers. There appears to be little noise generation in the arc column or at the anode of the discharge.

To an externally-applied signal, the column appears as a uniform impedance while the noise remains substantially unaffected. Apart from the first few centimetres beyond the constriction in the experimental tube, where propagation may be at about the electron thermal velocity, noise signals appear to travel at $>10^9$ cm/s. The noise contains one or two strong frequency components.

The fact that disturbances and externally-applied signals see the column as a spatially-uniform impedance, while the noise voltage does not, suggests that external signals modulate the drift current while the noise has a more complicated mechanism, possibly a modulation of the stream of fast electrons coming from the cathode space-charge region. That the hypothesis of ions oscillating in the potential minimum is inadequate to explain the noise generation fully is clear from the results with the cathode temperature-limited, when there is no space-charge minimum. Number density fluctuations still persist with excursions not significantly different from space-charge-limited conditions.

The impedance/frequency characteristics determined experimentally are consistent with the simple model postulated. This is perhaps over-simplified since it does not take separate account of the cathode region and the positive column. Further consideration of this point is required.

Future work should be directed towards analysis of the noise frequency spectrum and the influence of factors such as tube and electrode geometry on it. Also, attention should be directed towards the number density fluctuations which seemed to be sustained under such widely-varying cathode and discharge conditions.

Acknowledgment—The authors wish to thank DR. G. S. KINO for many helpful discussions.

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APPENDIX

High frequency impedance of a simplified model of the discharge

The impedance of the simple model of a discharge described in Section 3 will now be calculated, yielding the results quoted in equations (1) and (3). In this model the d.c. voltage V_a across the tube is independent of current I_a, so that $G_0 = I_a/V_a$, where

V_a is a constant and the rate of change of instantaneous conductance G is assumed to be given by the equation

$$\frac{dG}{dt} = \frac{(G_0 - G)}{\tau}. \tag{4}$$

In order to find out the high frequency conductance, the current due to a small applied h.f. voltage $\Delta V e^{j\omega t}$ will be found.

Substituting $G = I_a/(V_a + \Delta V e^{j\omega t})$ in equation (4) yields

$$\frac{dG}{dt} = \frac{G_0 \Delta V}{\tau V_a} \cdot e^{j\omega t} \tag{5}$$

whence by integration,

$$\Delta G = \frac{G_0}{j\omega\tau} \cdot \frac{\Delta V}{V_a}, \tag{6}$$

where ΔG is the peak amplitude of the variation in instantaneous conductance. Since G for harmonic quantities is normally defined in terms of the ratio of peak values of I and V, and their relative phase, and does not contain the time explicitly, the exponential term (modulus unity) is dropped from the equation.

Writing ΔG and ΔV as infinitesimals, equation (6) becomes

$$\left.\begin{aligned} \frac{G_0}{j\omega\tau V_a} &= \frac{dG}{dV} \\ &= \frac{dG}{dI_a} \cdot \frac{dI_a}{dV} \\ &= \left(\frac{V - I_a dV/dI_a}{V^2}\right)\frac{dI_a}{dV} \\ &= \frac{1}{V}\frac{dI_a}{dV} - \frac{G_0}{V}. \end{aligned}\right\} \tag{7}$$

The explicit time dependence has been dropped as above. Equation (7) may be written

$$\frac{dI_a}{dV} = \left(1 + \frac{1}{j\omega\tau}\right) G. \tag{8}$$

Now dV/dI_a is the h.f. impedance, and $1/G$ is the d.c. resistance, whence

$$Z = R_{dc} \cdot \frac{j\omega\tau}{1 + j\omega\tau}. \tag{9}$$

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NON-SYNCHRONOUS ACCELERATION OF IONS IN CYCLOTRONS

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Abstract—A particular frequency modulation programme is developed with the object of storing ions in the central region of a frequency modulated cyclotron. These ions could subsequently be captured into phase-stable orbits and accelerated in the usual manner, thereby increasing the mean beam intensity obtainable from such a machine.

First, a saw-tooth frepuency programme is considered in which the rate of frequency change is too fast for ions to be accelerated synchronously. It is shown that there is a bias towards the net acceleration of ions to higher energies. By modifying the shape of the frequency programme it is found that more ions can be extracted from the source than with the simple saw-tooth shape. A frequency programme resembling a blunt saw-tooth is found to give the largest gains. The perturbing effect of successive programmes is examined and an estimate made of the limiting number of ions which can be stored in the centre of a machine.

INTRODUCTION

IN this report a modification of a proposal by KELLER (1958) for the improvement of the beam intensity in a synchrocyclotron is considered. The original proposal was to extract and store ions in the central region of a synchrocyclotron by means of a small dee placed near the source to which noise power would be fed. The process of ion extraction and storage is of a stochastic nature. These ions would then be captured by the main accelerating system into phase-stable orbits, and so would be accelerated up to the full machine energy, the process being repeated each cycle of the main frequency modulation.

The modification considered here relates to the use of a programmed modulation of the signal fed to the small additional dee instead of a noisy signal.

1. *Ion motions*

The motions of ions subjected to various rates of frequency modulation are first considered. It is assumed that, to a first approximation, the synchronous frequency of a particle in the central region of a machine decreases linearly with increase in the instantaneous energy of the particle. The equations of motion are:

$$\frac{dE}{dt} = \frac{eV(t)}{2\pi} \cdot \omega(E) \cdot \sin \phi(E,t) \tag{1}$$

$$\frac{d\phi}{dt} = \omega(E) - \omega_{rf}(t) \tag{2}$$

where

$$\phi = \phi_0 + \int_0^t [\omega(E) - \omega_{rf}(t)]\, dt, \tag{3}$$

and it is assumed that

$$\omega(E) = \omega_0(1 - \alpha E). \tag{4}$$

Particle paths have been found for various rates of frequency modulation, df_{rf}/dt, and for different initial phase angles by a numerical method. For simplicity it has been assumed that the maximum permissible energy gain per turn is constant; this assumption is not valid for ions moving in the dee gap, discussion of which is postponed to Section 3.

Typical paths are shown in Fig. 1 of ions undergoing non-synchronous accelerations (NSA). Only one half of the trajectories are given since they are anti-symmetrical about the frequency axis. Each curve is characterized by the instantaneous phase of the ion, relative to the accelerating radio frequency, at the instant when the synchronous frequency of the ions and the applied frequency are equal. This particular phase is called here the *characteristic phase*. In Fig. 2 is shown the variation of the instantaneous phase of an ion as a function of the time from the onset of NSA, and in Fig. 3 is given a three-dimensional representation of the information contained in Fig. 1. An illuminating discussion of similar ion motions in the central region of a F.M. cyclotron, together with some experimental results, is to be found in an article by RICHARDSON *et al.* (1948).

It is apparent that the change in energy of an ion rapidly approaches a limiting value as the difference between the dee radio frequency and the orbital frequency of the particles increases. If particle interaction is neglected, then the limiting value of the total change in particle energy is simply twice the

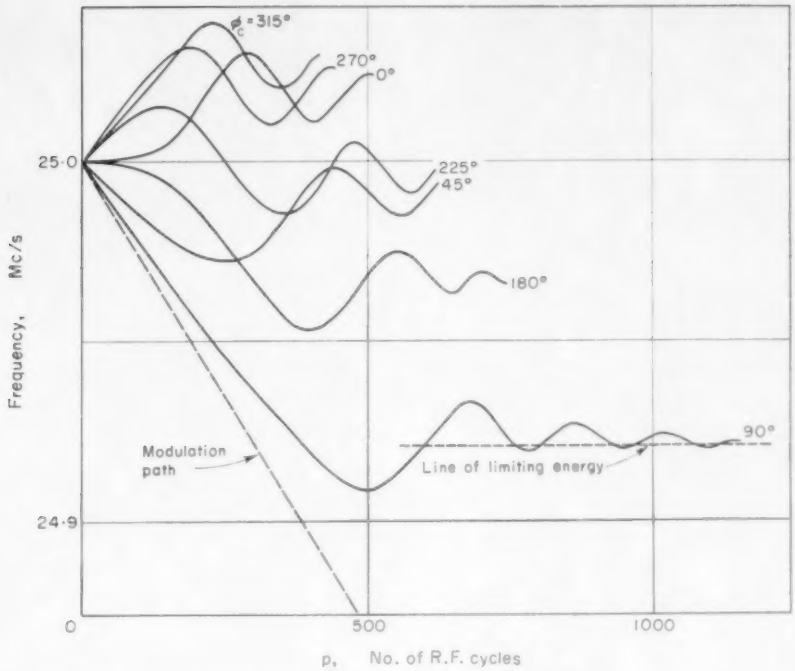

FIG. 1.—Curves showing the variation with time of the orbital frequency of ions undergoing non-synchronous acceleration. The maximum energy gain per turn is 4·1 keV and the rate of change of orbital frequency with particle energy is 54 c/s per keV.

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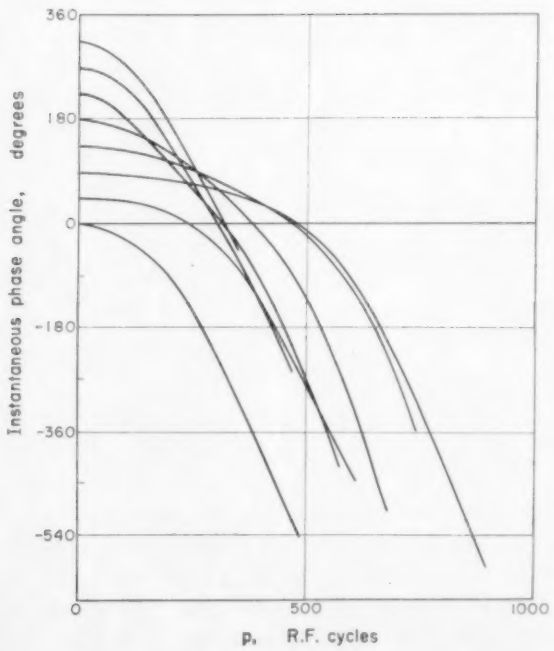

FIG. 2.—Variation of the instantaneous phase of an ion during NSA as a function of time for several different values of the characteristic phase angle.

energy change that occurs between $\omega(E) = \omega_{rf}(t)$ and $\omega(E) \gg \omega_{rf}(t)$. Curves showing the dependence of this energy change upon the characteristic phase of the particles are shown in Fig. 4 for several rates of frequency modulation. The curve number 4 is a limiting one in that it separates particles undergoing NSA from those that can be accelerated synchronously.

If particles are subjected to many such frequency modulation sweeps, and it is assumed that no phase correlation occurs between successive sweeps, then the particle will be accelerated in a manner analogous to stochastic acceleration. However, in contrast to normal stochastic acceleration in which a wide bandwidth of noise is used, the mean change in particle energy per frequency sweep can be quite large, but the frequency of such changes is greatly reduced.

For high rates of frequency modulation, and neglecting the possibility of phase bunching during such modulation cycles, the distribution of ions after a number of modulation cycles can be found using the methods applicable to stochastic acceleration. If the mean change in particle energy per modulation sweep is ΔE then it follows that after m sweeps the most probable total change in particle energy is

$$\Delta E_m = \sqrt{m}\Delta E. \qquad (5)$$

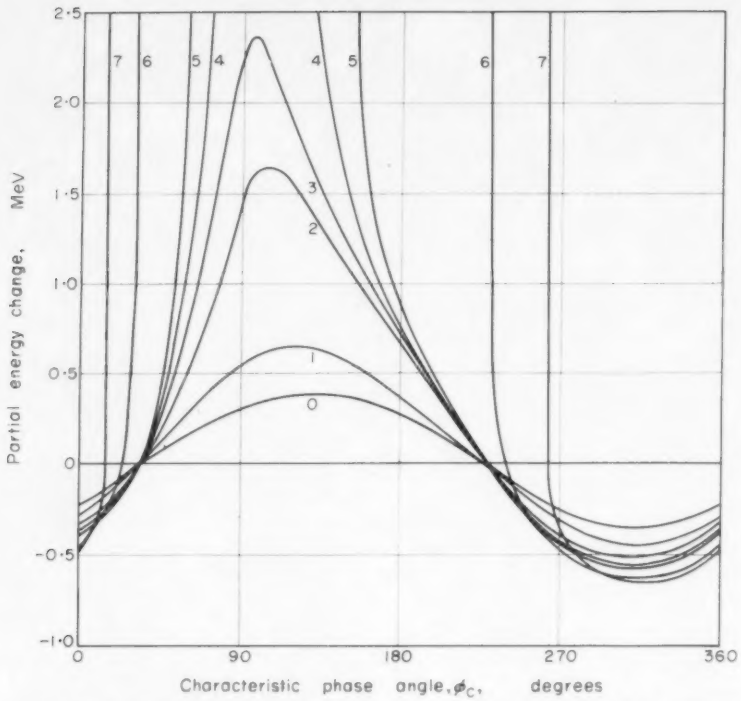

FIG. 4.—The variation of the limiting energy change as a function of the characteristic phase angle for different rates of frequency modulation. The rates of frequency modulation, normalized at the rate corresponding to a phase stability angle of 90°, are

Curve					
	0	4·65	4	1·000	ϕ_s 90°
	1	2·40	5	0·87	60
	2	1·20	6	0·50	30
	3	1·12	7	0·259	15.

The minimum depth of modulation is determined by the range of particle orbital frequencies within which particles are to be accelerated. If this is small then the number of modulation sweeps that can occur between successive sweeps of the main accelerating system will be large. In consequence the rate of diffusion of particles within the limits set by the extreme orbital frequencies will be large.

2. *Phase slip*

There is an important difference between NSA and stochastic acceleration which affects the efficiency of ion extraction from the source, namely, that large changes in phase occur in the NSA case between the dee voltage and the accelerated ions. Only during a part of each modulation sweep can ions be extracted and then accelerated to a finite limiting energy. In the case of stochastic acceleration, however, provided that the band-width of the applied noise power is large, phase slip is unimportant and ions are continuously extracted from the source.

Discussion of this effect in the NSA case is greatly simplified by the fact that ions become markedly bunched in phase about the $\phi = \pi/2$ point even after only one turn. Typical results can be found, therefore, by considering only those ions which have initial phase angles of approximately $\pi/2$. In Fig. 5 typical particle trajectories are given for ions starting at different times in a modulation sweep, from which it can be seen that ions can be extracted only during a limited part of each sweep. Figure 6 shows how the limiting energy gain depends upon the point in the modulation sweep at which the ions leave the source. The reference time is the point in a modulation sweep when the radio frequency is equal to the orbital frequency at the machine centre.

3. *Acceptance efficiencies*

The results of the above section are clearly shown in phase-acceptance plots of the type shown in Figs. 7, 8 and 9, corresponding to three different modulation rates. Also indicated in Fig. 8 are approximate contours enclosing regions of constant limiting particle energy. An interesting complication in Fig. 9 is the occurrence of a region in which ions can gain energy indefinitely, corresponding to synchronous acceleration. Contours of this region are given by the usual phase-energy (time) curves defining bucket size for a given phase stability angle, ϕ_s.

From these figures the behaviour of the acceptance

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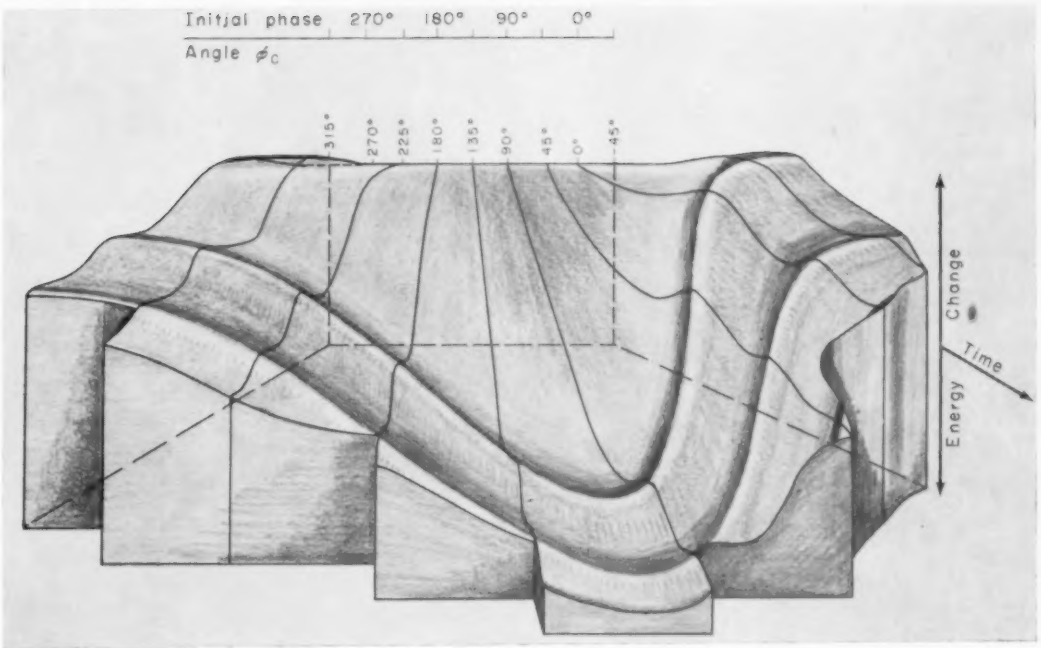

FIG. 3.—A perspective sketch showing the variation of particle energy with time for different characteristic phase angles.

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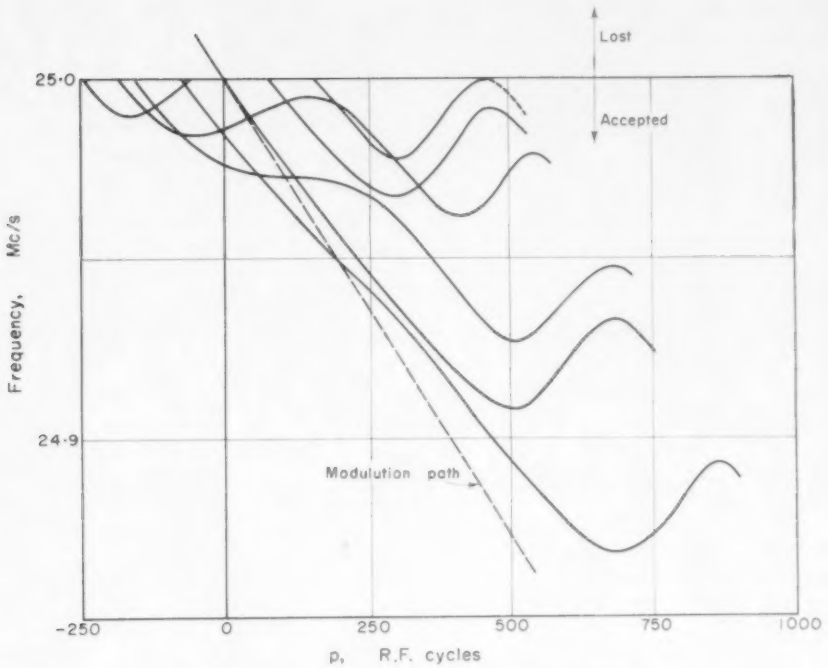

FIG. 5.—Plots of orbital frequency as a function of time for ions starting at different times in the modulation sweep. All starting phases are at 90° and the data is from Fig. 1.

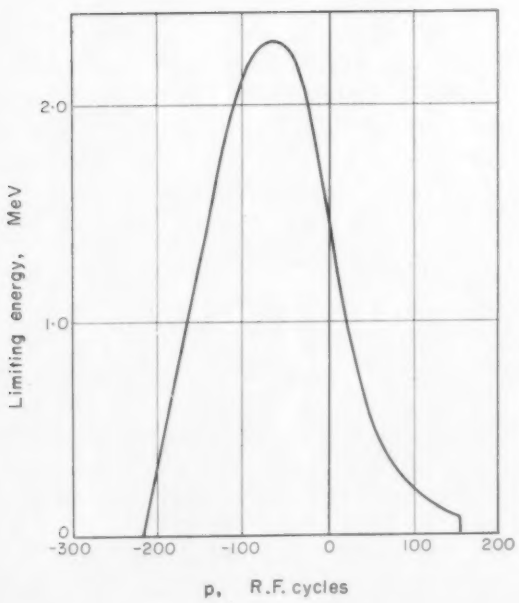

FIG. 6.—Variation of the limiting energy gains of the ion paths shown in Fig. 5. with the time of starting.

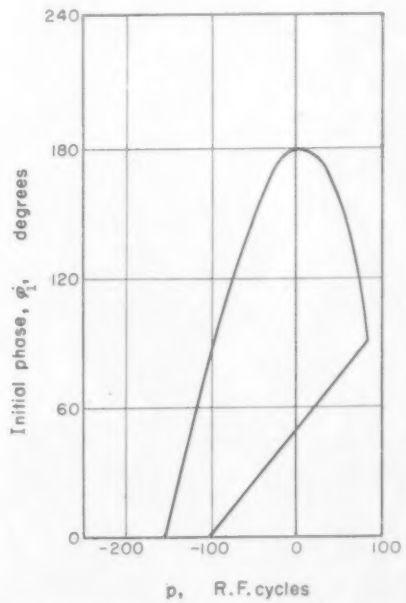

FIG. 7.—Curve showing the range of phases that ions can possess if they are to be accelerated as a function of the starting time. Data from curve 1 of Fig. 4.

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efficiency, defined as the fraction of each main modulation cycle during which ions are available for storage in the central region, as a function of the rate of modulation can be deduced. However, to obtain meaningful results, the effect of variation of energy

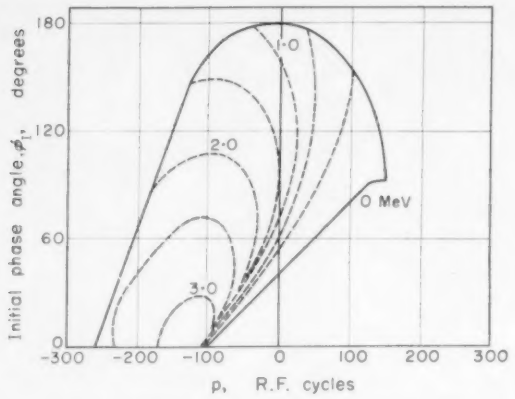

FIG. 8.—Similar to Fig. 7 but for curve 2 of Fig. 4.

gain per turn in the dee gap must be included and so this point is considered next.

Obviously, if the rate of modulation is held constant, the acceptance range per frequency sweep will be considerably reduced as the maximum energy gain per turn decreases. It is natural, therefore, to consider changing the rate of modulation for frequencies near the orbital frequency at the machine centre so as to compensate for the variable energy gain per turn in that region.

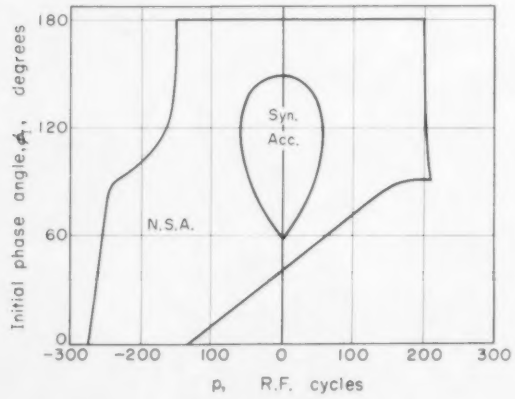

FIG. 9.—Similar to Fig. 7 but for curve 5 of Fig. 4.

Differentiating equation (4) with respect to the number of radio frequency cycles p gives

$$\frac{dE}{dp}\Big/\frac{df}{dp} = -\frac{2\pi}{\omega_0 \alpha}, \tag{6}$$

which is constant if α is held constant. If the time scale is changed by the factor $1/X$ then the frequency scale is changed by X and from equation (6) the scale of dE/dp changes by X^2, so that the energy scale must be changed by the factor X. Hence, it is possible to use the ion paths computed for a given scale factor when examining the ion motions in the central region simply by changing the scale factors in accordance with the changes in the energy gain per turn.

Assuming the relation between particle energy and orbit radius to be

$$E = A \,.\, r^2, \tag{7}$$

where A is defined as

$$A = E_1/D^2, \tag{8}$$

and the maximum energy gain per turn to vary as r/D, then for $r < D$

$$\frac{dE}{dp} = \sin\phi \,.\, \sqrt{\frac{E}{E_1}} \,.\, U. \tag{9}$$

The scale factor is then simply

$$X = \left(\sin\phi \sqrt{\frac{E}{E_1}}\right)^{1/2}. \tag{10}$$

If ϕ changes only slightly in the initial acceleration period then equation (9) can be integrated, and defining $p = p_1$ when $E = E_1$ then

$$\sqrt{\frac{E}{E_1}} = \frac{p}{p_1}. \tag{11}$$

Hence, by varying the rate of frequency modulation in the central region as p/p_1 the ion paths are unaltered. Consequently, the ranges of the phases that can be accepted are unaltered. The ion paths, however, are no longer antisymmetrical about the frequency axis.

Combining equations (6), (9) and (11) gives the dependence of f_{rf} upon p

$$f_1 - f_{rf} = \frac{\sin\phi\,\omega_0 \,.\, \alpha \,.\, U}{4\pi p_1} \,.\, p^2, \quad r < D. \tag{12}$$

One such plot is shown in Fig. 10 for the particular conditions indicated; this particular case is shown also in Fig. 11 with linear co-ordinates, illustrating the variable rate of modulation near the centre. In addition, two other particle paths are shown for ions leaving the source when the applied frequency is less than the orbital frequency at the centre.

In the NSA system the number of modulation cycles in each of the main frequency modulation cycles is relatively small. So it is necessary for ions to receive energy changes of more than a certain minimum if the rate of diffusion is to be prevented from becoming too low and thus giving a low ion density in the storage space available.

The approximate magnitude of this limiting minimum energy change is found readily from the two modulation frequencies, the mean energy of the stored ions and the equation (5). The adoption of such a minimum energy change enables a realistic estimate of the acceptance efficiency to be made for a given set of parameters. In Fig. 11 the appropriate limiting minimum energy is indicated. It is apparent that the number of r.f. cycles during which ions are extracted from the source and receive limiting mean energy changes greater than the specified minimum value is relatively small. For the particular case shown in Fig. 11 ions are acceptable for about 125 r.f. cycles in each modulation sweep, If there is little disturbance of stored ions by successive sweeps then the acceptance efficiency is approximately 0·065 when $F = 200$, as in the Harwell 110″ cyclotron. This value for the acceptance efficiency can be compared with that of a normal synchrocyclotron, such as the Harwell machine, namely 0·01. Hence, the maximum possible gain in beam intensity would be about six, assuming that the processes of capture and subsequent acceleration were ideal.

At the onset of NSA, increasing the frequency of the modulation sweeps reduces the acceptance efficiency, as the energy change per sweep curve becomes more symmetrical about the phase-angle axis (see Fig. 4). After reaching a minimum value, the efficiency increases with further increase in the modulation rate, finally approaching a limiting value corresponding to purely stochastic acceleration. This upper region is not considered here.

However, by modifying the modulation curve an improvement in the acceptance efficiency can be produced. Let each modulation sweep be preceded by a section in which the applied frequency is held constant and equal to the orbital frequency at the machine centre. During this part of each modulation sweep, ions are extracted as in a conventional cyclotron, in which the usual phase slip considerations apply.

Typical particle paths for this scheme are shown in Figs. 12 and 13 for two different rates of frequency modulation, from which it is seen that ions are made available for subsequent acceleration over a greatly increased number of r.f. cycles. The extent of the constant frequency section which is of use is determined simply from the condition that the maximum energy of the useful ions must not decrease too much.

A complication that occurs in this scheme arises from the marked perturbation of ion-paths by the extended constant frequency section of the modulation sweeps in subsequent modulation cycles. To examine this effect it is assumed that the ions extracted during any given modulation sweep become distributed in phase prior to the next modulation sweep.

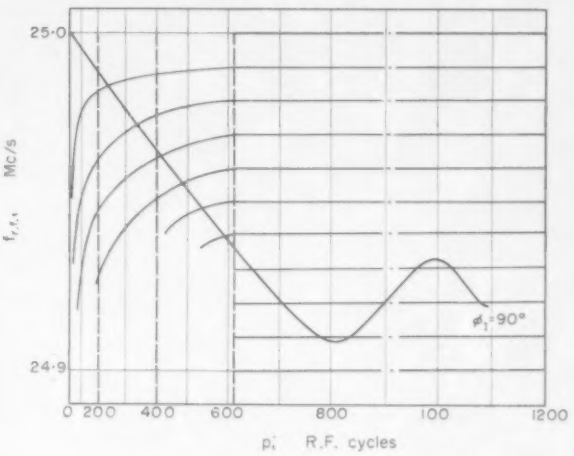

FIG. 10.—Plot of ion path in frequency-time indicating the non-linear scales used to examine ion motion near the machine centre. For this curve $E_1 = 1{\cdot}27$ MeV., $p_1 = 620$ cycles and $df_{rf}/dp = 258$ sec^{-1}. The starting phase is 90°.

An estimate of the acceptance efficiency can be made for this scheme by finding the fraction of those ions, extracted during one modulation sweep, that are not returned to the source during the following modulation sweep. In fact it is only necessary to examine the effect of the constant frequency part of the following modulation programme since the varying frequency section is strongly biased in favour of ion acceleration.

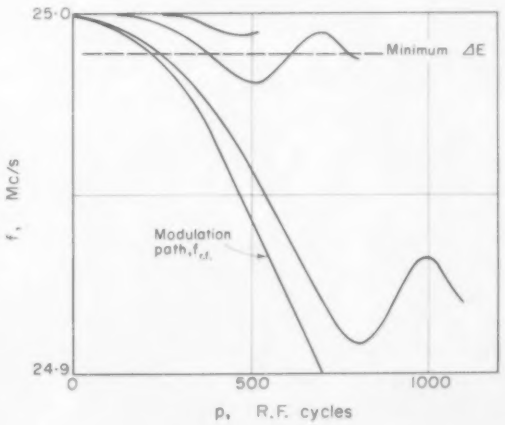

FIG. 11.—The same ion path as shown in Fig. 10 but with linear scales. Two other paths are shown for ions starting at a later time, all initial phases being 90°.

First, the number of r.f. cycles is found in which ions are accelerated to given limiting energies during a single modulation programme (see Fig. 14). This data

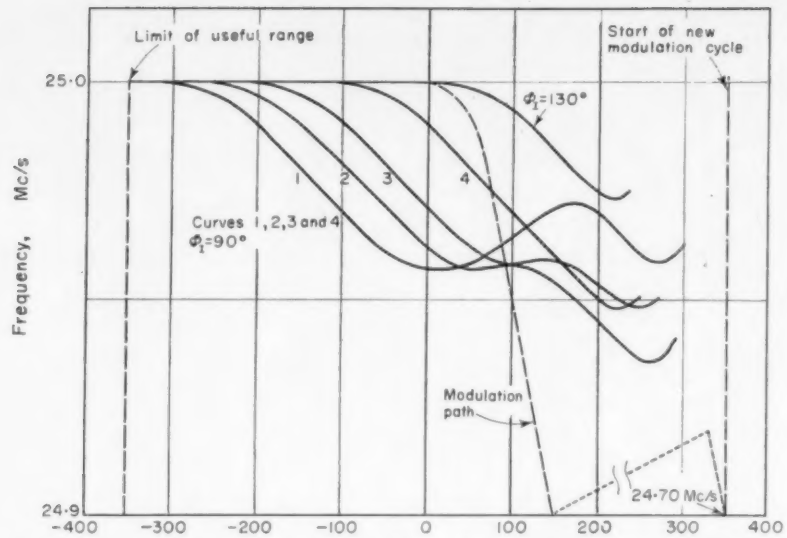

FIG. 12.—Ion paths in frequency-time showing the effect of using a constant frequency section before the usual frequency modulation sweep. The maximum energy gain per turn is 4·1 keV.

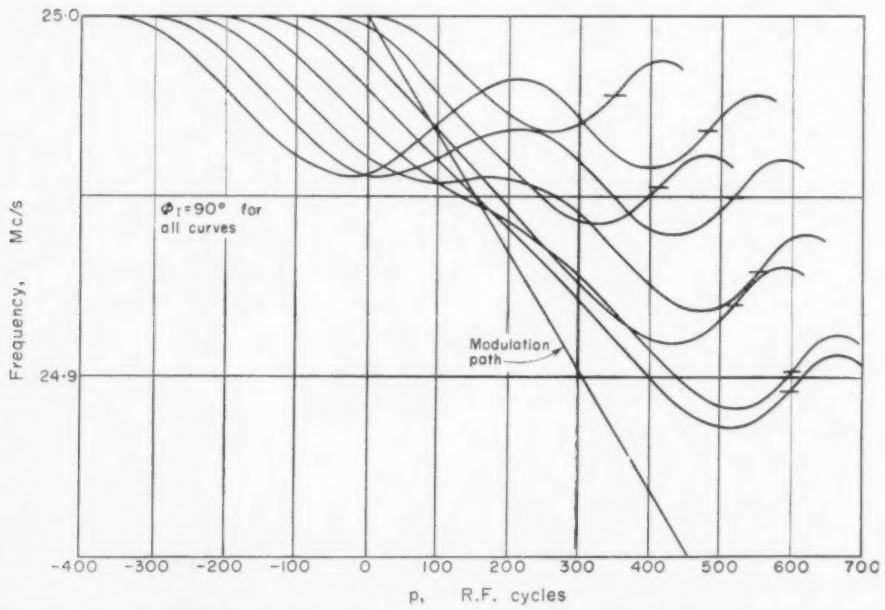

FIG. 13.—Ion paths in frequency-time similar to those in Fig. 12 but for a slower rate of frequency modulation.

is then transferred into a histogram showing the relative numbers of ions reaching given intervals of limiting particle energy as, for example, in Figs. 15 and 16. Next, by computing the limiting path of an ion which is just returned to the source, as a result of the extended application of the constant frequency section (see Fig. 17) the limiting phases and energies of returning ions can be found. In Fig. 18 these critical phases and energies are shown for two cases corresponding to the prolonged and restricted application of the constant frequency section. Based upon the assumption that the phases of the ions after each modulation programme become distributed, the initial distribution histogram of particles with energy can be corrected to allow for the ions lost by the constant frequency perturbation. These corrections are also indicated in Figs. 15 and 16; the doubly-shaded portions show the effect of prolonging the constant frequency part of the modulation programme.

Comparison of the corrected histograms shows that

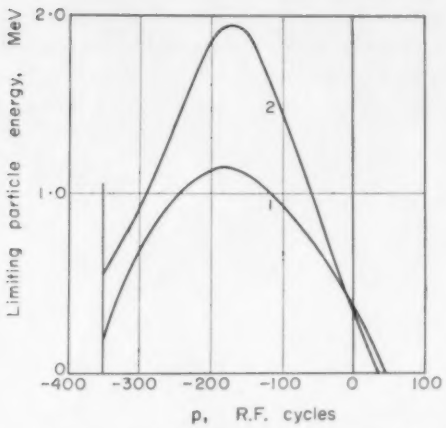

FIG. 14.—Curves of limiting particle energy as a function of the starting time in a modulation programme. The data for curve 1 is given in Fig. 12 and that for curve 2 in Fig. 13.

the fraction of ions lost in each successive frequency programme depends upon the rate of frequency change in the varying frequency section. To minimize the number of ions lost, a slow rate of frequency change is required. However, to satisfy the conditions implied in equation (5) the rate of change of frequency must not be made too low. The frequency programme shown in Fig. 13 would appear to be a reasonable compromise.

If it is assumed that the fraction f of ions lost in successive programmes is constant then the total number of ions that can be stored in this manner is given by the sum of a geometrical progression series with the constant ratio $(1-f)$, of which the sum is $1/f$, multiplied by the number of ions extracted in each of the modulation cycles. For the data given in Fig. 16,

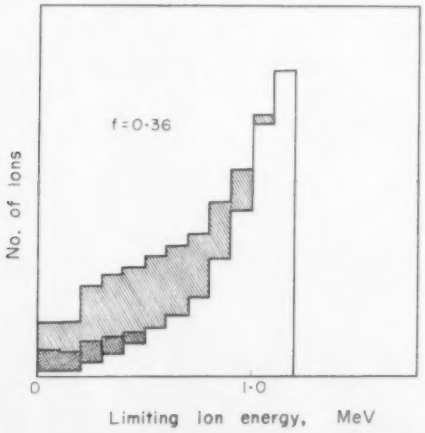

FIG. 15.—Histogram of the relative number of ions in a given energy range as a function of the limiting energy after one modulation programme. The shaded sections show the relative ion losses caused by the following constant frequency section of the next modulation programme. Data from curve 1 of Fig. 14.

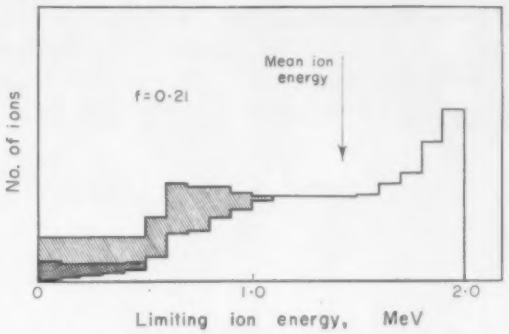

FIG. 16.—Similar histogram to that of Fig. 15 but for curve 2 of Fig. 14.

$f = 0{\cdot}21$ and so the acceptance efficiency, for $F = 200$, is only $\dfrac{390 \,.\, 200}{25 \,.\, 10^6 \,.\, 0{\cdot}21} \approx 0{\cdot}015$. This rather low value of the acceptance efficiency just found, however, is unrealistic since the varying frequency section of successive modulation cycles rapidly moves a large fraction of the stored ions into a region outside the influence of the constant frequency section. For example the frequency programme shown in Fig. 13, used in obtaining the above acceptance efficiency of 0·015, increases the mean energy of the stored ions by approximately 0·4 MeV per sweep. In addition the constant frequency section is only able to remove ions from those stored which have energies less than about 1 MeV, and most of these are removed after the first perturbation. Consequently, the total number of ions stored is not given simply by the geometrical progression series that was assumed initially.

Some idea of how the fraction f varies with the number of frequency sweeps can be obtained by recalculating its value after the action of only one complete modulation cycle upon an initial ion distribution. Allowance is made first for the initial fraction

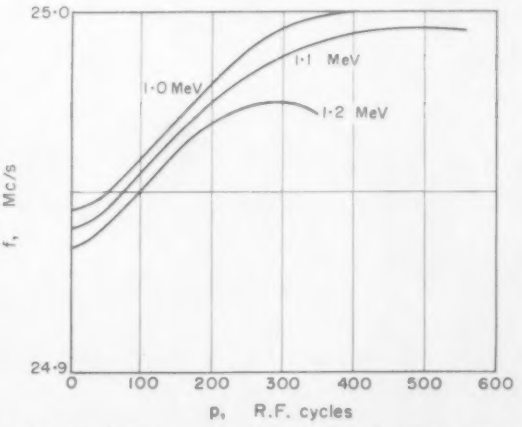

FIG. 17.—Frequency-time plots for ions during the constant frequency section of a modulation programme. Maximum energy gain per turn is 4·1 keV.

f_1 of ions which are lost in the constant frequency section of the following modulation cycle. By using the appropriate curve describing the energy changes occurring during the changing frequency section, signal on a small additional dee at the centre of a synchrocyclotron, a reasonably large number of ions can be extracted and stored in the central region. If all the ions made available in this

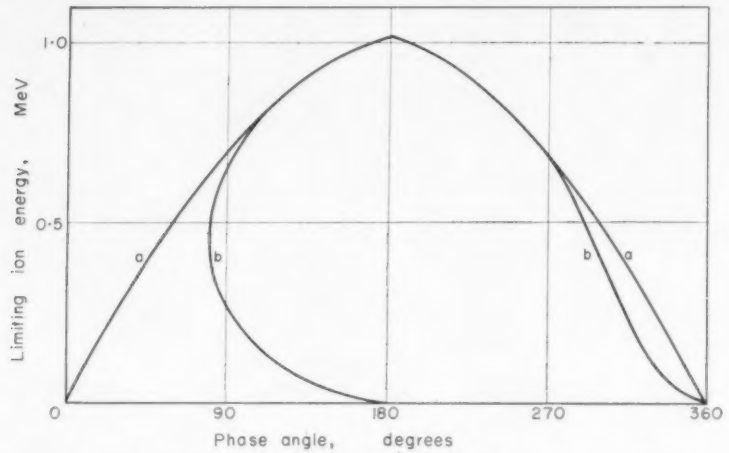

FIG. 18.—Curves of limiting phase and energy of ions which are returned to the source during the constant frequency section. Curve *a* corresponds to the prolonged application of the constant frequency section and curve *b* to only 350 cycles of the radio frequency signal.

obtained from Fig. 4, and assuming that the ions are still approximately distributed in phase, a new ion distribution histogram can be found. One such histogram is shown in Fig. 19, starting with the initial histogram of Fig. 16. The fraction of ions lost after the action of one more constant frequency section is shown also; in this particular case the value of the new fraction f_2 is 0·12, approximately. If this new value of f is now assumed to remain constant for successive modulation cycles and the original value is retained for the initial loss, then the acceptance efficiency is increased to 0·028.

The repeated application of this method would lead to even higher values for the acceptance efficiency and so a value of 0·03 can be adopted as a reasonable minimum in this particular case.

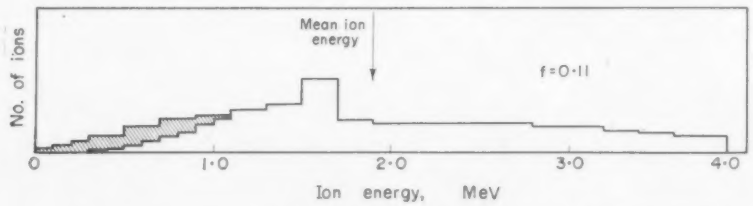

FIG. 19.—Histogram of the relative number of ions in a given energy range after two modulation sweeps. Data from curve 2 of Fig. 14.

4. *Conclusions*

By using a programmed frequency modulation way could be captured into phase-stable orbits and then accelerated to full energy, gains of up to 3 in the mean beam intensity would result.

Careful frequency programming is necessary if large efficiencies are to be achieved but this is alleviated by the relative simplicity of the modulation curve. A modulation curve closely resembling a 'blunt-saw-tooth' wave-form appears to give the largest gains that can be obtained in this method of ion storage. Although the acceptance efficiency with this method is only a very small fraction of that which might be expected with purely stochastic acceleration and storage, the two methods might in practice be competitive. The reasons for this are the adverse conditions of high ion density at the centre and small initial rate of energy gain in the purely stochastic case.

5. *Notation*

A = a constant of the machine, $2\pi^2 m f_0^2$
D = distance of accelerating dee from ion source
E = energy, electron volt units
E_1 = energy of ion with orbit radius D
f = frequency
f_0 = synchronous frequency at machine centre
f_{rf} = frequency of signal applied to the dee
p = number of radio frequency cycles
r = radius of ion orbit
t = time
U = peak voltage on dee, volts
V = voltage on dee, volts
$\alpha = -\dfrac{d\omega(E)}{dE}$, assumed constant
ω = angular frequency of particle
ω_{rf} = angular frequency of r.f. signal on the dee
ω_0 = angular frequency of particle at machine centre
ϕ = phase angle
ϕ_c = characteristic phase angle
ϕ_1, ϕ_0 = initial phase angle
ϕ_s = synchronous phase angle
m = number of modulation cycles
F = main modulation frequency of synchrocyclotron.

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RICHARDSON J. R., WRIGHT B. T., LOFGREN E. J. and PETERS B. (1948) *Phys. Rev.* **73**, 424.

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Plasma Physics (Journal of Nuclear Energy, Part C): 1961, Vol. 3, p. 196. Pergamon Press Ltd. Printed in Northern Ireland

BOOK REVIEW

Plasma Acceleration, KASH S. W. (Ed.) Stanford University Press, 1960. 117 pp., $4.25.

THIS is a collection of nine papers presented at a symposium sponsored by the Lockheed Missiles and Space Division of their Palo Alto Research Laboratory in December 1959. The symposium was the fourth in a series on the general subject of magnetohydrodynamics, but particular emphasis was placed on the more definite applied theme 'the acceleration of plasma for propulsion for interplanetary travel'. The embryonic stage of the development of plasma accelerators for this application is illustrated by the fact that only two of the papers reported any experimental work in the field. These were the description by John Marshall of his coaxial accelerator and the report by Starr and Naff of some measurements of the acceleration of vaporized wires and electrode material in T tube and coaxial geometries. While these results are of scientific interest and are perhaps suggestive as to ultimate possibilities in space propulsion, they do not easily lend themselves to an engineering extrapolation to practical devices.

The remaining papers present calculations varying widely from the general ('Scaling Relations for Plasma Devices' by G. S. Janes) to the particular ('Detailed electrode geometry design calculations for an electrostatic ion accelerator' by Raymond Fox).

Although the papers in this collection are individually interesting, the volume cannot claim to present an integrated and definitive review of the subject matter. Hence, its principal use is in providing the reader with a preliminary indication of some of the concepts which are now being explored in the field of plasma acceleration and the type of experiments which are in progress or are being contemplated.

R. D. MILLER

Plasma Physics, (Journal of Nuclear Energy, Part C), 1961, Vol. 3, pp. 197 to 202. Pergamon Press Ltd. Printed in Northern Ireland

ACCELERATION OF IONS IN A CYCLOTRON WITH AZIMUTHAL VARIATION OF THE MAGNETIC FIELD*

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(*Received* 6 *August* 1959)

Abstract—Experiments are reported on the acceleration of charged particles in the azimuthally varying magnetic field of the 1·5 metre cyclotron at the Institute of Atomic Energy, Academy of Sciences, USSR.[1]

The creation of a magnetic field of the sector type with about ±15 per cent flutter, for a potential between the dees of about 15 kV, permitted the acceleration of deuterons up to energies of 19 MeV. A study of the motion of the ions in the terminal orbits indicates the possibility of extracting the greater portion of a beam of ions with an energy considerably in excess of 20–22 MeV, by means of an electrostatic deflexion system. Relationships are obtained characterizing the process of acceleration in conditions of an azimuthally varying magnetic field. Experiments are described concerning adjustment of the magnetic field shape by current windings installed inside the accelerator chamber.

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INTRODUCTION

IN order to create vertical stability of ions accelerated in the normal cyclotron, it is necessary that the intensity of the magnetic field be decreased as a function of the radius. However, in 1938 THOMAS[2] had already shown that the motion of charged particles can also be stabilized in the case where the magnetic field increases with radius, by the presence of an azimuthal variation. The suggestion by THOMAS opened up new prospects of charged particle accelerators to give high currents at high energies. However, no progress was made at that time, since the creation of a magnetic field of the required shape presented an extremely complex problem, and the discovery by Veksler (1944) and McMillan (1945) of the synchrocyclotron principle distracted the attention of physicists from this suggestion.

In recent years, many authors have studied theoretically and experimentally the motion of charged particles in a cyclotron with azimuthal variation of the magnetic field.[3,4,5,6]† A number of theoretical papers have also been presented in the Institute of Atomic Energy of the Academy of Sciences, USSR.

In 1957, on a one-fifth scale model of a 1·5 metre cyclotron, experiments were carried out on the production of a magnetic field with azimuthal variation of the sector type. On this model the combined effect of iron shims and current-carrying adjusting coils was studied, presenting special interest for the design of a variable-energy cyclotron. The experiments showed that the combination of iron and current winding adjustment may ensure production of a given field shape over a broad range of field values.

Flutter values of ±15 per cent were obtained using three sectors. This choice is extremely simple from the point of view of engineering construction.

Preparations for acceleration of ions in the 1·5 metre cyclotron were commenced in 1958, at the conclusion of the experiments on the model. The sectors and the necessary iron and current shims were constructed and new pole pieces, which form the vacuum chamber covers were made.

The length of the existing dee stem on the cyclotron meant that the accelerating voltage wavelength could not be less than 26·7 m. This circumstance has determined the maximum energy to which deuterons and molecular hydrogen ions may be accelerated.

CONSTRUCTIONAL DETAILS

The new pole pieces are 1·5 metres in diameter, and their thickness is 80 mm. Machining of their surface was carried out within a tolerance of 0·08 mm. The tolerance on machining the iron sectors, whose height is 30 mm and whose angular spread is about 60°, amounted to 0·1 mm. All the iron adjusting shims were prepared to the same tolerance. In order to exclude high-frequency losses in the iron elements, their surfaces were electrolytically coated with a layer of copper to a thickness of about 70 microns.

Figure 1 shows a drawing of the accelerator chamber with the pole pieces. Sealing of the covers was accomplished by the usual means. The sectors (4)

* Translated by W. E. JONES from *Atomnaya Energiya* **8**, 201 (1960).

† The spiral ridge cyclotron is not discussed here.

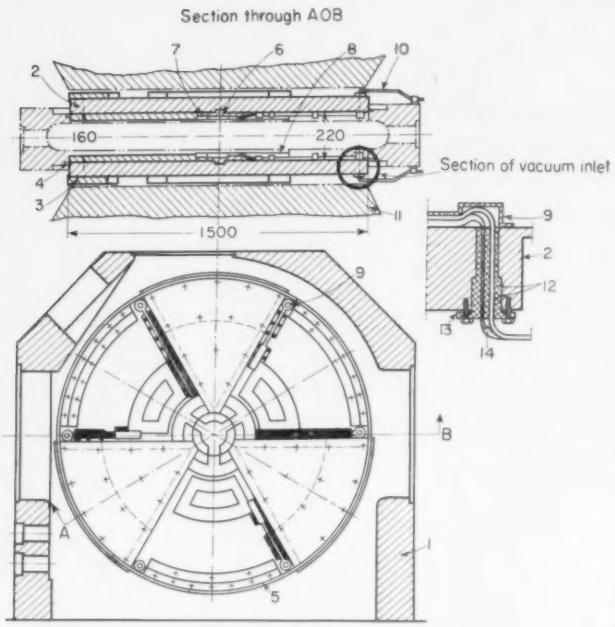

FIG. 1.—The accelerating chamber with iron shims and current-carrying adjusting windings. 1—Accelerator chamber; 2—Covers of accelerator chamber; 3—External shim; 4—Sectors; 5—Internal adjusting; 6—Central disks; 7—Central adjusting windings; 8—Adjusting windings within the cavities; 9—Copper screens; 10—Vacuum inlet supplying windings; 11—Electromagnet poles; 12—Insulators; 13—Rubber packing; 14—Vinylchloride tubing.

were bolted to the covers (2). The external (3) and internal (5) iron shims were secured by the same method (the external shims were large-sized). As is clear from the figure, the elements (5) were fixed in the cavities between the sectors and served for increasing the intensity of the magnetic field at the peripheries. Iron disks (6) with a diameter of 200 mm were located in the centres of the covers. For achieving fine adjustment at radii of 190–260 mm, bolts with a head in the shape of a small disk could be screwed into threaded holes in the sectors, serving as the adjusting element. Any one of the bolts securing the sectors to the cover may be, if the need arises, replaced by a bolt with a disk-shaped head. Up to four adjusting windings are located on the internal surface of each cover. The central adjusting windings (7) are constructed in two sections (each with seven turns) each with a separate electrical supply. One of the sections permits variation of the magnetic field value within the central portion, and the second one movement of the magnetic median plane.

The central adjusting winding is illustrated in two projections of Fig. 2. The casing of the windings (2) milled out from brass, is water cooled by means of copper tubes. In order that the zinc should not be

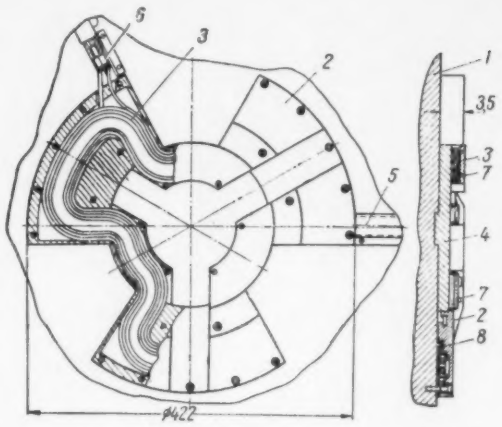

FIG. 2.—Central adjusting winding. 1—Cover of accelerator chamber; 2—Casing; 3—Winding of copper tubing; 4—Central disk; 5—Copper screen; 6—Demountable vacuum coupling; 7—Water tubes for cooling casing; 8—Clamping plate.

atomized by sparkover, the casing was electrolytically coated with a thick layer of copper. In the figure, both sections of the copper tubes can be seen, laid one on the other. The demountable vacuum coupling (6) permits, if necessary, the removal of the windings together with the casing. The windings (8) (with 14 turns in each, see Fig. 1) located within the cavities allow adjustment of the first harmonic of the magnetic field.

Figure 3 shows a plan and cross section of this winding. All windings are fabricated from copper tubing with a diameter of $4 \times 0{\cdot}5$ mm, laid in two banks, and they are water cooled, as are the casings for the windings. All windings are insulated by vinylchloride tubing, the out-gassing of which proved to be completely acceptable.

A cross section of the vacuum inlet is separately depicted in Fig. 1. All tappings for the windings are covered by copper screens, screwed into the cover of

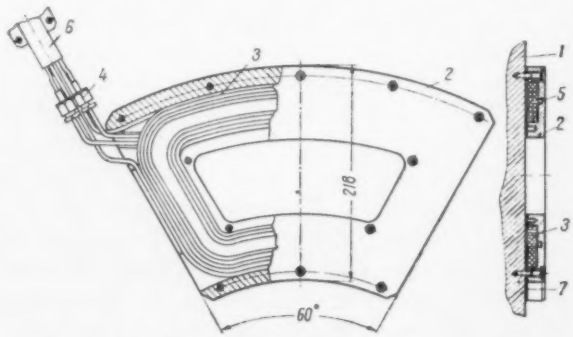

FIG. 3.—Adjusting windings within the cavities. 1—Cover of accelerator chamber; 2—Casing; 3—Winding; 4—Demountable vacuum coupling; 5—Water tubes for cooling casing; 6—Copper screen; 7—Cover of casing.

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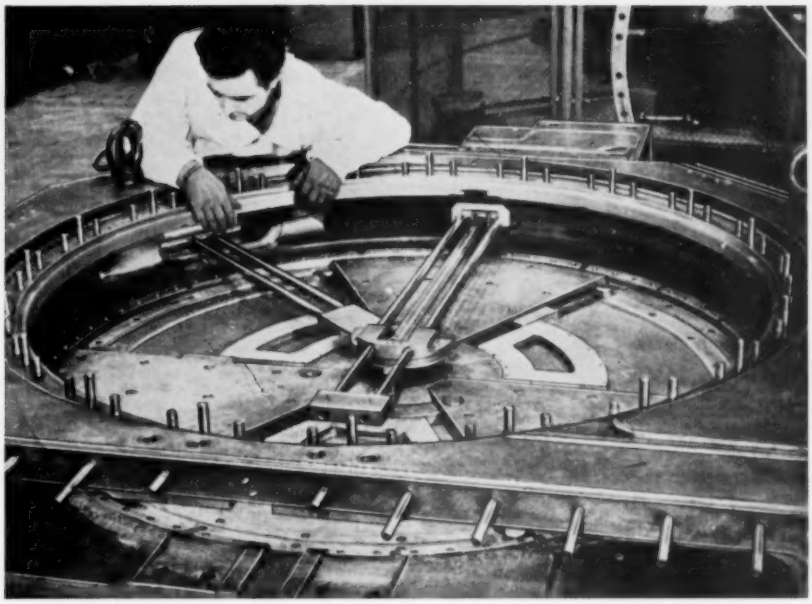

FIG. 4.—Accelerator chamber with top cover removed. The apparatus for measuring the magnetic field can be seen.

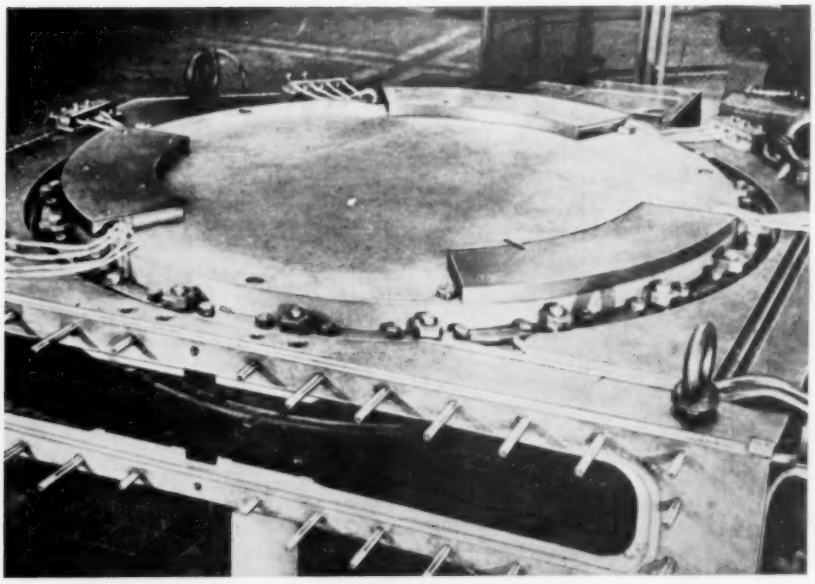

FIG. 5.—Accelerator chamber. On the top of the cover can be seen the outlets of the windings and the external adjusting shims.

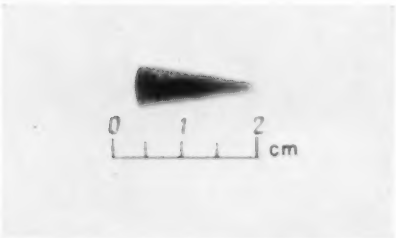

FIG. 13.—Photograph of the beam, obtained by superposition of a copper plate exposed to deuterons.

the chamber. Thus, because of the presence of the casings and of the screens, the windings are completely shielded from the effects of high-frequency currents and sparkover. Figure 4 shows the accelerator chamber with top cover removed, and Fig. 5 shows the top cover installed in position.

To study the characteristics of the accelerated ion beam, two screening probes were designed and manufactured. Before striking the measuring electrode, the ions pass through an aluminium filter, preventing the impact of low-energy charged particles on the measuring electrode. Knowing the thickness of the filter it is possible to obtain a good estimate of the energy of the ions at any radius during acceleration. A disadvantage of the screening probes is the 'blind' space, formed with the front wall of the casing and with the gap between this wall and the measurement electrode. This blind space reduces the reading of the probes, cutting the initial portion of the beam by approximately 1·5 mm. In the experiments, a probe was also used installed inside one of the dees. As well as for beam measurement, this probe served for screening the beam in order to determine the zone of the centres of trajectories by the three-probe method. In order to study the vertical distribution of intensity of the ion beam, a multi-electrode probe was used, in which were distributed thermocouples via a copper screen with slits. By means of a telephone selector switch the thermocouples were connected with the measuring equipment.

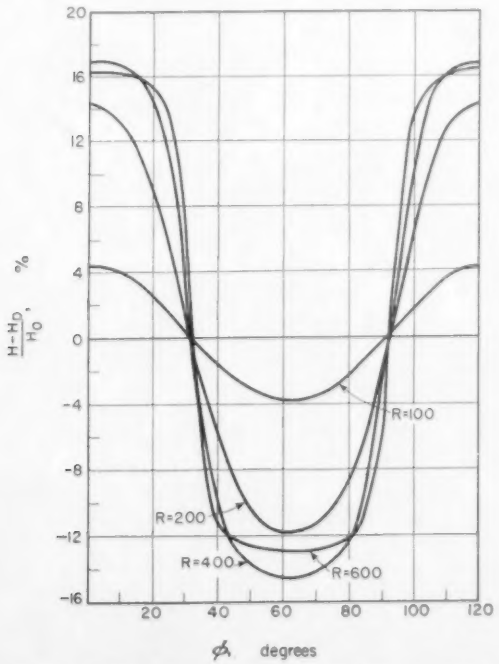

FIG. 6.—Dependence of magnetic field intensity on angle ϕ.

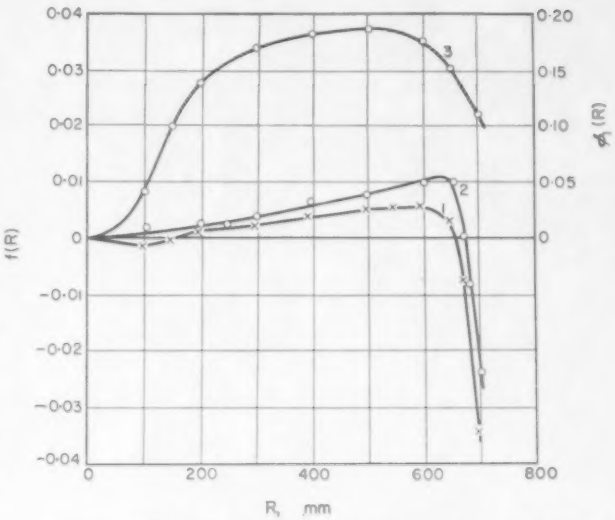

FIG. 7.—Radial variation of the mean magnetic field and the amplitude of the principal harmonic. 1—$f(R)$ for $H_0 = 14{\cdot}7$ kilo-oersted; 2—$f(R)$ for $H_0 = 14$ kilo-oersted; 3—$\Phi(R)$ for $H_0 = 14{\cdot}7$ kilo-oersted.

For obtaining the ions, the usual open type source is used. The intensity of the high-frequency field extracting the ions from the source was increased by installation on the dees of fillers with a length of 200 mm. Extraction of the ions takes place in bothd ees.

To reduce activation of the acceleration chamber components, all experiments were carried out under pulsed operation with a duty-ratio range from 5 to 500. The ion current per pulse was measured by means of a calibrated oscillograph, and the value of the mean current either by integration or by a thermal method. In all quoted measurements, the values of currents per pulse are given. For the purpose of studying the ion motion at large radii in the region where the fringe field of the magnet might cause a perturbation, the deflexion system was removed from the dee.

CHARACTERISTICS OF THE MAGNETIC FIELD AND ACCELERATION OF THE IONS

For measuring the vertical component of magnetic field intensity in the median plane, the usual arrangement was employed, with two coils connected in opposition. A ballistic galvanometer served as the measuring apparatus. The accuracy of measurement amounted to 0·05 per cent of the central field H_0.

In Fig. 6 is shown the dependence of magnetic field intensity on the azimuthal angle ϕ for various radii R. From these curves were obtained functions $f(R)$, and the amplitude of the principal harmonic $\Phi_1(R)$, shown in Fig. 7.

The quantity

$$f(R) = \frac{1}{2\pi}\int_0^{2\pi} \frac{H(R,\phi) - H_0}{H_0}\, d\phi$$

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characterizes the radial variation of the mean magnetic field on the median plane.

The excellent agreement between the field measurements on the cyclotron and the design data obtained

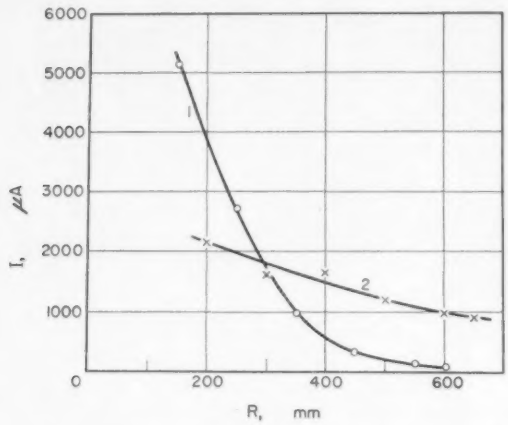

FIG. 8.—Variation of probe current with radius of installation of the probe. 1—For the normal cyclotron; 2—For the cyclotron with azimuthal variation of the magnetic field.

on the model should be noted. This agreement permitted to begin operation without any alteration in the design of the covers, sectors, or shims.

Acceleration of molecular hydrogen ions and deuterons was accomplished by two schemes of operation, differing in the radial variation of the mean field. (Acceleration of molecular hydrogen ions was convenient, in that it caused by far the least activation of the accelerating chamber components). Two different sets of operating conditions of the magnetic field were chosen, with the aim of explaining the effect of the law of increase of the magnetic field upon the motion of the accelerated ions.

In Fig. 8, curve 2 shows the variation of measured probe current with radius for the cyclotron with azimuthally-varying magnetic field; for comparison, curve 1 was obtained in the same cyclotron and in the same circumstances, but without azimuthal variation.

The relatively small decrease of intensity with radius in the cyclotron with azimuthal variation is explained by the absence of phase loss and the stronger vertical focusing.

The vertical distribution of current intensity in the beam and position of the magnetic median plane, were studied by means of the multi-electrode thermocouple probe. Figure 9 shows such curves obtained at two different radii.

It should be noted that the tolerances with which the covers and sectors were made proved to be adequate, in that after assembly the maximum currents at the final radius were obtained at once without any adjustment of the magnetic field.

Figure 10 shows the dependence of the ion current upon the accelerating voltage ('triggering characteristics'), obtained for energies of 19 MeV with $H_0 =$ 14·0 kilo-oersted. The energy of the accelerated ions in the given case was computed from the mean orbit radius, measured by means of the three probes. The point with a triangle corresponds to the computed triggering voltage, on the assumption that the initial phase is $+45°$. The 'triggering characteristics' for the same ion energy with a magnetic field $H_0 = 14{\cdot}7$ kilo-oersteds is practically indistinguishable from the curve shown.

The effect of the central adjusting windings on the acceleration process of the ions was studied. Using these windings, it was possible to study the effects of increase or decrease in the mean field value, at small radii. In all cases when the increase of the field created by the windings in the centre of the chamber did not exceed $+0{\cdot}5$ per cent of H_0, the increase of current on the probe was insignificant. For greater

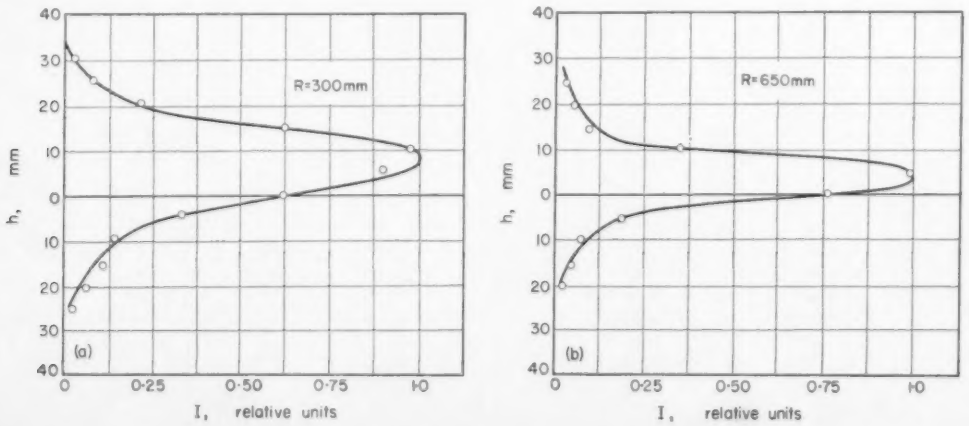

FIG. 9.—Vertical distribution of intensity in the ion beam.

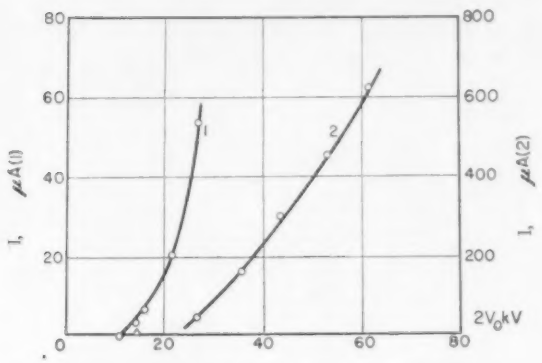

FIG. 10.—Dependence of the probe current on the dee-potential (2); initial portion of the curve on an enlarged scale (1).

variations of the field in the centre (of both signs), the probe current generally decreased.

For verification of the value in the energy of the accelerated ions, computed from the mean radius of orbit, the experiments were set according to determination of the range of the ions in aluminium. The measuring electrode of the probe was covered with a sheet of thickness 190 mg/cm². The absorption curve obtained under these conditions is shown in Fig. 11. The value obtained for the average energy amounted to 21·5 MeV. The calculations agreed with those obtained by experiment to an accuracy of 3 per cent. The energy spread of the ions, determined from the curve, was equal to about ±1·5 per cent. It should be noted that for normal cyclotrons this value is about ±3 per cent.

The motion of the ions at large radii was studied, and Fig. 12 shows the radial dependence of the accelerated ion current.

Figure 13 shows a print of the beam at a radius of 00 mm, obtained by superposition of an exposed copper plate on a photo-emulsion. Attention is drawn to the very small height of this beam.

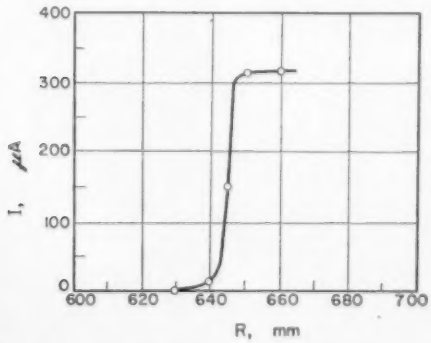

FIG. 11.—Absorption curve obtained with an aluminium filter.

Figure 14 shows the curve obtained by a photometric of the beam print. The steep decrease in intensity on the right hand (Figs. 13 and 14) is caused by the fact that at this point the ions began to impinge on the front section of the dee. The large width of the beam (more than 15 mm) is explained by the loss of radial stability due to a steep decrease in magnetic field.

This circumstance permits extraction of the beam by means of an electrostatic deflexion system using electric field intensities considerably less than in normal cyclotrons. Consequently, from the acceleration chamber of a cyclotron with azimuthal variation of the magnetic field, ions may be extracted with energy considerably in excess of that obtained in normal cyclotrons (22–24 MeV).

CONCLUSIONS

Experiments on the study of the motion of ions in a 1·5–metre cyclotron with azimuthal variation of magnetic field, have given the following results,

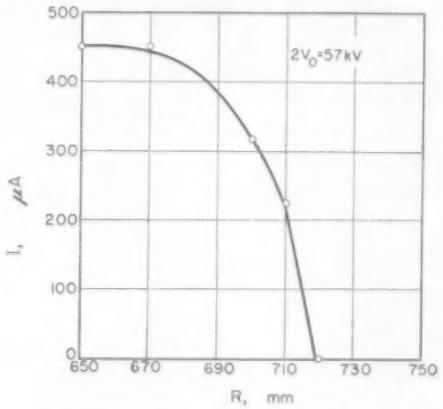

FIG. 12.—Dependence of current on probe radius.

extremely valuable for the planning of similar accelerators:

1. For a potential difference between the dees equal to 80 kV, molecular hydrogen ions and deuterons were successfully accelerated up to energies of about 21 MeV with an ion current at the final radius of about 1000 μA. At a maximum energy of 23·7 MeV, the current of the accelerated particles amounted to about 200 μA.

2. It was shown that creation of the shape of the magnetic field agreed completely with the shape of the field obtained on a one-fifth scale model. This is further related to the iron shims, incorporating fine adjustment bolts with disk-shaped heads.

3. Current-adjusting elements in the form of windings were introduced into the vacuum space and operated reliably during the time of all experiments.

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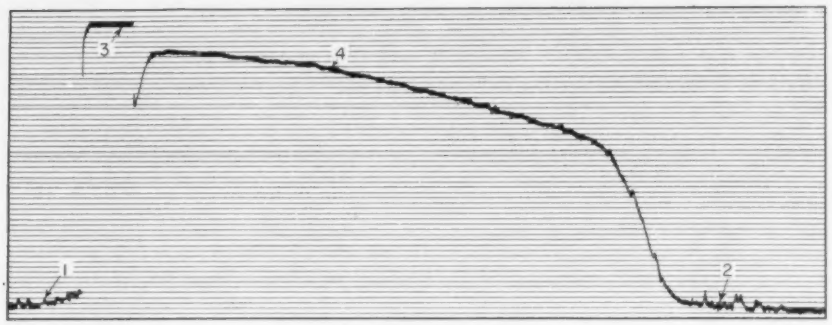

FIG. 14.—Results of a photometric of the beam print. 1, 2—'Natural background' of the plate; 3—Datum mark, obtained at the closure of the photo-element; 4—Curve of blackening.

The magnetic fields generated by these windings were studied.

4. As a result of the investigation of the variation of ion current at large radii of acceleration, valuable data were obtained for achieving extraction of the beam. Arising from this data, it is possible to confirm that by means of an electrostatic system, ions can be extracted with an energy considerably greater than the energy of the ions obtained in normal cyclotrons.

5. Experiments on the determination of the energy of the ions in the final radius of acceleration by means of aluminium filters, showed that the energy spread of the ions in the beam amounted to about $\pm 1{\cdot}5$ per cent.

Acknowledgments—In conclusion, the authors wish to thank N. D. FEDEROV, A. P. BABICHEV, A. S. KNYAZYATOV and V. K. ANOKHIN for their participation in the magnetic measurements, S. I. PROKOF'EV for active assistance in the fabrication of the accelerating chamber covers, N. N. KHALDIN for valuable advice and participation in constructional development, N. I. VENIKOV and all personnel servicing the cyclotron installation, I. M. SHNAPTSEV and A. G. YADYKIN for conducting the vacuum tests, M. A. EGOROV, V. M. KOMAROV, V. I. ANDREEV and V. S. KALYAYEV for the excellent performance of the mechanical assembly.

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ION CYCLOTRON RESONANCE IN DENSE PLASMAS*

L. V. Dubovoi, O. M. Shvets and S. S. Ovchinnikov

Abstract—The feasibility of heating plasmas by using the mechanism of ion cyclotron resonance has been investigated. It is shown, that in plasmas with a density of charged particles of 10^7–10^{11} cm^{-3} the effect of transverse ionic polarization fields can be reduced by the use of heating units which are short relative to the length of the plasma column, on account of the motion of electrons from the discharge along the lines of force of the external magnetic field. In a plasma with a low degree of ionization, a strong reduction is observed in the efficiency of transfer of energy from the high-frequency field to the ions with increase of their velocities. This is associated with cooling of the ions by neutral atoms.

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INTRODUCTION

In this paper, the results of experiments are given on the feasibility of heating plasmas by means of rapidly alternating electromagnetic fields. The mechanism of heating has been studied by using ion cyclotron resonance, which is of particular interest owing to the possibility of direct transfer of energy from high-frequency fields. Although the method of achieving this is apparently simple, some difficulties are encountered and the practical value of the chosen method depends largely on their being successfully overcome. Thus, by studying the properties of a hydrogen plasma in the region of ion cyclotron resonance,[1] the feasibility is confirmed of heating the ion component in the discharge to a temperature in the region of several kilovolts for a density of charged particles of 10^8–10^9 cm^{-3}. Further increase of density sharply reduces the efficiency of heating, because of the appearance of intrinsic plasma fields associated with the ion currents. The results of these experiments are in accordance with calculations in a paper,[2] according to which the criterion for the appearance of polarization effects in the region of cyclotron resonance is the disturbance of the inequality $\omega_0^2/\omega^2 < 1$, where ω_0 is the Langmuir frequency of the ions and ω is the frequency of the heating generator.

The discharge tube which specifies the plasma geometry,[1] was installed in a space-uniform high-frequency electric field (model of an infinite plasma column in an oscillating field, uniform in an axial direction) in accordance with the fundamental theoretical assumptions.[2] In systems of this type, for an intensity of the magnetic field of 10^3–10^4 gauss, heating of plasma with a density greater than 10^9 cm^{-3} becomes less efficient, since the only ions which interact are those in the surface layer of the plasma column, due to the strong skin effect associated with the external high-frequency field.

One of the possibilities for reducing the effect of intrinsic plasma fields is mentioned in a paper.[3] The method consists in the selection of a space distribution of the high-frequency heating field, such that the space charges giving rise to transverse ion currents are compensated by the motion of electrons along the lines of force of the external magnetic field. The simplest system of this type is an infinitely long cylindrical plasma column in a longitudinal magnetic field, with heating units in the form of short solenoids or condensers.[4] Ionic polarization fields arising within the limited volume of the heating units are neutralized by the longitudinal drift of the electrons.

In accordance with the theory,[3] the criterion $\omega_0^2/\omega^2 < 1$, characterizing the range for which polarization phenomena for a plasma in a uniform high-frequency field are possible, heating in the case of a non-uniform field converts it into $\omega_0^2/k^2c^2 < 1$, where $\mathbf{k}$ is a wave vector corresponding to a wavelength equal to twice the size of the heating unit in a longitudinal direction; c is the velocity of light *in vacuo*. Calculations show, that for a unit length of 10 cm, a skin effect will be observed only for densities greater than 10^{14} cm^{-3}. This shows the advantage of this type of system.

In the paper, the results of studies of various systems for the introduction of high-frequency energy into a plasma are given, in which this principle is applied.

A power pack of several hundred milliwatts was used as the high-frequency energy source. The dependence of the efficiency of absorption of high frequency energy in the region of ion cyclotron resonance for atomic hydrogen, on both the parameters of the plasma and the configuration of the electrodes of the high-frequency units was studied. The results obtained are in agreement with theory.[3,5]

* Translated by W. E. Jones from *Atomnaya Energiya* **8**, 316 (1959).

DESCRIPTION OF EQUIPMENT

The plasma was produced by an electrical discharge in which the electrons oscillated. The advantages of a construction of this type are a relatively high degree of ionization, simplicity of the ancillary equipment and an extensive range of operating pressures.

In contrast to the usual designs,[6] a version with two anode circuits was used, spaced along the axis of discharge.[7] As a result of this the space between the anodes is free from the electric fields maintaining the discharge, and the fields from direct currents flowing in the plasma. Such plasma parameters as the shape of the discharge column, intensity of the emitted light, and the discharge current were kept constant during the course of the experiments for a range of the magnetic field of 700–1900 gauss. This ensured that the plasma density was constant with variation in the magnetic field.

By using aluminium electrodes, the density of the discharge could be varied from 10^7 to 10^{12} cm^{-3} for a change of potential on the anodes from 0·5 to 3·5 kV. In all experiments the discharge current did not exceed 100–150 mA.

The discharge tube, of molybdenum glass, or quartz was located in a longitudinal magnetic field (an external view of the assembly is shown in Fig. 1) with a uniformity in the region of the high-frequency electrodes of not less than 1 per cent. The high-frequency energy supply system was an assembly of solenoids or condensers. Each element was switched in anti-phase with its neighbours. The number of units could be varied from one to four. Figure 2 shows a specimen of the tube with the sectionalized solenoids.

The lowest pressure in the assembly, using a liquid nitrogen trap was 1×10^{-5} mm Hg. No special measures were taken for preliminary outgassing of the tube. The pressure of hydrogen in the system was adjusted by varying the temperature of a palladium leak.

A diagram of the measuring circuit is shown in Fig. 3. A signal generator of the type GSS–6 or GS–27 was connected to a resonance loop, which enabled energy to be fed into the plasma by capacitative or inductive coupling. Measurements were carried out by a series resonance method. The signal for detection, proportional to the amplitude of the voltage on the loop, was fed to the input of the d.c. amplifier of the *Y*-plate of an oscillograph ENO-1; a signal, proportional to the intensity of the magnetic field was fed onto the *X*-plates. The method enabled a continuous trace to be produced of the shape of the high-frequency resonance absorption for the plasma, for the frequency range of the generator of 1·1–2·7 Mc/s.

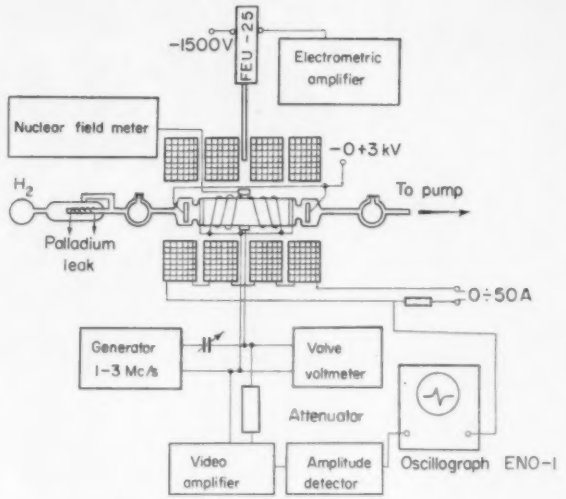

FIG. 3.—Diagram of measuring circuit.

The magnetic field was calibrated by the nuclear resonance method with an accuracy of 0·5 per cent.

The power consumed by the plasma was determined by means of calibration curves for the generator working on operating load.[4]

Figure 4 shows oscillograms of the resonance curves obtained by means of a sweep-generator, for no-load and active load, equivalent to transfer of power from the generator with 30 per cent efficiency. The general *Q*-value of the no-load curves was equal to 150–200. The symmetry of the resonance curves and freedom from parasitic resonances in the measuring circuit facilitate exclusion of errors associated with the reactivity introduced by the plasma in the vicinity of gyroresonance.

The concentration of charged particles in the discharge for a density of $n < 10^{11}$ cm^{-3} was determined by the method of dual high-frequency probes,[8] and for $n \simeq 10^{12}$ cm^{-3} by measurement of the transmission factor of a high-frequency signal with a wavelength of 3 cm.[7] The relationship between the plasma density and the discharge current was shown to be approximately linear. Extrapolation with respect to discharge current for a maximum value of the density in our configuration gives the value of $3 \cdot 10^{12}$ cm^{-3}.

The radial distribution of density in the discharge was estimated by the half-width of the curves for Langmuir resonance of the electrons, obtained by means of the high-frequency probe method for density measurements. The radial non-uniformity of the density obtained by this method did not exceed 15–20 per cent, which was in accordance with the quantitative results obtained by means of photometric evaluation of the visible light emitted by the discharge

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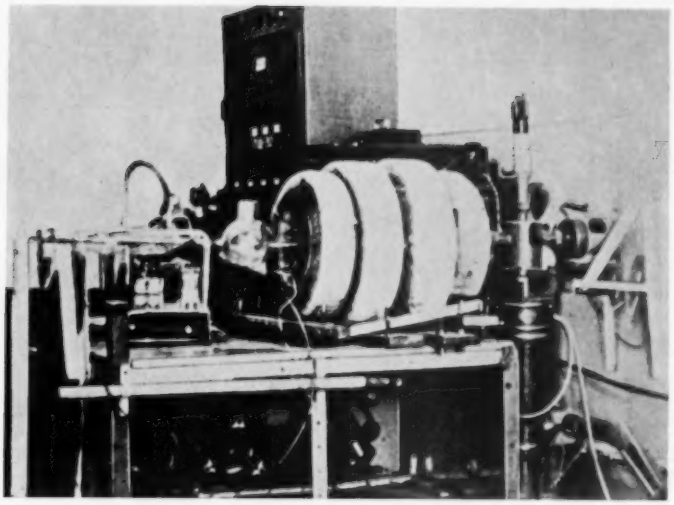

FIG. 1.—External view of assembly.

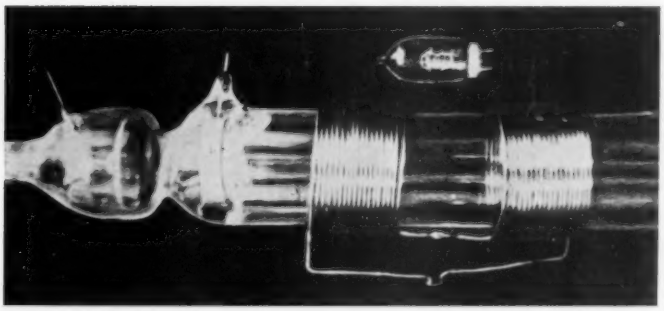

FIG. 2.—Discharge bulb with sectionalized solenoid.

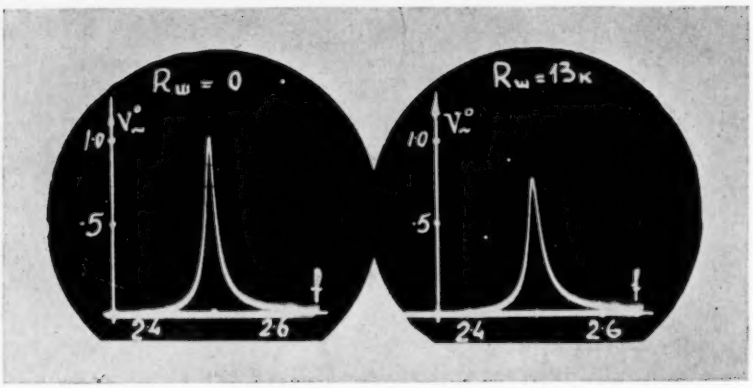

FIG. 4.—Calibration curves of the resonance circuit.

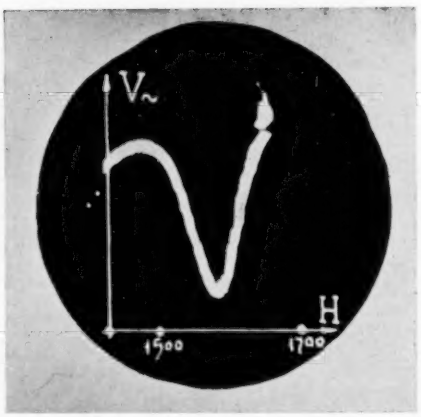

FIG. 5.—Oscillogram of the amplitude of potential on the loop from the plasma, as a function of the magnetic field.

in the range of wavelengths 4000–6000 Å. Optical measurements also show the presence of a sharp boundary between the plasma and the vacuum.

The method used by us to determine the density, even for orientations of the electrical component of the probing signal parallel to the magnetic field, is applicable, strictly speaking, only for the case of negligibly small intensities of the magnetic field, taking into account the actual electron temperature in the discharge. Consequently, the measurements should always be carried out in fields for which the inequality $\omega_{ce}^2/\omega_1^2 \ll 1$ is fulfilled, where ω_{ce} is the electron cyclotron frequency; ω_1 is the frequency of the generator. For frequencies less than 3000 Mc/s, corresponding to $n \leqslant 10^{11}$ cm^{-3}, the minimum magnetic field associated with a specific discharge does not allow the above inequality to be satisfied. Consequently, for control measurements, a method was used for determining the density of the electrons by the relationship between the dielectric constant of the plasma (obtained by the high-frequency probe method) and the magnetic field in the region of electron cyclotron resonance. The values of the density were found by comparing the measured relationships with the calculated ones given in the paper.[7] The data obtained by this method showed there to be satisfactory agreement with the results of fundamental density measurements.

EXPERIMENTAL RESULTS

Figure 5 represents a typical oscillogram of the relationship between the amplitude of the potential on the loop from the plasma, and the intensity of the magnetic field. The vertical deflexion of the beam is proportional to the Q-factor of the loop, the horizontal trace to the magnetic field. A decrease of potential is observed for values of the field corresponding to equality of the ion gyrofrequency and the generator frequency, associated with absorption of high-frequency energy by the plasma. The well marked resonance character of the absorption can be clearly seen.

The data from the oscillograms was subsequently converted into absolute values of the plasma loading on the generator. Using simple procedures, which followed directly from calculations,[5] it was shown that under our conditions an efficiency of energy transmission from the generator to the plasma of 30–40 per cent could be obtained relatively simply. Since the behaviour of the width of the resonance curve, directly characterizing the time of interaction of the accelerated particles with the high-frequency fields plays the principal role in proving the feasibility

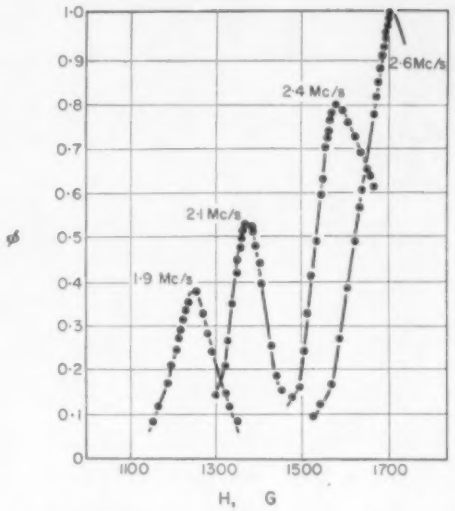

FIG. 6.—Absorption resonance curves of a high-frequency plasma for various generator frequencies. ($p = 6 \times 10^{-3}$ mm Hg; $n \simeq 10^{10}$ cm^{-3}).

of heating the ions by the method of cyclotron resonance, no attempt was made to increase further the matching of the generator with the plasma.

The variation of the range of resonance absorption as a function of the various plasma parameters is discussed below. Curves are shown in Fig. 6 for the normalized value of the high-frequency power Φ, absorbed at the instant of resonance, as a function of the magnetic field for different generator frequencies. The measurements give an accurate equality for the frequency of the generator and the gyrofrequency of the atomic hydrogen ions ω_{ci} at the point corresponding to maximum absorption, within the range of plasma densities of 10^7–10^{10} cm^{-3}. The relationship between the amount of absorption at the instant of resonance and the generator frequency is approximately linear.

In Fig. 7, the variable parameter is the gas pressure. The relationship obtained for the half-width of the resonance curves $\Delta\omega_{ci}$ for pressures greater than 3×10^{-3} mm Hg is approximately linear and can be compared with the collision frequency of the atomic ions with neutral atoms[9] by the relationship

$$\nu_c = \frac{1}{\tau_c} = 1{\cdot}3 \times 10^8 p,$$

where p is the pressure (mm Hg); τ_c is the time between two collisions.

The effect of density on the magnitude of absorption of high-frequency power is shown in Fig. 8. The measured relationship between the absorption at the instant of resonance and n is approximately linear

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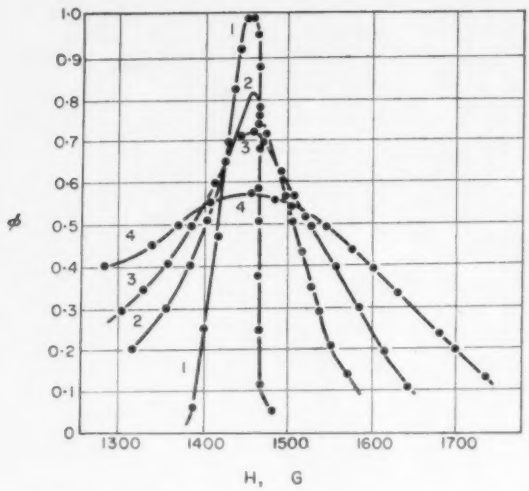

FIG. 7.—Absorption resonance curves for various pressures in mm Hg.
($f = 2{\cdot}2$ Mc/s), mm Hg.
(1) $p = 9 \times 10^{-4}$;
(2) $p = 5 \times 10^{-3}$; (3) $p = 9 \times 10^{-3}$;
(4) $p = 1{\cdot}5 \times 10^{-2}$.

within the range of densities 10^7–10^{11} cm^{-3}. With further increase in density, saturation is observed for $n \simeq 3 \times 10^{11}$ cm^{-3} followed by a decrease (Fig. 9) for $n > 6 \times 10^{11}$ cm^{-3}. At the same time, with the appearance of a non-linear relationship between the absorption maximum and the density, the primary resonance peak is divided into two, in which the absorption maxima are monotonically displaced with increase of n, in opposite directions from the value of the magnetic field corresponding to resonance for a given generator frequency.

By decreasing the length of the space period of the high-frequency energy input system by a factor of two, the magnitude of the displacement of the absorption maximum is reduced by a factor of two or three for the same values of density.

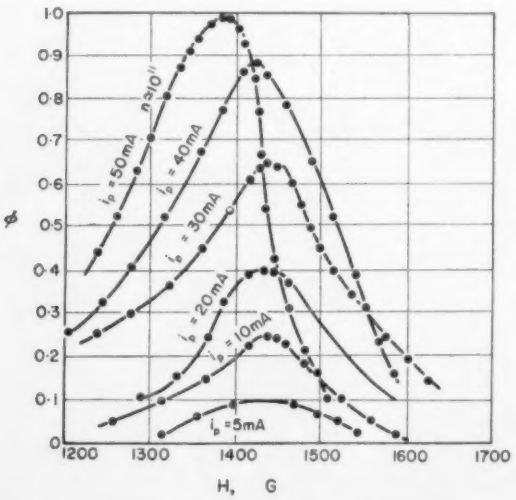

FIG. 8.—Resonance curves for various plasma densities.
($f = 2{\cdot}2$ Mc/s; $p = 6 \times 10^{-3}$ mm Hg).

DISCUSSION OF RESULTS

The absence of constant electric fields and of magnetic fields from direct currents in the region of the heating units, and the relatively uniform radial distribution of density of the plasma provide a sufficiently close approximation to the principal theoretical concepts[3,5] for the type of discharge used. In accordance with this theory, providing that the inequality $\omega_0^2/k^2c^2 < 1$ is satisfied, polarization phenomena in the plasma are absent. The absorption in the region of cyclotron resonance is proportional to the product of the plasma density and the generator frequency. The absorption maximum corresponds to exact equality of ω and ω_{ci}.

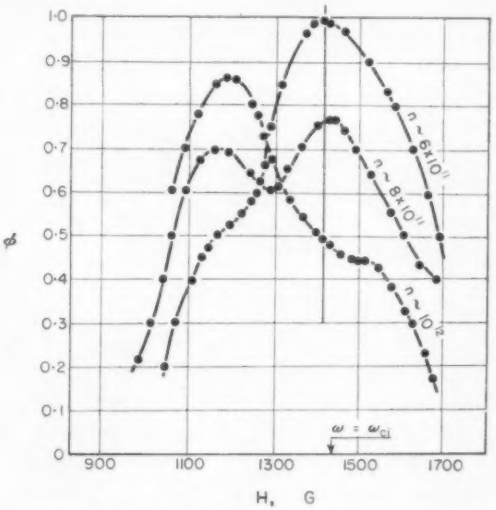

FIG. 9.—Absorption curves in the region of high densities.
($f = 2{\cdot}12$ Mc/s; $p = 10^{-3}$ mm Hg).

For greater densities (more accurately for $\omega_0^2/k^2c^2 \gg 1$), intrinsic fields originate in the plasma, a typical consequence of which is the displacement of the range of effective penetration of the high-frequency fields from the value of the magnetic field corresponding to equality of ω and ω_{ci}, in the region of $\omega > \omega_{ci}$[2,10] or $\omega < \omega_{ci}$.[3,5] If the displacement, equal to $\omega\omega_0^2/k^2c^2$, is somewhat greater, so that it exceeds the value of the half-width ΔH of the cyclotron resonance curve, heating of the ions ceases and the absorbed energy goes into the maintenance of a different form of weakly-damped hydromagnetic oscillations in the plasma column.[2,3,5]

For the system used to obtain the curves shown in

Figs. 8 and 9 ($k = \frac{1}{16}$ cm^{-1}), the value for the critical density above which the efficiency of resonance heating commences to drop, is equal to $\sim 5 \times 10^{11}$ cm^{-3} and the linear increase of high-frequency power absorption observed in the experiments for an increase of density n from 10^7 to 10^{11} cm^{-3}, corresponds to that anticipated. The measured value of the optimum density (5×10^{11} cm^{-3}) is also in agreement with the calculated value.

Associated with the fact that in polarization phenomena change of density is equivalent to a change of the square of the length of an elementary cell of the heating units, it is understandable that the relationship between the magnitude of the displacement of the absorption maximum and k agrees qualitatively with theory.

Unfortunately, quantitative comparison of the results of measurements for different values of the quantity k, is hampered by the appearance of Coulomb interaction for a density of charged particles greater than 10^{11}–10^{12} cm^{-3}, which can only be eliminated in heated plasmas where the ion energy is not lower than a few electron volts. For experiments with small values of k, it is necessary to have large volumes with a strong magnetic field.

Data for sharply differing values of k can be obtained from a comparison of the results of the experiments carried out with the results in the paper,[1] in which the case was investigated equivalent to $k = 0$, and the effects accompanying polarization of the plasma (destroying of the linear increase of high-frequency power absorption with increase of density, the appearance of the dependence of the half-width of the resonance curves and the position of the absorption maximum on the density) were observed when the inequality $\omega_0^2/\omega^2 < 1$ was not valid. At the same time, by looking for skin effects in the present work with $\omega_0^2/\omega^2 \simeq 1$ ($\omega_0^2/k^2c^2 \simeq 10^{-8}$), no marked deviations from those expected in accordance with the paper[3] were discovered, and the polarization effects described appeared as a result of the inequality $\omega_0^2/k^2c^2 < 1$ ceasing to be valid (for $\omega_0^2/\omega^2 \simeq 10^4$). Hence, despite the absence of direct proof of the penetration of high-frequency fields into the plasma in the systems investigated, the possibility of setting up designs with reduced skin effect is favoured both by the satisfactory agreement of the measured and calculated values of the density at which the effects accompanying plasma polarization appear, and also by the fact that the absorption maximum is displaced with increasing n and not broadened as would necessarily occur if there were heating only of the boundary layer which has a continuous density variation.*

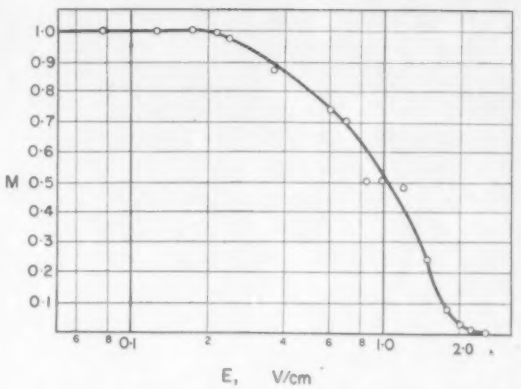

FIG. 10.—Effect of intensity of the electric field in the time of interaction of an ion with the high-frequency field.

The equation assumed for the case of total compensation of the field outside and inside the plasma, permits an estimation to be made of the degree of heating of the ions. In accordance with the paper,[11] as a result of the motion of a charged particle in crossed fields with a constant magnetic field H and an alternating electric field $E = E_0 \sin(\omega t)$, for exact resonance, an ion will traverse an orbit with a radius $R(t) = E_0/2H$ in a time t sec after commencement of acceleration. The value of t, equal to τ (the time of interaction of an ion with the high-frequency field between two successive collisions) can be found for a given experiment from the relationship $\Delta\omega\tau_c = 1$.[9] The product $HR(t)$, measured in this manner, can serve as a measure of the energy acquired by an ion in the heating process. For an intensity of the electric field of 0·2 V/cm, typical for the experiments carried out, the gain in energy of the ions by interaction with the probe signal ($\sim 10^{-2}$ eV) is small in comparison with the thermal energies of the particles in the discharge, and disturbance of the plasma in the process of measurement is immaterial.

For the purpose of investigating the possibility of increasing the ion energy, the relationship was obtained between the lifetime of an ion M (relative to the lifetime corresponding to a resonance width of 5 per cent) and the intensity of the electric field E, established by a parallel plane condenser (Fig. 10).

Estimation of the increase in velocity $v_\perp$, acquired by an ion in the heating process, relative to the thermal velocity v_0 corresponding to a temperature of ~ 300°K and representative for the majority of normal discharges, shows that at the instant of breakdown of the

* An attempt to measure directly the radial distribution of intensity of the high-frequency heating field by means of probes was unsuccessful because of the extremely strong distortion of the configuration and of the high-frequency properties of the discharge, accompanied by the appearance of a large amount of contamination from sputtering of the probes.

curve $M(E)$, $v_\perp \simeq v_0$ for an intensity of $E \simeq 0{\cdot}3$ V/cm. In accordance with the paper,[12] the value of τ_e is inversely proportional to $\sqrt{E}$ for $v_\perp > v_0$. However, the steepness of fall of $M(E)$ in the region of $E \simeq 1$ V/cm cannot possibly be explained by this only. The stronger dependence of $M(E)$ for $v_\perp > v_0$ can be explained by a process analogous to that suggested in the paper[13] and which is associated with the tendency of the plasma towards smoothing out the anisotropy of the type $v_\perp > v_0$ which originates in it during the heating process. The resulting increase in magnitude of the longitudinal component of the velocity $v_\parallel \simeq v_0$ with respect to the value $v_\parallel \simeq v_\perp > v_0$ associated with this, leads to the necessity for calculation of the complementary reduction of $\tau_e(E)$ on account of the decrease in the time $\tau_\parallel$ which an ion spent in the region of the heating units: $\tau_\parallel \simeq 1/kv_\parallel$.

Calculation of the relationship $\tau_e \sim 1/\sqrt{E}$ for $v_\perp > v_0$ with the assumption that $v_\parallel \simeq v_\perp$, leads to the possibility of a quantitative description of the observed dependence of $M(E)$. At the same time, the dependence of the time between two collisions $\tau_e \simeq 1/\sqrt{E}$ associated with a strong energy exchange between ions and neutral atoms, is sufficiently well-studied in theory by the mobility of the ions.[12] As a consequence of this, at insufficiently high degrees of ionization of the plasma, the neutral atoms not confined by the magnetic field will transfer a large quantity of energy to the walls of the chamber.

CONCLUSION

The results of experiments using space-periodic systems of high-frequency energy injection in the region of ion cyclotron resonance for cold plasmas with a density range of 10^7–10^{11} cm^{-3}, are found to be in satisfactory agreement with theory, predicting the possibility of reducing the effect of intrinsic polarization fields by the penetration of alternating fields for the case of heated plasmas which are totally ionized at a density of up to 10^{13}–10^{14} cm^{-3}.

The experiments show, that for successful heating of the plasma by the method of ion cyclotron resonance, it is necessary to attain the minimal concentration of neutral atoms.

The relationship between the behaviour of the plasma in the region of gyro-resonance of the ions and both the density of the particles and their degree of heating, can be employed for the development of methods of investigating the properties of gaseous discharges.

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HIGH FREQUENCY BEAM STACKING IN A CYCLIC ACCELERATOR*†

A. N. LEBEDEV

Abstract—When working out the theory of high frequency beam stacking in accelerators operating with a constant magnetic field it is important to take into account the perturbations of the beam which has already been stacked caused by later acceleration cycles. The main effect of these perturbations is to increase the energy spread of the beam and to alter its mean energy. This paper presents a general formulation of this problem and a solution is given for certain particular cases.

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1. EQUATIONS OF MOTION

IT is convenient to consider the change in energy of the particles in terms of the dimensionless variables

$$z = \frac{2\pi Q}{eV}\int_{E_0}^{E}\frac{dE}{\Omega(E)};\quad \Phi;\quad Q = \left|\frac{q\,.\,\Omega_0\Omega_0' eV}{2\pi}\right|^{1/2} \tag{1.1}$$

where $\Omega(E)$ is the frequency of revolution of particles having energy E; $\Omega' = \frac{d\Omega}{dE}$; q is the harmonic number and V is the peak accelerating voltage. The suffix 0 is used to denote values corresponding to the stacked energy. φ is the phase in which the particle finds itself in the accelerating field and $\omega(t)$ is the frequency of the field:

$$\dot{\varphi} = q\Omega - \omega(t). \tag{1.2}$$

We will assume that the energy of a particle in the beam which is being stacked is only a little different from the stacked energy E_0, i.e. we can linearize the expression for Ω

$$\Omega(E) = \Omega_0 + \frac{\Omega_0' eV\Omega_0}{2\pi Q}z. \tag{1.3}$$

We shall see shortly that for the vast majority of particles this assumption is justified. Let us further assume that near the stacked energy the frequency of the accelerating field varies according to a linear law, increasing or decreasing with time.

With these assumptions the equations of motion have the following simple form.[1]

$$\left.\begin{aligned} \dot{z} = \cos\varphi = -\frac{\partial H}{\partial\varphi};\quad H = -\sin\varphi + \\ + \frac{\Omega_0'}{|\Omega_0'|}\frac{z^2}{2} - \tau z\cos\varphi_s; \\ \dot{\varphi} = \frac{\Omega_0'}{|\Omega_0'|}z - \tau\cos\varphi_0 = \frac{\partial H}{\partial z}; \\ \cos\varphi_s = \frac{d\omega}{dt}\left(\frac{2\pi}{q\Omega_0\Omega_0' eV}\right). \end{aligned}\right\} \tag{1.4}$$

Here and in what follows we have used as the independent variable the dimensionless time

$$\tau = Qt \tag{1.5}$$

and we have chosen a time scale such that $\omega(0) = q\Omega_0$.

It can be seen from the expressions (1.4) that the variables z and φ are canonically conjugate so we can immediately make some general conclusions about the behaviour of the beam from Liouville's theorem.[1,2] In fact, so long as the frequency is a linear function of time, a closed (for $|\cos\varphi_s| < 1$) phase trajectory region exists on the phase cylinder (z, φ) which moves with a velocity equal to $\cos\varphi_s$. We will call the area of this stable region S and suppose that it is filled to a mean density ρ with the particles being accelerated. At the end of each stacking cycle the frequency rapidly rises to the original value, so that the particles are left with energies close to the stacked energy. According to Liouville's theorem the average density in phase space of the beam being stacked cannot exceed ρ and consequently we can estimate its energy spread:

$$\delta_n Z \geq \frac{Sn}{2\pi} \tag{1.6}$$

where n is the number of stacking cycles. The inequality in (1.6) becomes an equality only in the event that the whole phase space in region of the stacked energy is filled with particles to a density of ρ, or, as we say, in the event of 'density packing' of the separatrices. We shall be considering the deviations from equality in the following sections.

Later we shall require to use the phase trajectory equation. From equation (1.4) it follows that in terms of the canonical variables $y = \frac{\Omega_0}{|\Omega_0|}z - \tau\cos\varphi_s$ and φ this equation has the form:

$$y^2 - 2(\sin\varphi - \varphi\cos\varphi_s) = \text{const}. \tag{1.7}$$

2. VARIATION OF PARTICLE ENERGY DURING A SINGLE STACKING CYCLE

After one stacking cycle all the particles suffer a displacement in the z-direction the magnitude of which

* Work done in 1958.

† Translated by D. L. ALLAN from *Atomnaya Energiya*, **9**, 189 (1960).

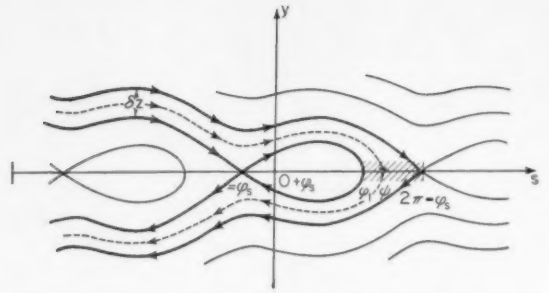

FIG. 1.—Phase trajectories of particles in a stacked beam.

depends on the final co-ordinate z_f and on the phase ψ with which the particle must have passed the separatrix (with $y = 0$). At the point ψ the condition $q\Omega(z) = \omega(\tau)$ is fulfilled, after which the sign is reversed so that $\dot{\varphi}(\psi) = 0$.* It is easy to see that ψ cannot assume any arbitrary value in the interval $(0 - 2\pi)$. This is demonstrated graphically in Fig. 1 which gives a schematic representation of the ascending separatrix and the phase trajectories in the unstable region. The region of values which ψ can assume is indicated in the figure by the shaded area: it is bounded on one side by $2\pi - \varphi_s$ and on the other by the point ψ_1, which is determined from the equation

$$\sin \varphi_1 - \varphi_1 \cos \varphi_s = -\sin \varphi_s + \varphi_s \cos \varphi_s. \quad (2.1)$$

We will express the co-ordinate shift Δ as a function of the final phase φ_f and ψ:

$$\Delta = \int_{\varphi_i}^{\varphi_f} \frac{2^{-1/2} \cos \xi d\xi}{[\sin \xi - \sin \psi - (\xi - \psi) \cos \varphi_s]^{1/2}} + \\ + \begin{cases} 2\frac{1}{2} \displaystyle\int_{\varphi_f}^{\psi} \frac{\cos \xi d\xi}{[\sin \xi - \sin \psi - (\xi - \psi) \cos \varphi_s]^{1/2}}; \\ 0 \end{cases} \quad (2.2)$$

the upper or lower value being taken according to whether the particle reaches the phase ψ or not. Since initially the separatrix is far away from the stacked beam we can set the initial phase φ_i in the lower limit of the integral in equation (2.2) equal to $-\infty$ (when $\cos \varphi_s < 0$, $\varphi_i = +\infty$).

If the separatrix does not vanish when the stacked energy is reached but passes near the beam, the expression (2.2) becomes

$$\Delta = \Delta_\infty = \sqrt{2} \int_{\mp\infty}^{\varphi} \frac{\cos \xi d\xi}{[\sin \xi - \sin \psi - (\xi - \psi) \cos \varphi_s]^{1/2}} \\ (\cos \varphi_s < 0) \quad (2.3)$$

i.e. the quantity Δ becomes independent of z_f.

* Since the frequency may return to its original value at $\tau = 0$ not all the particles will pass through the phase ψ.

We will now find the probability that the phase trajectory passes by the separatrix with phase ψ.† For this purpose we will consider phase trajectories in the zone lying between the two boundary trajectories which pass through the singularities of the phase cylinder (these are distinguished in Fig. 1 by the thick lines). For large values of $|\varphi|$ (at the beginning of the stacking cycle) the vertical extent of this zone is

$$\delta_0 z = 2\pi \left| \frac{\cos \varphi}{z - \tau \cos \varphi_s} \right|. \quad (2.4)$$

Since the closed phase space region for large values of $|z - \tau \cos \varphi_s|$ has practically no effect on the phase trajectories in the neighbourhood of the stacked energy, we can consider the density of the trajectories constant over the interval $\delta_0 z$. Therefore the probability that the phase trajectories will pass through the interval between z and $z + dz$ is

$$F(z)dz = \left| \frac{dz}{\delta_0 z} \right| = \frac{dz}{2\pi} \left| \frac{z - \tau \cos \varphi_s}{\cos \varphi_s} \right|. \quad (2.5)$$

It will be noticed however that each point in the zone $\delta_0 z$ corresponds to its own value of ψ so that from relation (1.7) we obtain

$$|z - \tau \cos \varphi_s| \, dz = |\cos \varphi_s - \cos \psi| \, d\psi. \quad (2.6)$$

(We will note that for particles in the unstable region $\cos \varphi_s$ is always $\leqslant \cos \psi$ when $\cos \varphi_s \leqslant 0$). From the expressions (2.5) and (2.6) it follows that the probability that a phase trajectory strikes the point ψ is different from zero over the interval $(\varphi_1 - 2\pi - \varphi_s)$ and

$$F(\psi)\, d\psi = \frac{d\psi}{2\pi} \left| \frac{\cos \varphi_s - \cos \psi}{\cos \varphi_s} \right|. \quad (2.7)$$

It is not difficult to show that this probability is normalized to unity.

We shall shortly require an asymptotic expression for Δ for large values of $|\varphi_f|$. After some simple manipulation we can obtain from equation (2.2) for the case $\cos \varphi_s \leqslant 0$ the following two relations

$$\Delta \approx \mp \frac{1}{[z|\cos \varphi_s|]^{1/2}} \frac{\sin |\varphi_f|}{\sqrt{|\varphi_f|}} \approx 0; \quad (2.8a)$$

$$\Delta \approx \Delta_\infty(\psi) \pm \frac{1}{[z|\cos \varphi|]^{1/2}} \frac{\sin |\varphi_f|}{\sqrt{|\varphi_f|}} \approx \Delta_\infty(\psi). \quad (2.8b)$$

depending on whether the particle did or did not pass through the phase ψ.

† A similar analysis has been given also by VOGT-NILSEN.[2]

3. EQUATION FOR THE DISTRIBUTION FUNCTION WHEN THE NUMBER OF STACKING CYCLES IS LARGE

A general equation describing the variation of the distribution function is the difference equation connecting two successive cycles.

$$P(z, n) = \int_0^{2\pi} P(z - \Delta, n - 1)F\, d\psi - \alpha P(z, n - 1) + \Phi(z) \qquad (3.1)$$

In formula (3.1) the probability $F(\psi)$ is given by expression (2.7) and $\Phi(z)$ describes the emergence of new particles from the ascending separatrix. If all cycles are strictly identical, $\Phi(z)$ is independent of n. The quantity Δ must be expressed as a function of z and ψ (hereafter we will omit the index). In the general case of an arbitrary z this can, in principle, be done if we express φ in terms of z and φ using equation (1.7) and substitute the value obtained into equation (2.2). In some cases one can simply use the asymptotic expressions (2.8). Such an approach is justified by the fact that in practice we require only that the spread does not exceed a certain value which is much greater than unity.* In this connexion we would observe that the behaviour of the distribution function at small values of $|z|$ is of no particular interest and it is sufficient to know how function $P(z)$ behaves for values $|z| \gg 1$.

The quantity α in equation (3.1) describes the probability that the beam is lost during the course of an acceleration cycle due to scattering by the residual gas and other processes:

$$\alpha = \frac{T}{\tau_0} \ll 1 \qquad (3.2)$$

where τ_0 is the average lifetime of the beam and T is the duration of a stacking cycle. There is a number of reasons why the usual method of reducing expressions (3.1) to a Fokker-Planck differential equation is not desirable in the present case. We shall see later, however, that the main features of the behaviour of $P(z)$ can be deduced immediately from equation (3.1).

4. PERTURBATION OF THE BEAM BY THE PENETRATING SEPARATRICES

We will consider first the behaviour of the stacked beam in the case where the separatrices pass through the beam and go off to infinity. Physically it is obvious that the mean energy of the beam must at the same time fall off at the rate $S/2\pi T$. In fact, each penetrating separatrix reduces the area of the phase cylinder under the beam by the amount S and this area is equal to the mean beam energy multiplied by 2π. In the case when $\cos \varphi_s < 0$, i.e. in the case of separatrices which pass through the beam in the downward direction, the same argument shows that the mean beam energy must be increased.

The problem we have set ourselves can be solved using equation (3.1) where, obviously, we must take $\Phi(z) = 0$ and $\Delta = \Delta_\infty$. The fact that Δ_∞ is independent of z greatly simplifies the problem and we are able to solve it completely.

Applying a Fourier transformation to equation (3.1) we obtain a solution in the form

$$P(z, n) = \int_{-\infty}^{+\infty} \exp\{i\lambda z - n \ln[1 + f(\lambda)]\} p(\lambda)\, d\lambda, \qquad (4.1)$$

where

$$p(\lambda) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} P(z, 0) e^{i\lambda z}\, dz \qquad (4.2)$$

and

$$f(\lambda) = \int_0^{2\pi} (1 - e^{i\lambda\Delta_\infty}) F\, d\psi. \qquad (4.3)$$

Some very tedious calculation is necessary before one can obtain numerical results from equation (4). One can, however, find out something about the asymptotic behaviour of the function $P(z, n)$ for large values of n. We note first that the value of $R(f)$ is always positive and is of the order of unity. Evaluating the integral (4.1) by the saddle method we obtain the asymptotic formula

$$P(z, n) \simeq \frac{N_0 e^{-n\alpha}}{\sqrt{2\pi n(\overline{\Delta_\infty^2} - \bar{\Delta}_\infty^2)}} \times \exp\left[-\frac{(z - n\bar{\Delta}_\infty)^2}{2n(\overline{\Delta_\infty^2} - \bar{\Delta}_\infty^2)}\right]. \qquad (4.4)$$

where N_0 is the initial number of particles.

Thus for large n the centre of the beam is displaced with a velocity $\bar{\Delta}_\infty/T$ independently of the initial distribution. At the same time the distribution tends to a gaussian distribution and its dispersion increases according to the relation:

$$D^2 = (\overline{\Delta_\infty^2} - \bar{\Delta}_\infty^2)n. \qquad (4.5)$$

Using equations (2.2) and 2.7) by direct integration one can satisfy oneself that when $\cos \varphi_s \gtrless 0$, $\bar{\Delta}_\infty = \mp \frac{S}{2\pi}$. Thus, the estimate based on Liouville's theorem which we gave in Section 1 applies completely to the motion of the centre of the beam. At the same time

* We would point out that the spread $\delta z \approx 1$ roughly corresponds to the amplitude of the phase oscillations.

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the penetrating separatrices 'wash out' the beam in accordance with equation (4.5).

The results we have obtained enable us to calculate the perturbations of the stacked beam caused by parasitic separatrices sweeping through the beam in the case of the multi-cascade system of stacking, and by downward moving separatrices in the case of an accelerator which passes through a critical energy.

5. THE ENERGY SPREAD OF THE STACKED BEAM

We will now consider the behaviour of a beam which is approached by particle-filled separatrices from below. We shall suppose that when the energy E_0 is reached the separatrix vanishes and the particles inside it are injected into the stacked beam. After this the cycle is repeated.

We will consider only a beam in the steady state; we shall not be concerned with the processes by which it was established. The initial equation will obviously be

$$P(z) = \int_0^{2\pi} P(z - \Delta) F \, d\psi - \alpha P + \Phi(z). \quad (5.1)$$

As we remarked in Section 3, our main interest is in the behaviour of $P(z)$ for large $|z|$. We can then put $\Phi(z) = 0$ in equation (5.1) since in accordance with its physical meaning $\Phi|z|$ has already fallen to zero when $|z| \approx 1$. Using the asymptotic formulae (2.8) for large positive values of z we obtain from (5.1):

$$P(z) \equiv 0. \quad (5.2)$$

The physical meaning of this result is immediately understood if we remember that the separatrix can only go up to the stacked energy and therefore phase trajectories with $z \gg 1$ are almost unperturbed and lie close to the curve $z = \text{const}$. This implies that for large values of z the energy of the particles is constant but, since the source function is centred in the region $|z| \leqslant 1$, the particles cannot in general reach large values of z. In some positions particles with large negative values of z will be found by which move past the separatrices. Using an argument similar to that given in Section 4 one can calculate that the particles will move downwards with a velocity $\bar{\Delta}_\infty/T$ and in the stationary state the lower boundary of the beam will have the value $\bar{\Delta}_\infty/\alpha$. This value can also be obtained from Liouville's theorem using the hypothesis of 'density packing' of the separatrices.

We will seek a solution for large negative values of z in the form

$$P(z) = \text{const.}\ e^{\lambda z}, \quad (5.3)$$

where $\lambda > 0$. Substituting expression (5.3) into (5.1) and taking into account that $\alpha \ll 1$ we find after some fairly simple manipulation that

$$\frac{1}{\lambda} = \frac{|\bar{\Delta}_\infty|}{\alpha}\left(1 + \frac{\alpha\overline{\Delta_\infty^2}}{2\bar{\Delta}_\infty^2}\right). \quad (5.4)$$

The first item in this expression agrees exactly with that calculated according to Liouville's theorem. The corrections to it are small and of the order α.

Since the quantity $1/\lambda \gg 1$, the majority of the particles are concentrated in the region where the asymptotic expression (5.3) is applicable. In fact, according to Liouville's theorem the function $P(z)$ is restricted from above so that there are few particles with small values of $|z|$ ($< |z|2\pi\rho$). Therefore one can use the asymptotic expressions without introducing any significant error:

$$P(z) = \begin{cases} 0; \\ \dfrac{s\rho\lambda}{\alpha} e^{\lambda z}, \end{cases} \quad (5.5)$$

where $z \lesssim 0$ through the region where z is varying.

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COHERENT RADIATION FROM THE ELECTRONS IN A SYNCHROTRON*

IU. M. ADO and V. V. ELIAN

Abstract—The coherent radiation emitted by the electrons in a synchrotron has been studied experimentally at a wavelength of 10 cm for various electron distributions within the phase stable region. This wavelength corresponds to the 50th harmonic of the revolution frequency of the electrons. In the majority of cases these experiments confirm the theory of coherent radiation from electrons but deviations from the theory are observed for electron distributions which approach uniformity. The emitted radiation has been used to measure the frequency of the electron phase oscillations and also the adiabatic damping of the phase oscillation amplitudes. These experiments were carried out on the 280 MeV synchrotron at the Physics Institute of the USSR Academy of Sciences, which has betatron start-up.

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1. INTRODUCTION

DURING their circular motion in a cyclic accelerator the electrons radiate electromagnetic waves having frequencies equal to multiples of the revolution frequency.[1] In the region where the wavelength is comparable to the dimensions of the electron bunch this radiation is coherent and has a character which depends upon the electron distribution around the orbit.[2–6] This distribution is taken into account by introducing a form factor F_{nm} into the equation giving the intensity of the coherent radiation

$$W_{nm} = w_{nm} N^2 F_{nm}, \tag{1}$$

where w_{nm} is the intensity of the radiation emitted by a single electron which is executing phase oscillations having a frequency Ω; N is the number of electrons being accelerated; n is order of the harmonic and $m = 0, 1, 2, 3 \ldots$ characterizes the side frequencies about the n-th harmonic.

The form factor F_{nm} has been computed for various types of stationary distributions $f(\Phi)$ of the electron phase oscillation amplitudes:[5]

for $$f(\Phi) = \frac{\Phi}{2\Phi_0}\sqrt{(2\Phi_0)^2 - \Phi^2} \tag{2}$$

$$F_{n0} = \frac{\sin^2(2n\Phi_0)}{(2n\Phi_0)}; \tag{2'}$$

for $$f(\Phi) = \delta(\Phi - \Phi_0) \tag{3}$$

$$F_{n0} = J_0^2(n\Phi_0); \tag{3'}$$

for $$f(\Phi) = \frac{\pi\Phi}{2\Phi_0^2}\, e^{-\pi/4}\left(\frac{\Phi}{\Phi_0}\right)^2 \tag{4}$$

$$F_{n0} = e^{-(2n^2\Phi_0^2/\pi)} \tag{4'}$$

where Φ_0 is the mean amplitude of the phase oscillations in the bunch.

If $m \neq 0$, then $F_{nm} = 0$ for all three types of stationary distribution.

As the electrons in the synchrotron are accelerated the phase oscillations die away while simultaneously the intensity of the coherent radiation varies in the manner described by one of the form factors (2′), (3′) and (4′) according to the character of the electron distribution with respect to Φ. For the form factors (2′) and (3′) these variations are of an oscillatory nature—a feature which was first observed during some coherent radiation studies using the 16th and 24th harmonics in the 5 MeV synchrotron at the Physics Institute of the USSR Academy of Sciences. The phenomenon was used to measure the adiabatic contraction of the electron bunches.[6]

When the electron phase oscillation amplitude distribution is non-stationary F_{nm} is not necessarily zero when $m \neq 0$.[5] In addition, the signal from the radiation is amplitude modulated at the phase oscillation frequency, a circumstance which enables the frequency of the electron phase oscillations to be measured experimentally.

2. APPARATUS

The radiation emerged from the synchrotron chamber through an 8 cm diameter tangential pipe section and was picked up by a superheterodyne receiver. The general arrangement of the apparatus is shown schematically in Fig. 1. A horn antenna (1) and a mixer (2) were placed in close proximity to the accelerator while the heterodyne unit (9) (incorporating a K-11 klystron), the IF amplifier (3) and the stabilized power packs (4) and (8) were placed about 3 m away from the accelerator magnet in order to avoid trouble due to the alternating magnetic field. The IF unit was a six-stage resonant amplifier having a band-width

* Translated by D. L. ALLAN from *Atomnaya Energiya* **9**, 455 (1960).

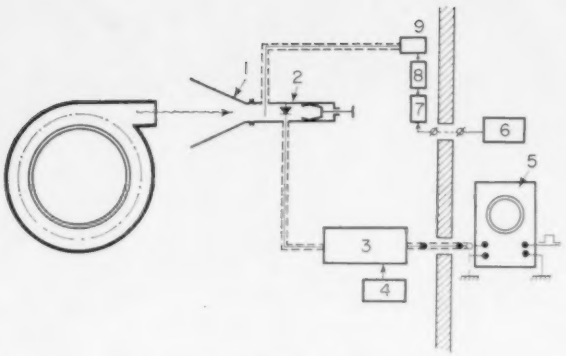

FIG. 1.—Block diagram of apparatus.

of 0·5 Mc/s and a gain $\sim 5 \cdot 10^5$. The intermediate frequency was 25 Mc/s. After detection, the signal was displayed on an OK-17M oscillograph unit (5) with camera attachment. The oscillograph sweep was synchronized to the accelerator magnetic field and time markers were displayed on the oscillograph trace. The receiver could be tuned over a frequency range of $\sim$10 Mc/s, this range being limited by the operating range of the klystron. The klystron frequency was altered by changing the reflector voltage, an operation which was carried out with the aid of the pair of selsyn motors (6) and (7).

3. EXPERIMENTAL RESULTS

Oscillograph studies of the radiation were made with different electron distributions in the phase stable region.

In the case of a synchrotron with betatron start-up it is possible to change the distribution by switching the high frequency accelerating voltage on at an instant of time which is displaced from the optimum time. Figure 2 shows the experimentally determined relation between the relative number of particles N which are captured into the synchrotron regime as a function of the time at which the accelerating field is switched on. The optimum time for applying the accelerating field corresponds to $N = 100$. In the regions I and III the electrons fall inside the phase stable region on phase trajectories which lie close together and consequently the amplitude distribution of the electron phase oscillations approximates to a δ-function. In region II the phase stable region is filled more uniformly and the electron Φ-distribution follows equation (2) quite closely. In region I the number of captured electrons depends upon the frequency of the accelerating field and therefore when measuring the frequency one should operate at a point where N has the greatest value in this region.

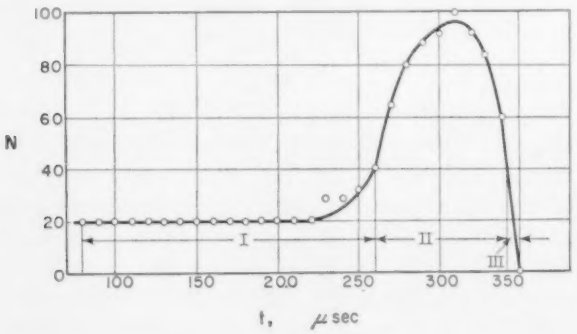

FIG. 2.—Experimentally determined dependence of the number of particles N captured into the synchrotron accelerating regime as a function of the time at which the high frequency accelerating field is switched on. Zero time corresponds to the instant the magnetic field passes through zero.

Figure 3 presents some typical oscillograms obtained when the accelerating field is applied in region II. It will be seen that in the betatron accelerating regime there is no radiation when the electron energy $E \leqslant 4$ MeV; radiation appears only after the bunches have been formed. When $E > 180$ MeV the radiation intensity undergoes a sharp decrease and it falls below the noise level of the equipment. The shape of the radiation pulse depends on the conditions of capture in the synchrotron regime; i.e. it depends upon the amplitude and frequency of the accelerating field, on the rise-time of the field and upon the instant it is switched on. Under certain conditions the radiation is partially amplitude modulated: an example of such modulation is indicated by an arrow in Fig. 3(b). Further oscillograms of this region, taken with a fast sweep, are shown in Fig. 4 for three different peak accelerating voltages U. The ratio of the modulation frequencies deduced from oscillograph trace measurements are

$$F_1 : F_2 : F_3 = 0{\cdot}85 : 1 : 1{\cdot}15.$$

This agrees well with the theoretical phase oscillation frequencies:

$$F_1 : F_2 : F_3 = \sqrt{U_1 \sin \varphi_s 1} : \sqrt{U_2 \sin \varphi_s 2} : \sqrt{U_3 \sin \varphi_s 3}$$
$$= 0{\cdot}84 : 1 : 1{\cdot}10,$$

where φ_s is the equilibrium phase.

This result demonstrates that the amplitude modulation is due to the phase oscillation of particles having a distribution which is non-stationary with respect to the amplitude of these oscillations.

Table 1 presents the phase oscillation frequencies F derived from the oscillograph traces of Fig. 4, the maximum amplitudes α_{max} of the phase oscillations obtained from the known relation[7] between F and α_{max} and also the maximum sizes ψ_{max} of the phase stable regions.

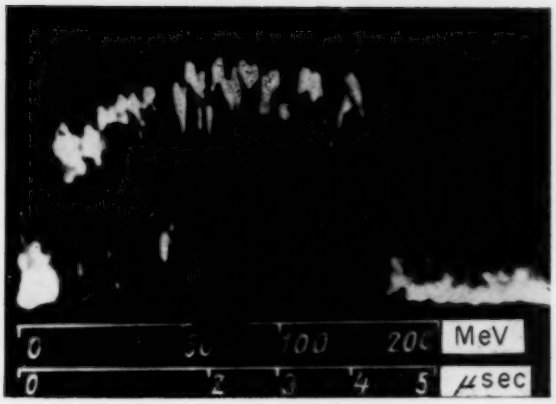

FIG. 3(a).

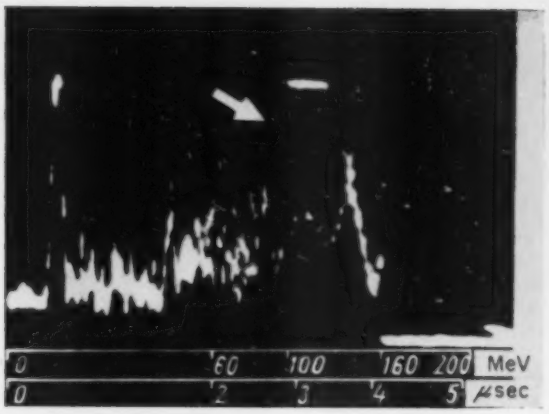

FIG. 3(b).

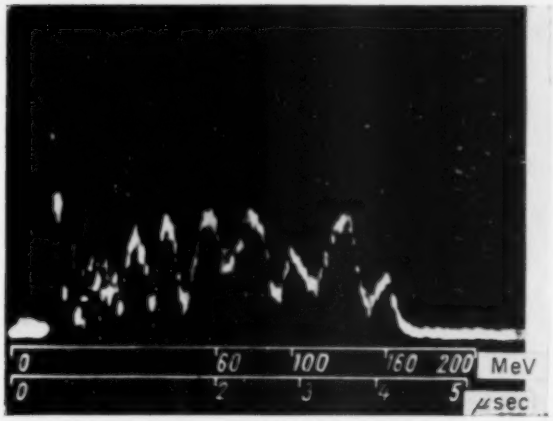

FIG. 3(c).

FIG. 3(a–c).—Oscillograms of the coherent radiation.

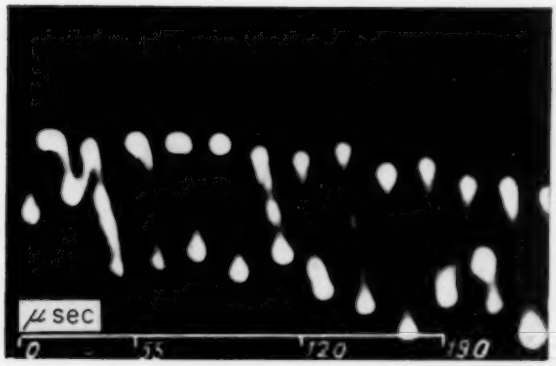

FIG. 4(a).

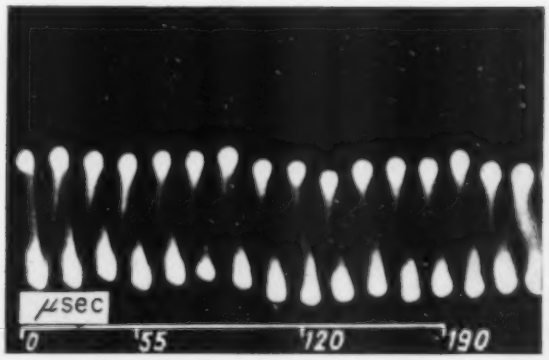

FIG. 4(b).

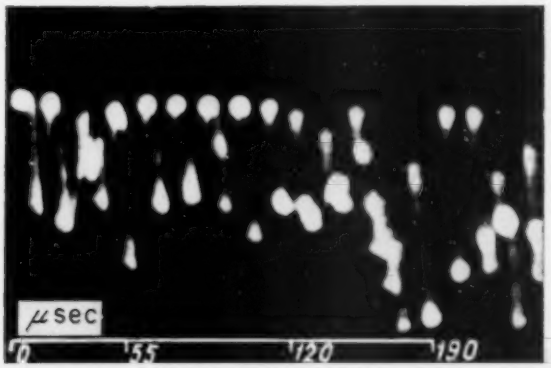

FIG. 4(c).

FIG. 4(a–c).—Oscillograms of the coherent radiation intensity amplitude modulated at the phase oscillation frequency. (a) With $U = 800$ V; (b) with $U = 1000$ V; (c) with $U = 1200$ V.

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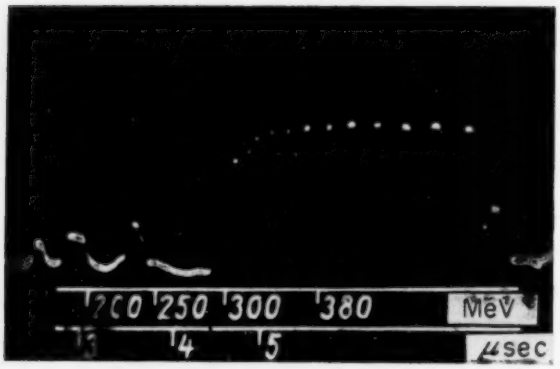

FIG. 5(a).

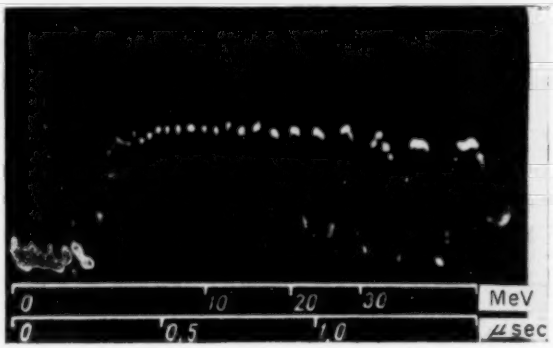

FIG. 5(b).

FIG. 5(a,b).—Slow oscillatory modulation of the radiation intensity.

TABLE 1.—DATA FROM THE PHASE OSCILLATION FREQUENCY MEASUREMENTS

U, V	$\sin \varphi_s$	F, kc/s	α°_{max}	ψ°_{max}
800	0·77	52 ± 5	92 ± 9	100 ± 10
1000	0·87	63 ± 6	115 ± 11	120 ± 12
1200	0·91	84 ± 6	127 ± 12	130 ± 13

For the purpose of these calculations we took $E = 80$ MeV, the exponent of the magnetic field gradient as 0·65 and the revolution frequency of the electrons was taken to be 58 Mc/s. A comparison of α_{max} with ψ_{max} shows that it is the electrons grouped in the vicinity of the separatrix of the phase diagram which contibute to the modulation.

The radiation has an oscillatory character only under the special conditions of bunch formation in the transition regime when the particles have a distribution with respect to phase oscillation amplitude which approaches a δ-function. For example, when the accelerating field is switched on at an instant corresponding to region I of Fig. 2, the radiation has the energy dependence shown in Fig. 5. However, shortly after the commencement of the synchrotron regime, the radiation becomes either quite irregular (i.e. noise: see, for example, Fig. 3a) or partially regular (Fig. 3c). The regularity may begin to break up at various electron energies from $E = 8$ MeV (Fig. 5a) to $E = 60$ MeV (Fig. 5b), according to the frequency of the accelerating field. This behaviour is evidently associated with a redistribution of the electrons in the phase stable region during the process of acceleration.

The slow oscillations of the radiation intensity (see Fig. 5) are due to the adiabatic damping of the phase oscillations. Similar oscillations have been observed by PROKHOROV.[6]

Figure 6 presents graphs showing the decay in the amplitude of the phase oscillations calculated by the adiabatic invariance method.[7] The figure includes also the experimental values derived from the oscillograph traces (see Fig. 5).

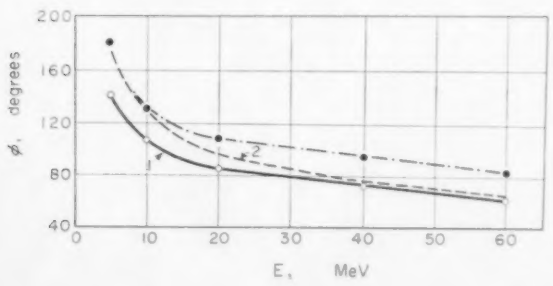

FIG. 6.—Adiabatic damping of the phase oscillations. (1) and (2) Theoretical curves; ○ and ●—experimental results for $\Phi_{0\,init} = 140^\circ$ and $\Phi_{0\,init} = 180^\circ$.

While the experimental points agree very well with the calculated curves for $\Phi_{0\,init} \leqslant 140^\circ$, they do not agree with the curves for $140^\circ \leqslant \Phi_{0\,init} \leqslant 180^\circ$. This means that in region I of Fig. 2 the electrons which are captured into the synchrotron regime lie on phase trajectories having amplitudes less than 140°.

The frequency band width on either side of the 50th harmonic was also measured. For radiation having an oscillatory character (Fig. 5) it was found that the frequency band width of this harmonic was less than 0·5 Mc/s. This confirms the conclusion reached theoretically that for a type (3) distribution the side frequencies of the radiation are damped out, i.e. $F_{nm} = 0$ when $m \neq 0$.

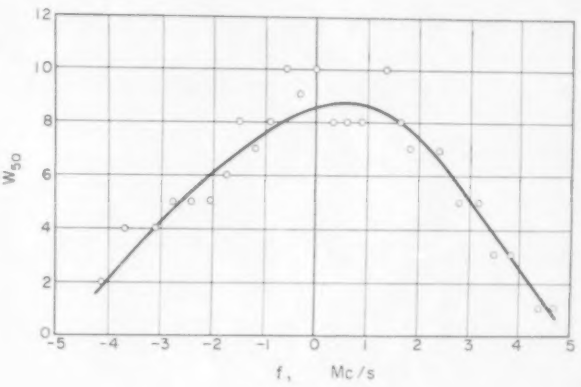

FIG. 7.—Relative intensity distribution of the radiation over the band of frequencies centred on the 50th harmonic for the case where the accelerating field is applied at the optimum instant of time.

The noise type of radiation (see Fig. 3) embraces a broad band of frequencies (Fig. 7), i.e. for this case $F_{nm} \neq 0$ when $m \neq 0$. Up to now no one has given a theoretical explanation of this effect. We suggest that a good qualitative explanation of the broad frequency band on either side of the harmonics of the radiation emitted by electron bunches having a type (2) distribution can be found in the fluctuations which must inevitably occur in the distribution of the electrons with respect to phase oscillations amplitude. Such fluctuations are not considered in the theory.

A sharp drop in the intensity of the radiation for electron energies exceeding 180 MeV is explained by the fact that at that stage of acceleration the electron distribution with respect to phase oscillation amplitude approaches the gaussian form (4). At the same time the quantity F_{n0} (4′) is very much smaller than (2′) and (3′). The formation of a type (4) distribution has been confirmed in some other experiments[8] on the 280

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MeV synchrotron at the Physics Institute of the USSR Academy of Sciences.

4. CONCLUSIONS

(1) Observation of the coherent radiation emitted by the electrons provides a simple means of determining the moment at which electron bunches begin to form in a synchrotron with betatron start-up.

(2) For a non-stationary distribution of the electrons in the separatrix the radiation can be used to measure the frequency of the phase oscillations.

(3) The coherent radiation of the 50th harmonic can be used to study the damping of the phase oscillations only when the amplitude distribution of the oscillations is of the δ-function type. For other types of distribution one should use lower harmonics.

(4) The theory of the coherent radiation emitted by the electrons in a synchrotron is in agreement with the experimental observations only in certain particular cases. To obtain a more generally applicable theory it will be necessary to take into account the effect of fluctuations in the electron distributions upon the degree of coherence.

Acknowledgments—In conclusion the authors thank PROF. P. A. CHERENKOV, PROF. M. S. RABINOVICH, PROF. A. M. PROKHOROV and L. V. IOGANSEN for valuable discussion.

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LETTERS TO THE EDITORS

An electrostatic method of extracting electrons from a betatron*

(*Received* 19 *February* 1958)

THE electrostatic method of extracting the electrons from a betatron[1–3] has definite advantages over the magnetic extraction method. With an electrostatic field, considerable electron deflexions can be obtained quite easily over small portions of the orbit and it is possible to localize the electric field so that the electrons remain undisturbed up to the moment they enter the deflecting device (which we shall call the condenser). Furthermore, the electrostatic method avoids the necessity of using auxiliary coils and high-power pulse circuits to expand the orbit at the end of the acceleration cycle. The height of the condenser can be made greater than the vertical spread of the beam at the point towards the end of the acceleration cycle where the orbit is expanded.

The dimensions and general design of the deflecting condenser are governed by the electron beam parameters, especially the vertical spread of the beam and the step of the expanding spiral trajectory. To obtain the required information about the beam, KCl or NaCl crystal probes can be used. These are placed at various radii normal to the beam trajectory and the colouration

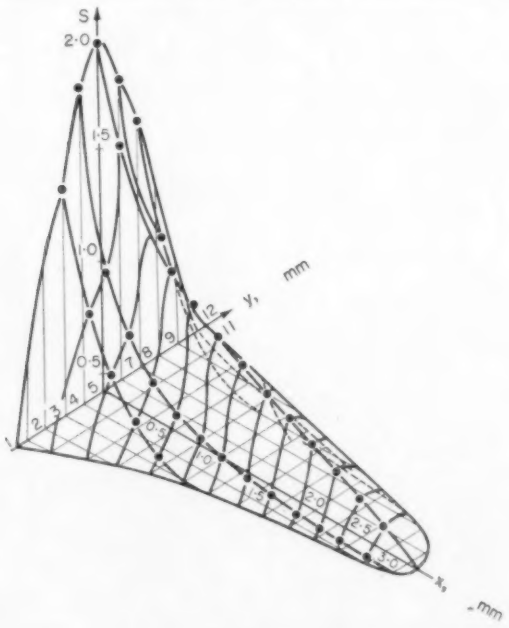

FIG. 1.—Results of photometric measurements on a crystal irradiated inside the betatron vacuum chamber.
S: optical density of crystal discolouration; *x*: distance from crystal edge measured radially from the centre of the accelerator; *y*: vertical co-ordinate of beam ($y = 6$ corresponds to the median plane of the accelerator).

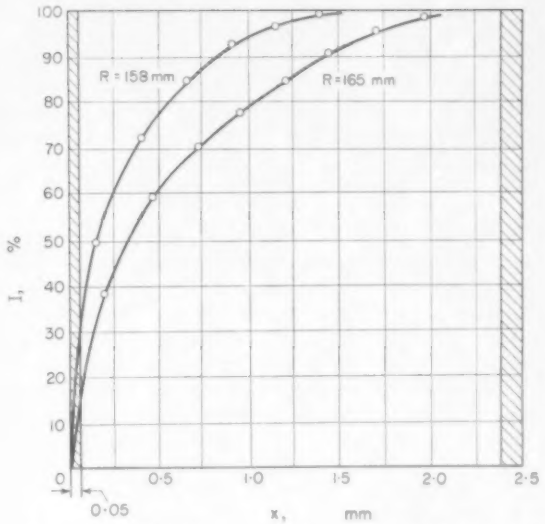

FIG. 2.—Distribution curves of beam current (I) against x-co-ordinate. These curves are used to determine the step of the spiral and the loss of electrons at the intercepting plate of the condenser, with this unit located at $R = 158$ and $R = 165$ mm.
The cross-hatched areas represent the condenser electrodes.

produced by the electrons is proportional to the number of electrons which strike the crystals. A microphotometer is used to measure the optical transmission of the crystal and from these measurements the distribution of electrons across the beam is deduced. Figure 1 shows the spatial distribution of the electrons in the 15 MeV betatron at the Tomsk Polytechnic Institute. The vertical spread of the beam is less than 9 mm and the number of electrons falls off rapidly towards the edge of the crystal. By integrating the spatial distribution graphically, an estimate can be made of the number of electrons which are lost at the deflecting condenser and a suitable choice of the condenser plate spacing can be made (Fig. 2). At a radius $R = 160$ mm and for a magnetic field exponent $n = 1$, the step of the spiral trajectory is 2 mm while at a radius $R = 165$ mm it is 2·5 mm. In the region $R = 160$–165 mm the electron is 16 per cent and the extraction efficiency is about 80 per cent with an intercepting plate thickness of 0·05 mm.

On the basis of these measurements of the electron distribution across the beam, we designed the beam deflecting condenser for the 15 MeV betatron shown in Fig. 3. The intercepting plate is made of 50 μ tantalum foil stretched across stainless steel supports. This plate is at earth potential while another, made of stainless steel, is maintained at a high voltage. The edges of the plate are rounded and polished to increase the breakdown resistance of the gap and the complete condenser is mounted upon glass supports. A voltage of less than 40 kV was found to be sufficient to deflect 15 MeV electrons.

When deciding upon the voltage to be applied to the condenser it is necessary to consider the electric leakage field of the condenser and its effect on the electron orbit at the moment of

* Translated by D. L. ALLAN from *Atomnaya Energiya* **7**, 487 (1959).

injection. The disturbance due to the leakage field can be so strong that, with the injector located outside the equilibrium orbit, a condenser voltage of ~2 kV is enough to upset the capture of the electrons into the acceleration regime. By screening the condenser it is possible to maintain normal betatron operation with plate voltages up to 6 kV, but if the voltage is raised to for example, 9 kV, the number of electrons captured into the acceleration regime becomes quite negligible (in fact, 1000 times fewer than normal). Although by using more complicated screening it would be possible to obtain an adequate radiation intensity with tens of kV on the condenser, this would lead to a greater loss of electrons at the condenser and therefore to a reduction in yield. It is, in fact, more expedient to dispense with a d.c. or a.c. supply for the condenser and go over to a pulsed voltage supply.

A suitable pulse generator presents no special problems; it should merely provide a pulse of sufficient amplitude and the voltage should remain constant while the accelerated electrons are being pulled out by the condenser. We considered two types of pulse generator. In the first, a pulse shaping line is discharged through a thyratron into a pulse transformer and in the second the voltage is supplied by a step-up transformer with a highly saturated core (a high-voltage peaking transformer). If it is desired to change the energy of the extracted electrons by pulling them out at different phases of the acceleration cycle, the peaking transformer should be supplied through a phase shifter. A supply of this type is very simple and reliable—factors which are of considerable importance when the betatron is to be operated by personnel who have no specialized technical qualifications.

Using this apparatus it was possible to extract an electron beam from the 15 MeV betatron of the Tomsk Polytechnical Institute. The principal beam parameters are as follows: the mean beam current is 4×10^{-9} A (0·27 μA in the pulse), the cross section of the beam at 2 cm from the exit port (made of 0·03 mm aluminium foil) is 6 × 10 mm and the angular divergence of the beam after passing through 17 cm of air is 5° in the vertical plane and 8·5° in the horizontal plane. The extraction efficiency is ~60 per cent. The fact that this is 20 per cent less than the figure mentioned above is due to the fact that the intercepting plate of the condenser became slightly buckled during the rolling process. The employment of a better quality foil will certainly raise the extraction efficiency.

During these investigations we noticed that there was an electron yield when no voltage was applied across the condenser. These electrons emerged as a well focused beam 6 mm in diameter having an intensity which depended upon the position of the deflecting condenser (Fig. 4). The beam disappeared completely when the condenser was removed from its usual operating position in the accelerator chamber. This electron yield arises as a result of the interaction of the electrons with the current induced in the plates. The extraction efficiency is low (~6 per cent) but is still higher than that attained by using a magnetic shunt.

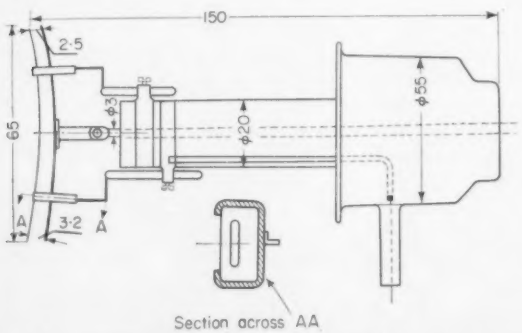

FIG. 3.—Diagram of deflecting condenser (dimensions are given in mm).

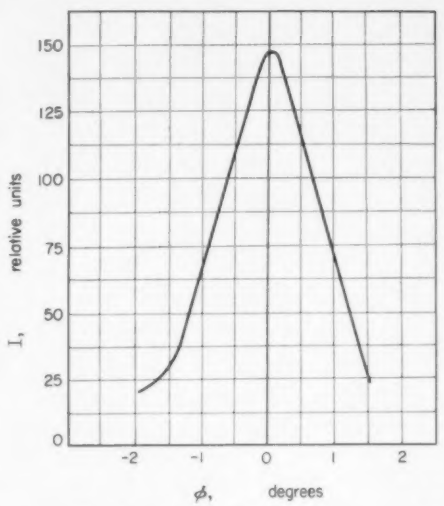

FIG. 4.—The dependence of the electron current I upon a displacement of the condenser through an angle ϕ from the optimum position.

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Mass-spectrographic and spectroscopic studies of hydrogen discharge from an ion source*

(*Received* 27 *November* 1959)

THERE is considerable interest in maximizing the relative content of either protons or molecular hydrogen ions in the currents from hydrogen ion sources. Previously reported[1] mass-spectrographic studies of ions issuing from the slit source of the one and a half meter cyclotron of the USSR Academy of Sciences showed that from an area 20 × 2 mm, currents of about 60 mA containing 95 per cent of protons or 80 per cent of molecular hydrogen could be obtained. Subsequently optical spectroscopic methods were used to study the relative H^+ and H_2^+ ion densities in the ion source and also the relative densities in the discharge plasma of the neutral particles H and H_2. Figure 1 shows diagrammatically the ion source and its electrical supply circuit. The hydrogen spectrum was studied with a Zeiss three-prism glass spectrograph equipped with a camera objective of 84 cm focal length. In the region of 6500 Å the reciprocal of linear dispersion for this instrument is 38 Å mm^{-1}.

* Translated by J. ADAM from *Atomnaya Energiya* **8,** 44 (1960).

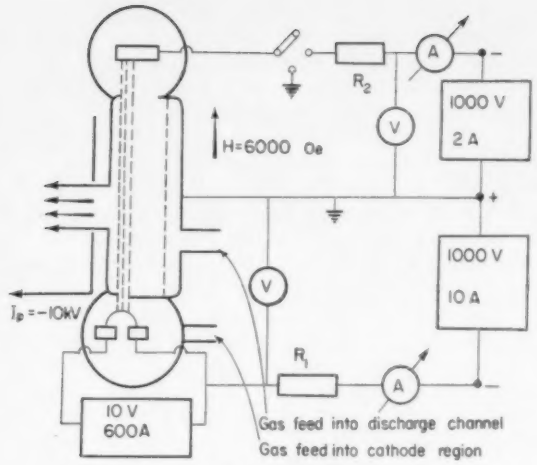

FIG. 1.—The ion source and circuit.

The light from the ion source passes through a rectangular opening 15 mm high and 2 mm long oriented along the discharge channel. To study the emission from atomic hydrogen we used the first line of Balmer's series (6562·79 Å). Half of the sum of 6031·90 Å and 6018·29 Å line intensities was taken as the measure of molecular hydrogen emission. The intensities of spectral lines can be used for determining the concentration of neutral particles, although one must remember that these intensities characterize the numbers of particles present in a given state of excitation.[2] Under constant excitation conditions the relative concentrations of neutral particles, N_H/N_{H_2}, can be followed by measuring the relative intensity changes of atomic and molecular hydrogen lines. If, however, the excitation conditions vary, the relative intensity J_H/J_{H_2} characterizes the ratio $N_H\sigma_H/N_H\sigma_{H_2}$, where σ_H and σ_{H_2} are the effective cross sections for the excitation of H and H_2 respectively. Since σ_H does not vary very strongly within the electron energy interval used in these experiments,[3] the changes in relative intensity J_H/J_{H_2} can be taken to represent the changes in relative concentration of neutral particles in the hydrogen discharge.

Experiments indicate that the H^+ and H_2^+ ion yields from the source differ markedly, depending on the position at which the gas

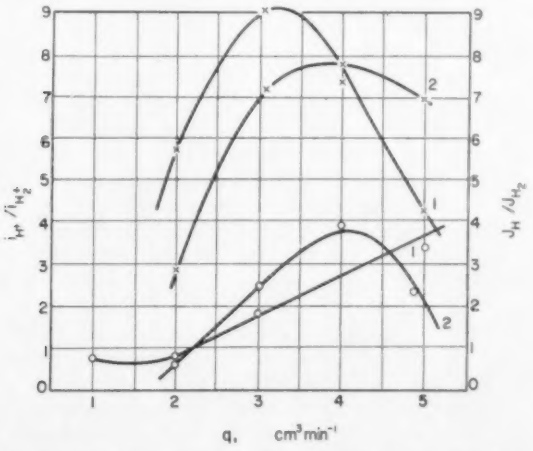

FIG. 2.—The ion current ratio (1) and the line intensity ratio (2) as functions of the gas feed rate. Discharge parameters $I = 3$ A, $U = 160$ V.

is fed into the system. Figure 2 shows the ratios of ion current $i_{H^+}/i_{H_2^+}$ and line intensities J_H/J_{H_2} as functions of gas flow rate for two types of feed. In all figures curves defined by circles refer to experiments in which the gas was fed into the cathode region and curves defined by crosses refer to experiments in which the gas was fed into the gas discharge channel. The curves of relative intensity, J_H/J_{H_2}, which characterize the dissociation process, show that dissociation is greatest when the gas is fed into the gas discharge channel. To explain the observed trends in the values of $i_{H^+}/i_{H_2^+}$ and J_H/J_{H_2}, Fig. 3 shows the absolute values of the ion currents i_{H^+} and $i_{H_2^+}$ and of the line intensities J_H and J_{H_2}, each as a function of the gas

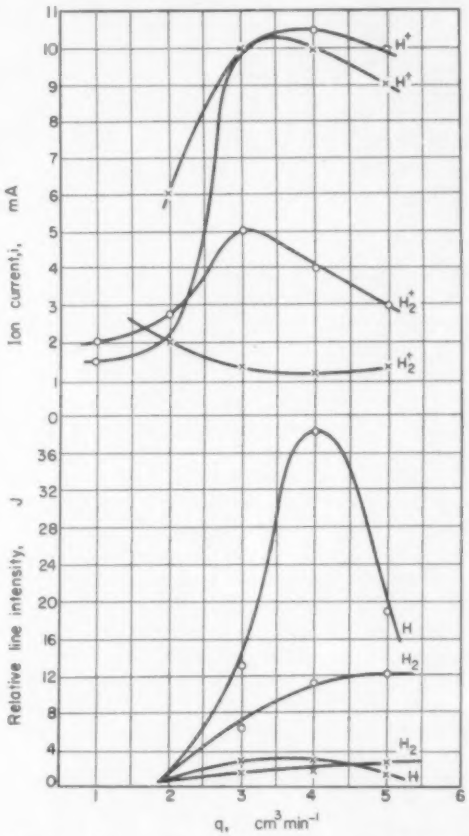

FIG. 3.—Absolute values of ion currents and spectral line intensities as functions of the gas feed rate. Discharge parameters $I = 3$ A, $U = 160$ V.

flow rate. The intensities of the lines in the first spectrum have been taken as unity, and the intensities in subsequent spectra have been referred to them. For comparison Fig. 4 shows the ratios $i_{H^+}/i_{H_2^+}$ and J_H/J_{H_2} as functions of discharge current both when the gas is fed into the gas discharge channel and when it is fed into the cathode region. In the latter case the difference between H^+ and H_2^+ ion yields is greatest.

The intensity ratio curve indicates that when the gas is fed into the gas discharge channel the relative concentration of atomic hydrogen increases rapidly with increasing discharge currents. The relative concentration of H^+ and H_2^+ ions varies with the discharge current in a similar manner.

If the gas is fed into the cathode region then as the discharge current becomes larger the dissociation of hydrogen molecules and the relative concentration of protons initially decrease and

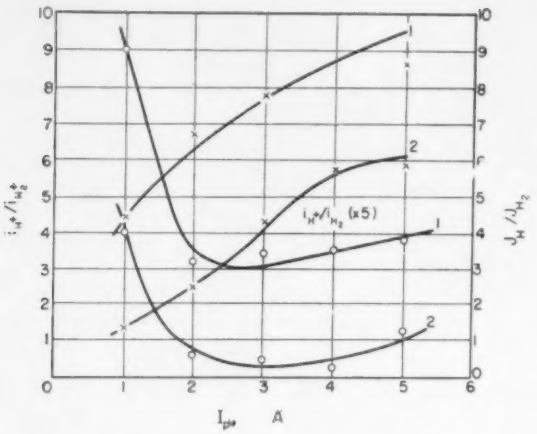

FIG. 4.—The ion current ratios (1) and the line intensity ratios (2) as functions of discharge current when the gas is fed into the discharge channel ($q = 4$ cm³/min, $U = 180$ V) and when the gas is fed into the cathode region ($q = 2$ cm³/min, $U = 160$ V).

then increase. The variation in the absolute intensities of the spectral line and in the ion currents show that the minima found in the curves of Fig. 4 are due to a rapid drop in the concentration of atomic hydrogen and to a consequent increase in the yield of molecular hydrogen ions. The decrease in H and H^+ as the discharge current increases characteristically occurs when the gas is fed into the cathode region. The causes of this effect are not clear.

Figure 5 shows the currents and intensity ratios as functions of the discharge voltage. The curves of relative line intensity are similar in shape to the ion current curves; when the gas is fed into the discharge channel the relative intensity curve reaches saturation at 200 V, but the ion current curve continues to go up.

When the gas is fed into the cathode region the curve of line intensity is displaced by about 40 V towards the low voltage side. This indicates that in the plasma the maximum dissociation occurs at a lower voltage than the maximum ionization. The line intensities and the absolute values of the ion currents show that changes in the degree of dissociation of molecular

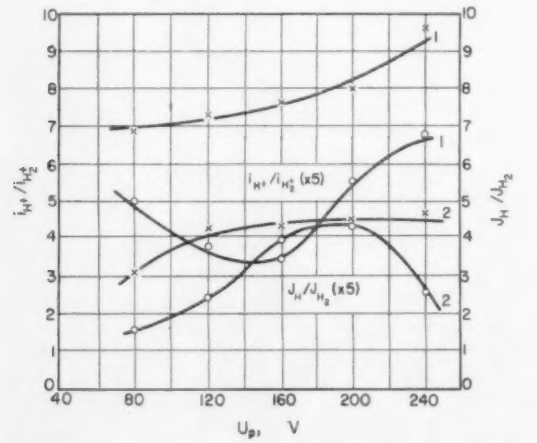

FIG. 5.—The ion current ratios and line intensity ratios as functions of discharge voltage when the gas is fed into the discharge channel ($q = 4$ cm³/min, $I = 2$ A) and when the gas is fed into the cathode region ($q = 2$ cm³/min, $I = 2$ A).

hydrogen and in the relative concentration of protons as the discharge voltage changes depend on the increasing or decreasing concentrations in the plasma of atomic hydrogen.

It follows from the accompanying experimental data that in an ion source the degree of dissociation and the yield of protons and of molecular hydrogen ions depend on where gas is fed into the system. Gas feed rates, discharge voltage and current are the most important discharge parameters.[4] The similar shapes of curves for the dissociation and ionization processes indicate that proton formation is a two stage process beginning with dissociation of H_2 and proceding by ionization of the hydrogen atoms. The higher proton yield observed when the gas is fed into the discharge channel arises because under these conditions more molecules are available to interact with electrons than is the case when the gas is fed into the cathode region. It is also possible that when the gas is fed into the discharge channel the gas particles interact with less energetic electrons and that the dissociation process is efficient under these conditions.[5]

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On the stability of plasma bunches in a wave guide*

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IN order to realize the method proposed by VEKSLER[1] for accelerating plasma bunches by means of radiation it is necessary to investigate their stability in the field of the accelerating wave. We confine ourselves to the consideration of bunches whose dimensions are small in comparison with the wavelength. In this case the field is quasi-stationary in the region occupied by the bunch and the electric vector causes electrostatic swelling of the bunch. In contrast the magnetic vector produces compressive forces in a conducting bunch. From this point of view it appears appropriate to accelerate conducting bunches by fields of the type of a symmetrical *H*-wave in a circular wave guide, in which only the longitudinal component of the magnetic vector differs from zero on the axis of the tube.[2] With this particular case in mind we consider here the problem of the deformation of an ideal conducting bunch in a quasi-stationary uniform magnetic field.

We employ the following simple idealization: we assume that the bunch will always be a uniform spheroid, the axis of which is parallel to the external field. As generalized coordinates we shall take the volume of the spheroid, V, and the ratio of the polar semi-axis to the equatorial semi-axis, q. Using the well-known solution of the problem of an ellipsoid in a

* Translated by N. KEMMER from *Atomnaya Energiya* **8**, 134 (1960).

uniform field,[2] we find the 'magnetic' part, $W_M(V, q)$, of the potential energy of the system. The hydrodynamical part of the potential energy $W_H(V)$ is determined from Poisson's equation $pV^\gamma = \text{const}$. Finally it is easy, from our assumption that the spheroid is homogeneous, to express also the kinetic energy K in terms of V, q and $\dot{V}, \dot{q}$:

$$W_M = \frac{2}{3}\frac{PV}{1 - M(q)}; \qquad W_H = \frac{PV_e}{\gamma - 1}\left(\frac{V_e}{V}\right)^{\gamma-1};$$

$$K = \frac{m}{90}\left(\frac{3V}{4\pi q}\right)^{2/3}\left\{(q^2 + 2)\frac{\dot{V}^2}{V^2} + 4(q^2 - 1)\frac{\dot{V}}{V}\frac{\dot{q}}{q} + 2(2q^2 + 1)\frac{\dot{q}^2}{q^2}\right\},$$

where $P = 3H_0^2/16\pi$ is the average over the surface of the magnetic pressure for a sphere (H_0 is the effective amplitude of the external field). $M(q)$ is the demagnetization factor (for a sphere $M = \frac{1}{3}$). $V_e = 4\pi a^3/3$ is the 'equilibrium' volume at which $p = P$; m is the mass of the bunch. The bunch has not an equilibrium shape, because W_M is a monotonically decreasing function of q. The assumed idealization has a physical meaning only as long as the velocities are small compared to the velocity of sound in the plasma. Thus we can consider only the initial stage of the deformation of the bunch. Assume for instance that at the initial moment the bunch is at rest and has the shape of a sphere of volume V_0. Putting $q = 1 + \eta$, $V = V_0(1 + x)$, and solving the linearized Lagrange equations for small η and x, we obtain

$$\eta = \frac{3}{2}\nu\left(\frac{t}{\tau}\right)^2; \; x = -\frac{15}{2}\nu(1 - \nu^{-\gamma})\left(\frac{t}{\tau}\right)^2. \qquad (1)$$

Here the radial velocities of the poles and of equatorial points of the bunch are equal respectively to

$$v_P = c\nu(5\nu^{-\gamma} - 3)\frac{t}{\tau}; \; v_E = c\nu(5\nu^{-\gamma} - 6)\frac{t}{\tau}, \qquad (2)$$

where $c = (PV_e/m)^{1/2}$ is the isothermal sound velocity, $\tau = a/c$, and $\nu = V_0/V_e$. Evidently equations (1) and (2) are valid only if $t \ll \tau$. For $t \approx \tau$ free acoustical vibrations are excited in the bunch and these decisively influence its stability. As the field in the wave guide is transversely inhomogeneous, a radial force appears if the bunch deviates from the axis of the tube, and acts on the bunch as a whole. Assume that the transverse displacement r is small compared to the radius of the wave-guide section, but large compared to the dimensions of the bunch; then the external field (both magnetic and electric) in the region of the bunch can as before be assumed to be homogeneous and we can find the force of interest to us from the change of the quasi-stationary potential energy of the bunch in these fields. For a spheroid this force is

$$F_r = \frac{VH_0^2}{4\pi}\frac{\kappa^2 + Mk^2}{1 - M^2}r,$$

where k is the wave number and κ is the critical wave number. Thus in the field of the symmetric H-wave the position of the bunch on the axis is unstable and for radiational acceleration an auxiliary focusing field is necessary. The defocusing force F_r is already large for very small r, compared to the accelerating force F_z. For instance for a spherical bunch of radius a, $F_r/F_z > 0{\cdot}16rR^2/a^3$ (R is the radius of the tube).

We note that in the field of a symmetrical E-wave with effective amplitude E_0 on the axis we have

$$F_r = -\frac{VE_0^2}{8\pi}\frac{(1 + M)\kappa^2 - 2M^2k^2}{M(1 - M^2)}r,$$

so that for $k^2 < (1 + M)\kappa^2/(2M^2)$ (for a sphere $k < \sqrt{6}\kappa$) the force is a focusing one. However in such a field an electrostatic swelling of the bunch takes place.

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Similarity solutions of plasma equations*

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THE electromagnetic containment of a fully-ionized neutral plasma of singly charged ions can be described by the following system of equations:

$$\left.\begin{aligned}
&\operatorname{curl} \boldsymbol{E} = -\frac{\partial \boldsymbol{H}}{\partial t}, \quad \operatorname{div} \boldsymbol{D} = 0, \quad \boldsymbol{D} = \varepsilon_0 \boldsymbol{E},\\
&\operatorname{curl} \boldsymbol{B} = \boldsymbol{i}, \quad \operatorname{div} \boldsymbol{H} = 0, \quad \boldsymbol{B} = \mu_0 \boldsymbol{H},\\
&\boldsymbol{i} = \frac{1}{H^2}[\boldsymbol{H}\nabla P], \quad \operatorname{div} \boldsymbol{J} + \frac{\partial N}{\partial t} = 0,\\
&\boldsymbol{J} = N\frac{[\boldsymbol{EH}]}{H^2} - \frac{\nu}{m\omega^2}\nabla P,\\
&P = 2NkT, \quad \nabla T = 0, \quad \boldsymbol{H}\nabla P = 0,
\end{aligned}\right\} \qquad (1)\dagger$$

where

$$\omega = -\frac{e}{m}H, \quad \nu = \frac{e^2 a}{m\mu_0}\frac{N}{T^{3/2}},$$

$$a = \frac{4\sqrt{2\pi}\cdot m^{1/2}\mu_0}{3e^2k^{3/2}}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\Lambda, \quad \Lambda = \ln\frac{p_{\max}}{p_{\min}} \approx 20.$$

The plasma temperature T is assumed to be a known function of time, $T(t)$.

The boundary conditions on the surface Σ of the plasma are written in the form

$$N|_\Sigma = 0, \quad \boldsymbol{H}|_\Sigma = \boldsymbol{H}_1(t). \qquad (2)$$

From equations (1) it follows that

$$\nabla P + \nabla Q = (\boldsymbol{H}\nabla)\boldsymbol{B}, \qquad (3)$$

where

$$Q = \tfrac{1}{2}\boldsymbol{HB}.$$

The problem of the electromagnetic containment of a plasma, expressed by equations (1) and (2), has no stationary solutions and no solutions with a stationary distribution of density, but it has similarity solutions. Generally speaking there exist similarity solutions of the system (1) for a cylindrical plasma filament. However, the most interesting case is when the field is concentrated in a thin surface layer of plasma so that the thickness of the surface electromagnetic layer is considerably less than the

* Translated by N. KEMMER from *Atomnaya Energiya* **8**, 135 (1960).

† In our notation the meaning and terminology for the vectors $\boldsymbol{B}$ and $\boldsymbol{H}$ is the reverse of the usual one, thus agreeing with their physical meaning: in the presence of a medium the intensity $\boldsymbol{H}$ is the average value of the microscopic intensity.

radius of the filament. Then the curvature in the field is negligible and the problem can be formulated as follows: We have a plasma half-space $x < l$, whose boundary with the vacuum is the plane $x = l$, bounded by an electromagnetic field which penetrates partially into the surface layer of the plasma and which vanishes at $-\infty$ within the plasma. All quantities depend on a single co-ordinate x. One has to find the distribution of the plasma and the field as functions of time. This problem reduces to a single equation

$$\left[\left(\frac{a}{T^{3/2}}\frac{\varphi''}{\varphi'} - \frac{\psi^*}{\psi'}\right)(H_1^2 - \psi'^2)\right]' + T\left(\frac{H_1^2 - \psi'^2}{T}\right)^* = 0 \qquad (4)$$

with the boundary conditions

$$\psi(-\infty, t) = 0,\ \psi'(-\infty, t) = 0,\ \psi'(l, t) = H_1(t) \qquad (5)$$

(H_1 is the given value of the magnetic field H at the surface of the plasma.)

All quantities characterizing the plasma in the field can be expressed in terms of the function

$$\psi = \int_{-\infty}^{x} H(x, t)\,dx$$

(the 'quasi-potential of the field'):

$$\left.\begin{aligned} &H_y = h_y(t)\psi'(x, t), \quad H_z = h_z(t)\psi'(x, t),\\ &E_y = -(h_z(t)\psi(x, t))^*,\\ &E_z = (h_y(t)\psi(x, t))^*, \quad h_y^2(t) + h_z^2(t) = 1,\\ &E_\perp = -\psi^*, \quad S = -\mu_0\psi'\psi^*, \quad i = \mu_0\psi''. \end{aligned}\right\} \qquad (6)$$

The prime denotes differentiation with respect to x, the dot with respect to t; S is the x-component of the Umov-Poynting vector.

We find the solution of equation (4) in the form

$$\psi(x, t) = f(t)\varphi\left(\frac{x}{l(t)}\right) = f(t)\varphi(\xi),$$

where $\xi = x/l(t)$ and $l(t)$ is the co-ordinate of the plasma boundary, which varies with time.

Similarity solutions exist if the plasma temperature T and the magnetic field on the surface of the plasma H_1 vary according to a power law (p.l.) or an exponential law (e.l.):

$$\left.\begin{aligned} &T = C_T t^{\gamma_T}, \quad H_1 = C_H t^{\gamma_H} && \text{(p.l.)},\\ &T = C_T e^{\lambda_T t}, \quad H_1 = C_H e^{\lambda_H t} && \text{(e.l.)}. \end{aligned}\right\} \qquad (7)$$

In these cases the dependence of l and f on t is expressed by the formula

$$\left.\begin{aligned} &l = C_l t^{\gamma_l}, \qquad f = C_l C_H t^{\gamma_f},\\ &\gamma_l = \tfrac{1}{2} - \tfrac{3}{4}\gamma_T, \quad \gamma_f = \tfrac{1}{2} - \tfrac{3}{4}\gamma_T + \gamma_H \end{aligned}\right\} \text{(p.l.)},$$

$$\left.\begin{aligned} &l = C_l e^{\lambda_l t}, \qquad f = C_l C_H e^{\lambda_f t},\\ &\lambda_l = -\tfrac{3}{4}\lambda_T, \quad \lambda_f = -\tfrac{3}{4}\lambda_T + \lambda_H \end{aligned}\right\} \text{(e.l.)}, \qquad (8)$$

and the function $\phi(\xi)$ is determined by the equation

$$(1 - \varphi'^2)\varphi'\varphi''' - (1 + \varphi'^2)\varphi''^2 + b(1 - \varphi'^2)\varphi'^2 + c(1 + \varphi'^2)\varphi\varphi'' = 0, \qquad (9)$$

where

$$\left.\begin{aligned} &b = (\gamma_H - \gamma_T)\frac{C_T^{3/2}C_l^2}{a},\\ &c = \left(\tfrac{1}{2} - \tfrac{3}{4}\gamma_T + \gamma_H\right)\frac{C_T^{3/2}C_l^2}{a} \end{aligned}\right\} \text{(p.l.)}$$

$$\left.\begin{aligned} &b = (\lambda_H - \lambda_T)\frac{C_T^{3/2}C_l^2}{a},\\ &c = \left(-\tfrac{3}{4}\lambda_T + \lambda_H\right)\frac{C_T^{3/2}C_l^2}{a} \end{aligned}\right\} \text{(e.l.)} \qquad (10)$$

The boundary conditions are

$$\left.\begin{aligned} &\varphi_0 \equiv \varphi(-\infty) = 0, \quad \varphi_1 \equiv \varphi(1) \equiv \alpha,\\ &\varphi'_0 \equiv \varphi'(-\infty) = 0, \quad \varphi' \equiv \varphi'_1(1) = 1. \end{aligned}\right\} \qquad (11)$$

The suffix zero denotes the values of the quantities within the plasma for $\xi = \xi_0 = -\infty$; the suffix 1 denotes the value at the surface of the plasma at $\xi = \xi_1 = 1$.

For the quantities that characterize the plasma and the field we obtain the expressions

$$\left.\begin{aligned} &H = C_H t^{\gamma_H}\rho, \quad E_\perp = C_l C_H t^{\gamma_f - 1}(\gamma_l\xi\rho - \gamma_f\varphi),\\ &i = \mu_0\frac{C_H}{C_l}t^{\gamma_H - \gamma_l}\rho',\\ &S = \mu_0 C_l C_H^2 t^{\gamma_H + \gamma_f - 1}(\gamma_l\xi\rho - \gamma_f\varphi)\rho,\\ &P = \tfrac{1}{2}\mu_0 C_H^2 t^{2\gamma_H}(1 - \rho^2),\\ &N = \tfrac{1}{4}\mu_0\frac{C_H^2}{C_T}t^{2\gamma_H - \gamma_T}(1 - \rho^2),\\ &J = \tfrac{1}{4}\mu_0 a\frac{C_H^2}{C_l C_T^{5/2}}t^{2\gamma_H - \frac{7}{4}\gamma_T - \frac{1}{2}}\\ &\quad\times\left[\frac{\rho'}{\rho} + \left(\gamma_l\xi - \gamma_f\frac{\varphi}{\rho}\right)\frac{C_l^2 C_T^{3/2}}{a}\right](1 - \rho^2) \end{aligned}\right\} \text{(p.l.)}; \qquad (12)$$

$$\left.\begin{aligned} &H = C_H e^{\lambda_H t}\rho, \quad E_\perp = C_l C_H e^{\lambda_f t}(\lambda_l\xi\rho - \lambda_f\varphi),\\ &i = \mu_0\frac{C_H}{C_l}e^{(\lambda_H - \gamma_l)t}\rho',\\ &S = \mu_0 C_l C_H^2 e^{(\lambda_H + \lambda_f)t}(\lambda_l\xi\rho - \lambda_f\varphi)\rho,\\ &P = \tfrac{1}{2}\mu_0 C_H^2 e^{2\lambda_H t}(1 - \rho^2),\\ &N = \tfrac{1}{4}\mu_0\frac{C_H^2}{C_T}e^{(2\lambda_H - \lambda_T)t}(1 - \rho^2),\\ &J = \tfrac{1}{4}\mu_0 a\frac{C_H^2}{C_l C_T^{5/2}}e^{\left(2\lambda_H - \frac{7}{4}\lambda_T\right)t}\\ &\quad\times\left[\frac{\rho'}{\rho} + \left(\lambda_l\xi - \lambda_f\frac{\varphi}{\rho}\right)\frac{C_l^2 C_T^{3/2}}{a}\right](1 - \rho^2) \end{aligned}\right\} \text{(e.l.)}$$

Here we have introduced the function

$$\rho(\xi) = \varphi'(\xi). \qquad (13)$$

From a qualitative analysis of equation (9) with boundary conditions (11) one can prove the theorem that the functions ϕ and ρ are monotonic. In the neighbourhood of $\xi = 1$ there

exists a unique regular solution of equation (9) which, under the conditions (11), is expressed by the following expansion:

$$\varphi = \alpha - (1-\xi) + \tfrac{1}{2}\alpha c(1-\xi)^2 + \tfrac{1}{12}(b-c)(1-\xi)^3 - \frac{1}{288}\frac{b^2 - c^2 + 2\alpha^2 c^2(5b-c)}{\alpha c}(1-\xi)^4 + \ldots \quad \text{for } c \neq 0, \tag{14a}$$

$$\varphi = \alpha - (1-\xi) + \tfrac{1}{16}b(1-\xi)^3 - \frac{b^2}{540}(1-\xi)^5 + \ldots \quad \text{for } c = 0. \tag{14b}$$

A special class of solutions (14b) can be represented exactly with the aid of a quadrature. These solutions describe the expansion of the plasma in the case when the condition

$$I^* = \frac{1}{\mu_0}\frac{S_1}{H_1^2} \tag{15}$$

is satisfied, which signifies that the particles at the surface of the plasma move with the speed of the electromagnetic drift. It follows from the equations (15), (8) and (12) (for $\xi = 1$ and $\rho = 1$) that $\gamma_f = 0$ and $\lambda_f = 0$; this together with the relations (8) and (10) leads to the condition $c = 0$.

Under this condition equation (9) goes over into the equation for $\rho = \phi'$:

$$(1-\rho^2)\rho\rho'' + (1+\rho^2)\rho'^2 + b(1-\rho^2)\rho^2 = 0, \tag{16}$$

where

$$\left.\begin{aligned} b &= \tfrac{1}{2}(\gamma_1 - 2)\frac{C_i^2 C_T^{3/2}}{a} \quad \text{(p.l.)},\\ b &= \tfrac{1}{2}\lambda_1 \frac{C_i^2 C_T^{3/2}}{a} \quad \text{(e.l.)}. \end{aligned}\right\} \tag{17}$$

By introducing the variables $z = \rho^2$ and $y = \rho'^2/b$, equation (16) transforms into

$$(1-z)z\frac{dy}{dz} - (1+z)y + (1-z)z = 0. \tag{18}$$

The points $z = 1$, $y \neq 0$ are ordinary points of this equation and a unique solution $z = 1$ passes through them; this solution lacks physical meaning. The required solution passes through the singular point $z = 1$, $y = 0$ (a saddle point) where one of the solutions is $z = 1$ and has no meaning. A solution of equation (18) passing through the points $z = 1$, $y = 0$ is the function

$$y = -\frac{z}{(1-z)^2}\left(\ln z - 2z + \tfrac{1}{2}z^2 + \tfrac{3}{4}\right), \tag{19}$$

whence

$$\frac{d\rho}{d\xi} = \sqrt{b}\,\frac{\rho}{1-\rho^2}\sqrt{2\rho^2 - \ln\rho^2 - \tfrac{1}{2}\rho^4 - \tfrac{3}{2}}. \tag{20}$$

Here the positive sign has been taken in front of the square root because it follows from the boundary conditions and the monotonic character of the function ρ that the derivative $d\rho/d\xi$ is always positive. Since the function $2\rho^2 - \ln\rho^2 - \frac{1}{2}\rho^4 - \frac{3}{2}$ is positive in the whole range of variation of ρ

$$(0 \leqslant \rho \leqslant 1)$$

we have $b > 0$. Together with equations (17), (8) and (10) this gives

$$\gamma_1 > 2, \quad \gamma_H < -2, \quad \gamma_T < -2, \quad \gamma_f = 0 \qquad \text{(p.l.)},$$

$$\lambda_1 > 0, \quad \lambda_H < 0, \quad \lambda_T < 0, \quad \lambda_f = 0 \qquad \text{(e.l.)}.$$

The special solution of (14b) thus describes the process of expansion of the plasma when the magnetic field is diminishing and the plasma is cooling down.

The solution of equation (20) has the form

$$\xi = 1 - \frac{1}{\sqrt{b}}I(\rho), \tag{21}$$

$$I(\rho) = \tfrac{1}{2}\int_0^{1-\rho^2} \frac{x\,dx}{(1-x)\sqrt{-\ln(1-x) - x - \frac{1}{4}x^2}}. \tag{22}$$

The values of $I(\rho)$ are given in Table 1. The integral (22) can be expressed approximately by the formula

$$I(\rho) \simeq \sqrt{6(1-\rho)},$$

which is a good representation of the exact behaviour of $I(\rho)$ up to $\rho = 0{\cdot}5$; together with expression (21) it gives $\rho \approx b(1-\xi)^2/6$ in agreement with the expansion (14b). Comparison of the expansion (14a) with the results of exact numerical calculations shows that it is a good representation of the exact solution of equation (9) up to $\rho = 0{\cdot}5$.

TABLE 1.—VALUES OF THE FUNCTION $I(\rho)$

ρ	1	0·9	0·80	0·70	0·60	0·50	0·40	0·30	0·20	0·10
$I(\rho)$	0	0·70	1·02	1·26	1·53	1·76	2·00	2·25	2·55	2·96

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PAPERS TO BE PUBLISHED IN FUTURE ISSUES

J. K. Wright, R. D. Medford and B. Chambers: Equilibrium configurations of the plasma in theta discharges with reversed trapped magnetic fields.

R. F. Post: Impurity radiation from a high temperature plasma.

P. H. Rebut and J. P. Somon: L'hypothèse de pression scalaire dans la dynamique des plasmas sans collisions.

A. Kildal: Energy absorption in a longitudinal wave in a collisionless magnetized plasma.

D. J. BenDaniel: Plasma potential in a magnetic mirror system.

J. N. Tandon and S. P. Talware: Stability of the hard-core liquid model of the pinch.

R. J. Tayler: Necessary and sufficient conditions for the hydromagnetic Rayleigh-Taylor stability of a cylindrical plasma.

A. P. Bamford and G. H. Stafford: Feasibility of a superconducting proton linear accelerator.

R. J. Churchill: Re-ignition of high current spark discharges (Research Note).

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ERRATUM

Plasma Physics–Accelerators–Thermonuclear Research (*Journal of Nuclear Energy* Part C) **3,** 7 (1961).

The headings of Table 1 at the top left-hand column on p. 7 should read:

$$\beta/\eta, \quad \beta BR \text{ (gauss cm)}, \quad \beta N(\text{cm}^{-1}).$$

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PLASMA POTENTIAL IN A MAGNETIC MIRROR SYSTEM

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Abstract—An estimate is made of the plasma potential created by the more rapid initial loss of electrons from magnetic mirror confinement. The effects of this potential on the mirror ratio of the system is discussed. Calculations of this potential as a function of position in the magnetic mirror system are presented for two cases of interest.

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IN particle decay from a magnetic mirror system containing ions and electrons, macroscopic electric fields arise from the more rapid initial loss rate of electrons. These macroscopic electric fields then retard the electron loss but serve to increase the ion loss. A balance is expected to be created in which the relative loss rates of the two types of particles are the same.

KAUFMAN (1956) has investigated the problem starting from the assumption that the magnetic field configuration was a 'square well.' The configuration supposes that the magnetic field has a magnitude B_0 from $-\frac{L}{2}$ to $+\frac{L}{2}$ and then rises abruptly to the value B_c at the coils $\left(z = \pm\frac{L}{2}\right)$. He has shown that the mirror ratio for an ion of energy kT_i is diminished from its theoretical value. He finds that the effective mirror ratio R_e is given by

$$R_e = \frac{\frac{B_c}{B_0}}{1 + \frac{T_e}{T_i}\gamma} = \frac{R_a}{1 + \frac{T_e}{T_i}\gamma} \tag{1}$$

where $R_a = \frac{B_c}{B_0}$ is the mirror ratio in the absence of the potential and

$$\gamma = \frac{3}{2}\ln\left(\frac{T_i}{T_e}\right) + \frac{1}{2}\ln\left(\frac{m_i}{m_e}\right) = -\frac{e\phi_c}{kT_e}. \tag{2}$$

The quantity γ is the ratio of the electrostatic barrier height $|e\phi_c|$ to the electron temperature kT_e.

In actual practice the magnetic field in the mirror region rises smoothly from B_0 to B_c and thus whatever the electric fields which are developed in order to balance the electron and ion loss rates, they must exist over finite distances in the mirror system. Due to the nature of the ion trajectories in the mirror system, some ions may be reflected before they experience the full potential drop. POST (1961) has made some preliminary calculations and estimates that the mirror ratio is not seriously diminished. We shall present a more exact calculation which takes into account the spatial properties of the system and results in a reasonable estimate of both the total potential drop from the system and the actual spatial dependence of this potential within the mirror system. The effect of the plasma potential on the trajectory of the ions is developed in a natural and wholly consistent manner.

To estimate the plasma potential in an actual system, we will need constants of motion for a particle of charge q and of mass m near the axis in the presence of the magnetic field B and an electrostatic potential ϕ which is taken as zero at the midplane. From adiabatic theory for a cylindrically symmetric system, these can be shown to be

$$\frac{\frac{1}{2}mv_{\perp 0}^2}{B_0} = \frac{\frac{1}{2}mv_\perp^2}{B} \tag{3}$$

$$v^2 = v_0^2 - \frac{2q\phi}{m} \tag{4}$$

and

$$B_0 r_0^2 = Br^2 \tag{5}$$

where the subscript '0' refers to the value of the appropriate quantities at the midplane. Using the definition

$$v_\parallel = v\mu \quad (\text{or } v_\perp = v(1-\mu^2)^{1/2}) \tag{6}$$

we can write, from (3) and (4)

$$\mu^2 = 1 - (1-\mu_0^2)\left[\frac{B}{B_0}\frac{1}{1 - \frac{2q\phi}{v_0^2 m}}\right]. \tag{7}$$

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For convenience, we now define the dimensionless variables R, η, R_p and μ_c by the relations

$$R = \frac{B}{B_0} \tag{8}$$

$$\eta = -\frac{e\phi}{kT_e} \tag{9}$$

$$R_p = \frac{R}{1 + \eta \dfrac{T_e}{T_i}} \tag{10}$$

$$\mu_c = \sqrt{1 - \frac{1}{R_e}} = \sqrt{1 - \frac{1 + \gamma T_e/T_i}{R_a}}\,. \tag{11}$$

The quantity R is the ratio of the magnitude of the magnetic field at the point (z, r) to the value at the point $(0, r_0)$ where r_0 is given by equation (5).

The quantity R_p is an effective value of the local ratio R for ions of temperature T_i along a trajectory in the presence of a plasma potential. The effective ratio R_p increases monatonically in $|z|$ from the value unity at the midplane to R_e at the coils.

The value μ_c is the effective loss cone for ions of temperature T_i in the presence of both the magnetic field and the plasma potential. If the velocity vector of an ion is oriented within this cone when the particle crosses the midplane that particle will be lost from the mirror containment system when it reaches the coils.

The dimensionless potential η is the absolute value of the plasma potential at the point (z, r) in units of the electron temperature. The value of η is arbitrarily set at zero at the midplane and increases to the value γ at the coils. The spatial properties of the mirror system require the potential η to satisfy Poisson's equation, which is written

$$h_e^2 \nabla^2 \eta = \frac{n_i - n_e}{n_{0e}} \tag{12}$$

where n_i and n_e are the density of the ions and electrons respectively at the point (z, r_0) and n_{0e} is the density of the electrons at the point $(0, r_0)$. The symbol h_e represents the Debye length at the midplane of the system defined by

$$h_e^2 = \frac{kT_e}{4\pi n_{0e} e^2}\,. \tag{13}$$

Equation (12) is subject to the conditions the η is equal to zero at the midplane and equal to γ at the coils.

We shall estimate the densities in equation (12) by assuming physically reasonable distribution functions in phase space at the midplane for two species. From these distributions at the midplane the densities at any point in the system may be evaluated. This procedure is discussed briefly in the Appendix in order not to interrupt the continuity of our present discussion.

The ions at the midplane are assumed to be monoenergetic and distributed in angle in what has been termed the normal mode. This angular distribution, which is a function of the loss cone μ_c, has been found to be invariant under the effects of collisions in a magnetic mirror system (BENDANIEL, 1960). The assumption of a monoenergetic speed distribution is somewhat inconsistent with this angular distribution, insofar as collisions would likewise tend to broaden the speed distribution. The resultant speed distribution would then not be monoenergetic but would have a spread in speeds about a most probable speed (BENDANIEL, 1960). Use of the monoenergetic speed distribution is made here in the interests of calculational simplicity. The results to be derived are therefore subject to some errors when the ion temperature and the electron temperature are nearly equal but should become exact when the ion temperature is large in comparison with the electron temperature.

The electrons at the midplane are assumed to have a Maxwellian speed distribution which is isotropic, except that those electrons whose parallel component of velocity is sufficient to overcome the potential barrier are presumed to be lost immediately and are missing from the distribution function. The ions and the electrons are presumed to have different temperatures. The somewhat complicated expressions for the density of the ions and electrons at each point in the mirror system based on these assumptions are developed in the Appendix. The density of the ions is shown to be an explicit function only of the variables R_p and R_e. The density of the electrons at each point in the system is an explicit function of the variables η, γ, R and R_a. Hence the right-hand side of equation (12) is a function of η and thus this equation must be solved by an iterative procedure.

The overall potential γ is determined from the independent condition that the ion and electron relative loss rates are balanced. Particle loss is caused by collisions. In the case of the ions collisions tend to cause them to enter the loss cone. In the case of electrons, collisions tend to fill the tail of the Maxwellian distribution. We may write, at any point z, r in the system

$$\frac{1}{n_e}\frac{dn_e}{dt} = -K_e\left[(n_e)_{\text{av}} + (n_i)_{\text{av}}\right] = \frac{1}{n_i}\frac{dn_i}{dt} = -K_i(n_i)_{\text{av}} \tag{14}$$

The coefficients K_e and K_i are the loss rate constants.

The densities $(n_e)_{av}$ and $(n_i)_{av}$ are suitable average densities encountered by each species around their trajectories. Equation (14) implies that the ions are affected only by ion-ion interactions. The high-energy electrons, on the other hand, are affected by interactions with both ions and electrons.

The densities are sharply peaked at the centre section of the machine; hence, we may assume that $(n_e)_{av} = n_{0e}$, $(n_i)_{av} = n_{0i}$ in equation (14), where n_{0e} and n_{0i} are the densities at the midplane of the electrons and ions respectively.

For the parameter K_i, it may be shown that

$$K_i \approx \left(\frac{1}{T_i}\right)^{3/2} \frac{N(N+1)}{2} \frac{1}{(m_i)^{1/2}} C_1. \qquad (15)$$

The eigenvalue N depends only on the effective loss cone μ_c. A plot of N vs. μ_c is given on Fig. 1.

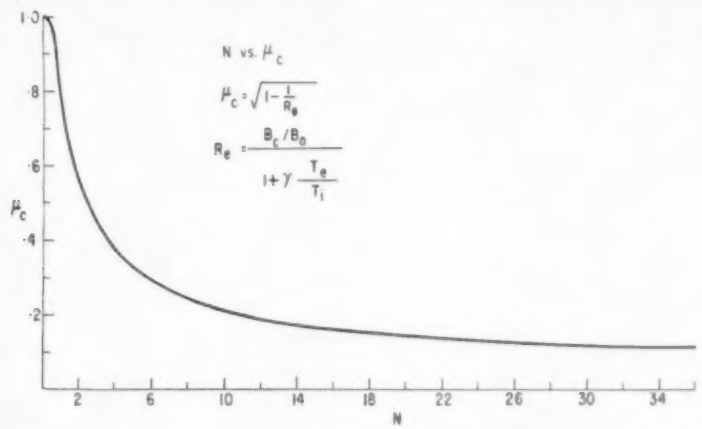

FIG. 1.—The variation of the eigenvalue N with the cosine of the loss cone angle, μ_c, is shown. The cosine of the loss cone angle is a function of the mirror ratio B_c/B_0, the total plasma potential γ and the ratio of the electron and ion temperatures T_e/T_i. The 'effective mirror ratio' is shown on Fig. 3.

Expression (15) emerges from a correlation of detailed investigations of mirror loss problems. This result is insensitive to the speed distribution of the ions and remains valid in the presence of a plasma potential provided the eigenvalue N is determined from the effective loss cone.

ROSENBLUTH *et al.* (1957) have shown that the tail of an incomplete Maxwellian at temperature T_e (such as the electron distribution assumed here) tends to fill from the available particles at a rate determined by the SPITZER energy relaxation time at that temperature. The leakage of electrons into the excluded tail of the distribution is accounted for in the expression for the loss rate constant K_e.

$$K_e \approx \left(\frac{1}{T_e}\right)^{3/2} \frac{\exp[-\gamma]}{(m_e)^{1/2}} \frac{1}{\gamma^{3/2}} C_1. \qquad (16)$$

In equations (15) and (16)

$$C_1 = \text{constant}. \qquad (17)$$

Putting these expressions into equation (14), we obtain the relation for γ.

$$\gamma = \frac{T_i}{T_e}\left(\frac{2}{N(N+1)}\right)^{2/3}\left(\frac{m_i}{m_e}\right)^{1/3}\left(\frac{n_{0e}+n_{0i}}{n_{0i}}\right)^{2/3} \times \exp\left[-\frac{2\gamma}{3}\right] \qquad (18)$$

or

$$\gamma + \frac{3}{2}\ln\gamma - \ln\left[\frac{2}{N(N+1)}\frac{n_{0e}+n_{0i}}{n_{0i}}\right] = \frac{3}{2}\ln\left(\frac{T_i}{T_e}\right) + \frac{1}{2}\ln\left(\frac{m_i}{m_e}\right). \qquad (19)$$

This relation is more general than KAUFMAN's result given by equation (2), because it accounts for more general magnetic field variations, and because it allows for differences between the electron and the positive ion loss processes. The additional terms on the left-hand side serve to prevent the value of γ from being unreasonably large for small mirror ratios. The potential γ as defined by equation (19) is a function of the temperature ratio T_i/T_e and the mirror ratio B_c/B_0.

We remark here that there is a fundamental limitation on the ratio T_i/T_e. The energy sharing time between two species is, according to SPITZER (1956):

$$t_{eq} \approx t_R\left(\frac{m_i}{m_e}\right)^{1/2}\left(\frac{T_e}{T_i}\right)^{3/2} \qquad (20)$$

and from mirror loss calculations, the containment

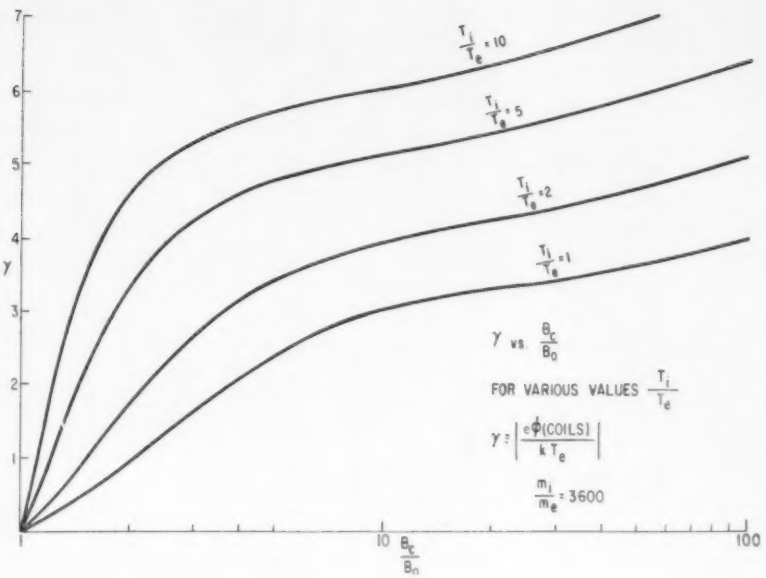

FIG. 2.—The variation of the total difference in potential between the centreplane of the mirror system and the coils is shown in units of the electron temperature. The ion-electron mass ratio is 3600.

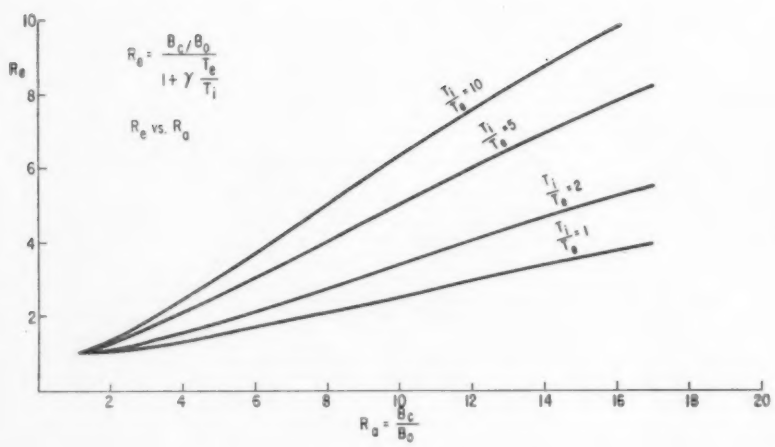

FIG. 3.—The 'effective mirror ratio' as a function of the actual mirror ratio B_c/B_0 is shown for various values of the ratio of ion to electron temperatures.

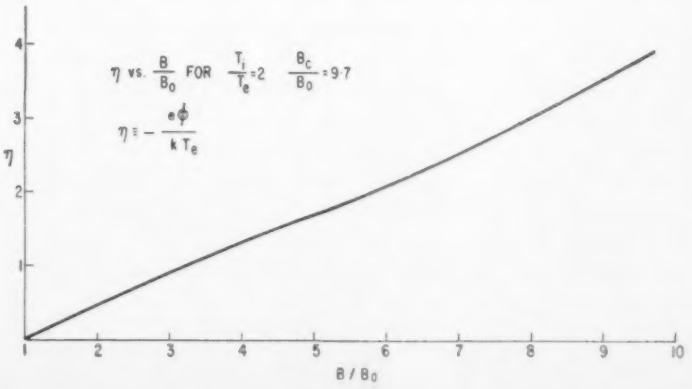

FIG. 4.—The spatial variation of the dimensionless plasma potential η as a function of the magnitude of the magnetic field, for a mirror ratio B_c/B_0 of 9·7 and a ratio of the ion to electron temperatures of 2.

time for the ions is given by

$$t_R \approx \frac{N(N+1)}{2}\, t(\text{containment}). \tag{21}$$

For

$$t_{eg} \geqq t(\text{containment}) \tag{22}$$

the following condition must obtain for ions and electrons

$$\frac{T_i}{T_e} \leqq \left[\frac{N(N+1)}{2}\right]^{2/3} \left(\frac{m_i}{m_e}\right)^{1/3} \tag{23}$$

otherwise electron-ion interactions will produce this condition in a time less than the average residence time of the ions. For $T_i/T_e < 10$ the separate temperature can certainly be preserved. More recent calculations (KILLEEN and HECKROTTE, 1960), indicate that the energy sharing between the ions and electrons may be somewhat slower than predicted by equation (20). This then may result in a less restrictive condition on the maximum possible ratio of ion and electron temperatures than indicated by equation (23).

We shall now describe an iterative procedure for solving equation (12). We assume $n_0 = n_{0e} = n_{0i}$. We see thus that equation (12) may be written

$$\frac{\Delta n}{n_0} = h_e^2 \nabla^2 \eta \tag{24}$$

where

$$\frac{\Delta n}{n_0} = \frac{n_i - n_e}{n_0} = G_i(R_p, R_e) - \exp(-\eta)\, G_e(\eta, \gamma, R, R_a) \tag{25}$$

The functions G_i and G_e are given by equations (42) and (49). In the problem at hand, we first assume that the action of the magnetic field dominates; the right-hand side of equation (24) is thus of order $h_e^2\gamma/L^2$, where L is the characteristic length of change of the magnetic field. Thus

$$\frac{\Delta n}{n_0} \approx \gamma\left(\frac{h_e^2}{L^2}\right). \tag{26}$$

Now, in mirror systems h_e^2/L^2 is of order 10^{-6} to 10^{-16} while γ is of order 10. Thus we may solve for the potential η using the equation

$$\Delta n(\eta) = 0, \tag{27}$$

as a first (and excellent) approximation. Physically, equation (27) means that when the Debye length is much smaller than the characteristic length of change of the magnetic field, the potential distribution must be such as to maintain charge neutrality. We may further note that since Δn depends on position in the system only through its dependence on $B/B_0 = R$, the solution for η obtained from equation (27) will be an explicit function only of the quantity R and the parameters R_a and T_i/T_e. To solve equation (27) for η, we use an iteration scheme, as follows. Choosing a value of R_a and T_i/T_e, we find γ from equation (19) and R_e from equation (1). We now choose a value of R_p somewhere between 1 and R_e and solve equation (27) by assuming G_e is equal to unity. With this value of η, say $\eta^{(1)}$, we next compute

$$R^{(1)} = R_p\left(1 + \eta^{(1)}\frac{T_e}{T_i}\right). \tag{28}$$

With this value of $R^{(1)}$ we evaluate $G_e^{(1)}$ and hence obtain a new estimate of η. Thus on the kth iteration, one would have

$$\eta^{(k)} = -\ln\left[\frac{G_i}{G_e^{(k-1)}}\right] \tag{29}$$

where

$$G_e^{(k-1)} = G_e(\eta^{(k-1)}, \gamma, R^{(k-1)}, R_a). \tag{30}$$

This iteration converges rapidly.

Having an estimate for η, one can go back and recompute the quantity $[h_e^2\, \nabla^2\eta]$ for a given magnetic mirror configuration and in this way, it may be verified that the correction to η arising from the right side of equation (24) is indeed small for cases of interest. A self-consistent solution is thus established and the assumption that the magnetic field dominates proves valid. This solution for η will describe a mirror system after the asymptotic angular distribution for the ions has been established.

Figure 2 gives a plot of γ as R_a for various values of T_i/T_e and for $m_i/m_e = 3600$. Figure 3 gives a plot of R_e versus R_a for various values of T_i/T_e. Figures 4 and 5 give plots of the potential η as a function of R for two cases of interest.

From Fig. 2 it may be seen that the plasma potential set up by the initial unbalance in the ion and electron loss rates will be several times the electron temperature. This result was established by KAUFMAN (1956).

Figure 3 shows that the effect of this potential is to diminish the mirror ratio of the system by a significant factor, the exact amount depending on the ratio B_e/B_0 and the ratio T_i/T_e. The loss of containment efficiency of the system, while not quite as severe as predicted by KAUFMAN, is nevertheless important.

Figures 4 and 5 indicate that an electric field will exist inside the mirror system, and thus affect the motion and containment of the ions severely. These curves were computed using the method outlined in the Appendix and equation (27) *et seq.* A first approximation to these curves may be obtained by correcting the results of KAUFMAN, (1956) for the ion temperature effects.

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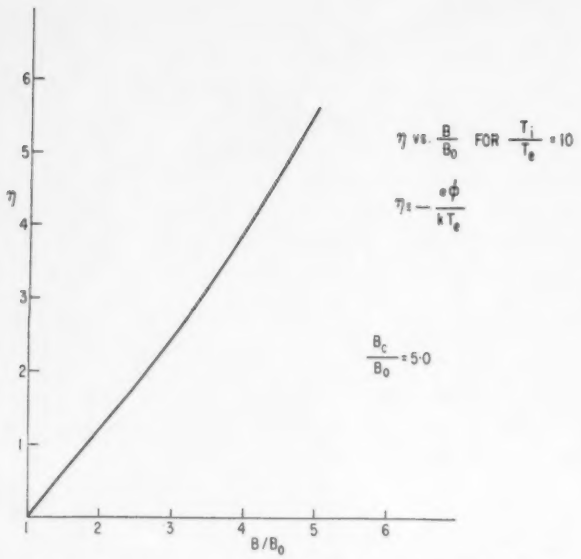

FIG. 5.—The spatial variation of the dimensionless plasma potential η as a function of the magnitude of the magnetic field, for a mirror ratio B_c/B_0 of 5·0 and a ratio of the ion to the electron temperatures of 10.

Finally, we may note the following qualitative conclusions. For a range of values of B_c/B_0 and T_e/T_i, η is nearly a linear function of B/B_0 (cf. Figs. 4 and 5) and $\nabla^2\eta$ becomes approximately proportional to $\nabla^2 B$. According to equation (24), this means that the residual plasma charge, after the plasma potential is established, will be positive where $\nabla^2 B$ is positive and negative where $\nabla^2 B$ is negative. This fact allows us to see by inspection that the net charge is positive in the regions of smaller magnetic field (i.e. 'inside' the mirror system) and negative near the coils; there is a net outward displacement of negative charge due to the tendency for more rapid loss of the electrons.

APPENDIX

The densities of the ions and electrons at any point in the mirror system are computed as follows.

The distribution function for each species of particle anywhere in a cylindrically symmetric mirror machine may be assumed to be independent of the azimuthal angle ϕ and is to be obtained from the relation

$$f(v, \mu, z, r) = f_0(v_0, \mu_0, 0, r_0) \tag{31}$$

using the transformations implied by equations (3), (4), and (5). Equation (31) assumes that the effects of collisions in a single period of the particle motion are small enough to be neglected and that a quasi-steady-state exists in the mirror machine. Using relation (31), one may obtain the density of each species in the mirror system from

$$n = \int_0^\infty \int_0^1 4\pi f v^2 \, dv \, d\mu. \tag{32}$$

For the ions we have assumed:

$$f_{0i} = n_{0i}(r_0)\, \delta\left(v_0 - \left[\frac{2kT_i}{m_i}\right]^{1/2}\right) M_1(\mu_0) \frac{1}{D_1} \tag{33}$$

where n_{0i} is the density of ions in the ring r_0 at the midplane

$$\delta\left(v_0 - \left[\frac{2kT_i}{m_i}\right]^{1/2}\right) \tag{34}$$

is a Dirac function for the speed distribution which is normalized so that

$$4\pi \int_0^\infty v_0^2\, \delta(v_0)\, dv_0 = 1. \tag{35}$$

The angular distribution $M_1(\mu_0)$ is (BENDANIEL 1960)

$$M_1(\mu_0) = 1 + \sum_{l=1}^{\infty} a_l \mu_0^{2l} \quad \text{for} \quad \mu_0 \leqq \mu_c \tag{36}$$

where μ_c is given by equation (11) and

$$a_l = \prod_{k=0}^{l-1} \frac{\left(-\frac{N}{2} + k\right)\left(\frac{N+1}{2} + k\right)}{(k+1)\left(k + \frac{1}{2}\right)} \tag{37}$$

and N is given in Fig. 1 as a function of μ_c.

The quantity D_1 is a normalization constant given by

$$D_1 = \int_0^{\mu_c} M_1 \, d\mu_0 \tag{38}$$

$$= \left(1 - \frac{1}{R_e}\right)^{1/2} + \frac{1}{2} \sum_{l=1}^{\infty} \frac{a_l \left(1 - \frac{1}{R_e}\right)^{l+\frac{1}{2}}}{l + \frac{1}{2}} \tag{39}$$

The density at the point z, r is given by

$$\frac{n_i}{n_{0i}} = \frac{1}{D_1} \int_0^{\sqrt{1 - \frac{1}{R_p}}} M_1(\mu_0)\, du \tag{40}$$

$$\frac{n_i}{n_{0i}} = \frac{1}{D_1} \int_{\sqrt{1 - \frac{1}{R_p}}}^{\sqrt{1 - \frac{1}{R_e}}} M_1(\mu_0) \frac{R_e}{R_p} \frac{\mu_0 \, d\mu_0}{1 - \frac{R_e}{R_p}(1 - \mu_0^2)} \tag{41}$$

or

$$\frac{n_i}{n_{0i}} = \frac{F_1(R_p, R_a)}{D_1(R_e)} \equiv G_i(R_p, R_e) \tag{42}$$

where

$$F_1 = \left(\frac{1}{R_p} - \frac{1}{R_e}\right)^{1/2} (R_p)^{1/2} + \frac{1}{2}\sum_{l=1}^{\infty} a_l \left[\sum_{j=0}^{l} \frac{\left(\frac{1}{R_p} - \frac{1}{R_e}\right)^{l-j+\frac{1}{2}} (R_p - 1)^j (R_p)^{-j}}{l - j + \frac{1}{2}} \times \frac{l!}{j!\,(l-j)!}\right] (R_p)^{1/2} \quad (43)$$

For the electrons, we have assumed

$$f_{0e} = n_{0e}(r_0)\, U_m(v_0)\, A(v_0, \mu_0)\, \frac{1}{D_2} \quad (44)$$

where n_{0e} is the density of electrons in the ring r_0 at the midplane

$$A(v_0, \mu_0) = 1 \text{ when } \mu_0 \leqq \sqrt{1 - \frac{1}{R_a}} \quad (45)$$

$$= 1 \text{ when } \mu_0 > \sqrt{1 - \frac{1}{R_a}} \text{ and } v_0 < v_m$$

$$= 0 \text{ when } \mu_0 > \sqrt{1 - \frac{1}{R_a}} \text{ and } v_0 > v_m$$

where

$$\tfrac{1}{2} m v_m^2 = e\phi_c\,(1 - R_a(1 - \mu_0^2))^{-1}. \quad (46)$$

The quantity D_2 is a normalization constant, given by

$$D_2 = 2\pi \left(2\frac{kT_e}{m}\right)^{3/2} \left[E_1(\gamma^{1/2}) + \left(1 - \frac{1}{R_a}\right)^{1/2} \times \exp(-\gamma)\, E_T\left[\left(\frac{\gamma}{\left(1 - \frac{1}{R_a}\right)}\right)\right]\right] \quad (47)$$

where $E_1(x) = \int_0^x \exp[-x^2]\,dx$;

$$E_T(x) = e^{+x^2}\left(\frac{\sqrt{\pi}}{2} - E_1(x)\right). \quad (48)$$

The density of electrons at the point r, z is given by

$$\frac{n_e}{n_{i0}} = \exp(-\eta)\,\frac{F_2}{D_2} \equiv \exp(-\eta)\, G_e(\eta, \gamma, R, R_a) \quad (49)$$

where F_2 is in a form exactly the same as D_2 with the replacements

$$\gamma - \eta \text{ for } \gamma \quad (50)$$

$$\frac{R_a}{R} \text{ for } R_a.$$

Acknowledgments—The author wishes to express his thanks to Drs. R. Post and S. Zwick for their many useful discussions of this material.

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EQUILIBRIUM CONFIGURATION OF THE PLASMA IN THETA DISCHARGES WITH REVERSED TRAPPED MAGNETIC FIELDS

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Abstract—The equilibrium state of a long tubular plasma with reversed trapped magnetic fields is discussed. It is shown that an equilibrium position stable to longitudinal perturbations exists for each value of the ratio of external to trapped flux providing the cross-sectional area of the plasma is less than half the total area of the system.

Variations in the containing magnetic field give rise to variations in the length and the cross-sectional area of the plasma tube. With observations confined to the end-view of the plasma and assuming that the tube did not vary in length, the observed variation in cross-sectional area would lead one to an assumption of an apparent value of $\gamma = 4$ for the plasma.

1. INTRODUCTION

RECENT experiments (GREEN, 1960) have indicated the possibility of achieving stable confined plasmas with theta discharges in a certain range of ambient pressures. The suggested explanation is that the plasma is contained in a set of closed field lines which form around it, due to the existence of a magnetic field core inside the plasma which is in the opposite direction to the confining field (Fig. 1). This magnetic field configuration could be achieved as follows.

A theta discharge is one where current flows in a single turn coil around a cylinder containing the gas being studied and the pinch occurs because currents are induced in the gas by transformer action with the external coil. It is often observed that breakdown does not fully occur on the first half-cycle of the oscillatory current pulse in the coil, but occurs on the second half-cycle. However, the gas is supposed to be sufficiently conducting during the first half-cycle for some of the magnetic field induced in the gas by the current flowing in the coil to be trapped and give rise to the field configuration shown in Fig. 1 on the second half-cycle.

The purpose of this paper is to analyse the condition for equilibrium of this configuration. The technique used is to estimate the energy U of magnetic field and plasma and to determine equilibrium configurations by the condition that U is stationary and the condition for stability by the condition that U is a minimum in a stable system.

The general solution to this problem is very difficult but it is possible to obtain a simple analytic solution for the extreme case when the plasma is much longer than its diameter.

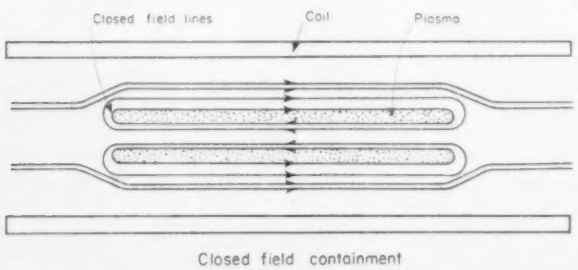

FIG. 1.—Confinement of plasma with reversed trapped magnetic field.

2. CONDITIONS OF EQUILIBRIUM

We assume the length l of the plasma is so great that the contribution to the magnetic energy of the field bending round the ends of the plasma is negligible. We examine the stability of the system to changes in l and it is clear that the changes in magnetic energy due to changes in field configuration at the ends can be made negligible in comparison with changes in field energy due to variations in l providing l is sufficiently great.

Consider a typical cross section of tube occupied by the plasma.

We denote the cross-sectional area of the discharge tube A, the area occupied by the plasma a_p and the area occupied by the trapped magnetic field inside the plasma tube by a_0. Then the area occupied by the field outside the plasma is simply $A - a_p - a_0$.

If the total length of the discharge tube is L and the length occupied by the plasma is l we have, apart from constant and effects,

$$U = \underbrace{p(A - a_p)l + PA(L - l)}_{\text{magnetic energy}} + \underbrace{\frac{pa_p l}{\gamma - 1}}_{\text{plasma energy}}.$$

We look for stationary values of U subject to the equation of conservation of flux

$$p(A - a_p - 2a_0)^2 = PA^2$$

and the isentropic compression equation for the plasma $p(a_p l)^\gamma$ is constant. It is convenient to introduce the dimensionless quantities

$$\mu = a_p/A$$

and

$$k = (A - a_p - a_0)/a_0.$$

It will be seen that μ is the fraction of the cross-sectional area occupied by the plasma and k is the ratio of the flux outside the plasma to the trapped flux.

The above three equations now become, respectively,

$$U = pAl\left\{1 - \mu - (1-\mu)^2\left(\frac{k-1}{k+1}\right)^2 + \frac{\mu}{\gamma - 1}\right\} + PAL \quad (1)$$

$$\frac{dp}{p} - 2\frac{d\mu}{1-\mu} = 0 \quad (2)$$

and

$$\frac{dp}{p} + \gamma\frac{d\mu}{\mu} + \gamma\frac{dl}{l} = 0. \quad (3)$$

Looking for stationary values of U subject to (2) and (3) we get

$$\begin{aligned}\frac{dU}{dl} &= 0\\ &= pA\left\{1 - \mu - (1-\mu)^2\left(\frac{k-1}{k+1}\right)^2 + \frac{\mu}{\gamma-1}\right\}\\ &\quad\times\left\{1 + \frac{l}{p}\frac{dp}{dl}\right\} + pAl\left\{-1 + 2(1-\mu)\right.\\ &\quad\left.\times\left(\frac{k-1}{k+1}\right)^2 + \frac{1}{\gamma-1}\right\}\frac{d\mu}{dl}\\ &= pA\left[1 - \left(\frac{k-1}{k+1}\right)^2(1-\mu)^2 - 2\mu\right] \quad (4)\end{aligned}$$

or

$$\frac{k-1}{k+1} = \frac{\sqrt{1-2\mu}}{1-\mu}. \quad (5)$$

This relationship is plotted in Fig. 2. It will be seen that there are no equilibrium states corresponding to $\mu > \frac{1}{2}$; that is where the plasma occupies more than half the cross-sectional area of the tube.

3. STABILITY OF LONGITUDINAL OSCILLATIONS

We have the condition that, for stability U must be a minimum, i.e. for stability

$$\frac{d^2U}{dl^2} > 0.$$

Differentiating (4) we have

$$\begin{aligned}\frac{d^2U}{dl^2} &= A\frac{dp}{dl}\left\{1 - \left(\frac{k-1}{k+1}\right)^2(1-\mu)^2 - 2\mu\right\}\\ &\quad + pA\left\{2\left(\frac{k-1}{k+1}\right)^2(1-\mu) - 2\right\}\frac{d\mu}{dl}.\end{aligned}$$

Using (2) and (3) this becomes

$$\frac{d^2U}{dl^2} = \frac{2\gamma\mu^2 pA}{l\{2\mu + \gamma(1-\mu)\}}. \quad (6)$$

It will be seen that $\frac{d^2U}{dl^2}$ is always positive and the equilibrium is therefore stable.

4. VARIATION OF LENGTH DURING THE COMPRESSION PHASE

In a theta discharge it is normal for the current in the external coil to vary almost sinusoidally. This means that the ratio k of external to trapped flux is constantly changing and if the rate of change of flux is small compared with the time required for a sound wave to traverse the system then the plasma will pass through a series of equilibrium configurations. On the assumption that no particles are lost from the system it is possible to calculate the variation in length of the plasma throughout the compression phase.

The isentropic compression equation is

$$p(a_p l)^\gamma = \text{constant}$$

and since the trapped reversed flux is constant

$$pa_0^2 = \text{constant}.$$

Hence $l \propto a_p^{-1} a_0^{2/\gamma}$.

In dimensionless units this becomes

$$l \propto \mu^{-1}(1-\mu)^{2/\gamma}(1+k)^{-2/\gamma}. \quad (7)$$

The variation of l with flux ratio k is shown in Fig. 3. It will be seen that appreciable changes in length will occur.

(N.B. It has been assumed here that appreciable changes in flux take place in a time much greater than the time for a sound wave to travel the length l of the plasma. If this condition is not satisfied, pressure waves will be generated and will travel along the length of the plasma.)

In some experimental work the compression of the plasma in the external magnetic field is observed. An apparent polytropic index G may be determined from the relationship:

$$p\mu^G = \text{constant}. \quad (8)$$

The value G will differ from γ since no account is taken

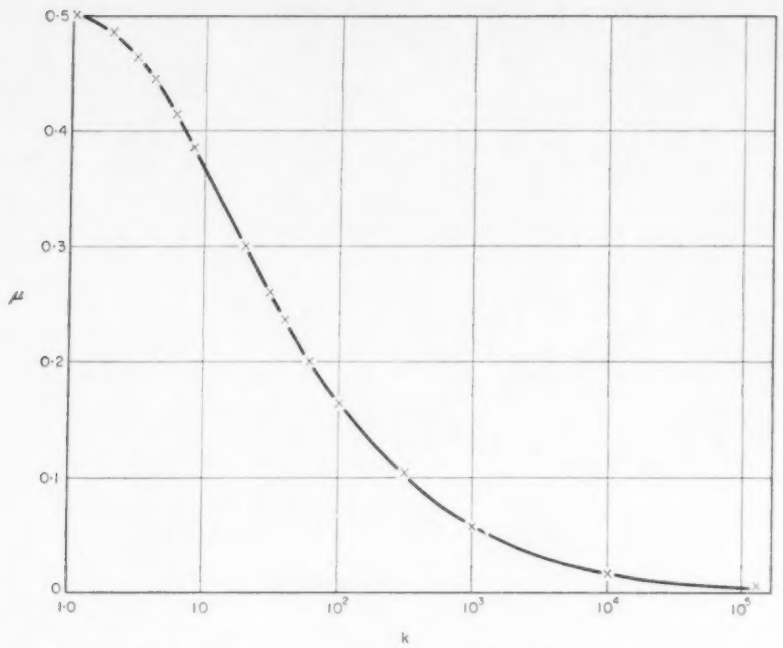

FIG. 2.—Equilibrium values of fractional area of cylinder occupied by the plasma as a function of the ratio of external to trapped flux, k.

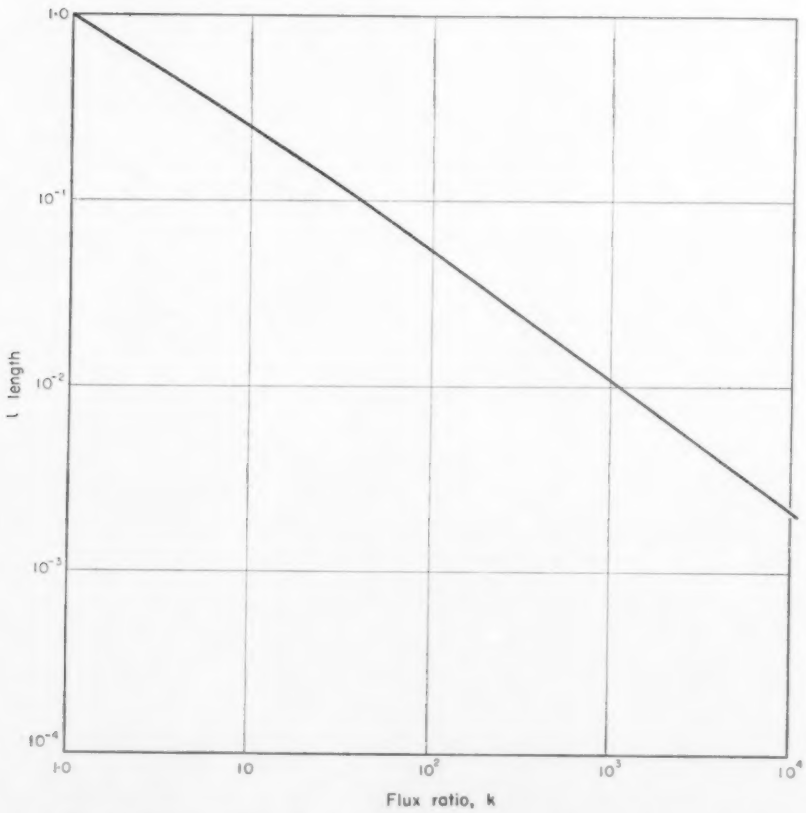

FIG. 3.—Variation of length of plasma tube as a ratio of external to trapped flux, k, is varied.

of changes in length. The solution of (5) at large k (small μ) is

$$\mu = 2k^{-1/2}$$

and, for small μ, using (7)

$$l\mu^{1-4/\gamma} = \text{constant}.$$

Since changes are adiabatic, $p(l\mu)^{\gamma}$ is constant

or $$p\mu^4 = \text{constant}. \qquad (9)$$

Hence, by comparison with equation (8) we find that the apparent polytropic index G is 4—a value independent of the plasma γ.

5. CONCLUSIONS AND DISCUSSION

We have shown that, on the simple model of a very long plasma, there is an equilibrium length of the plasma stable to longitudinal perturbations for each value of the ratio of internal to trapped flux providing that the cross-sectional area of the plasma is less than half the total area of the system. The general stability of the system has not been considered—it is clear that the field curvature at the ends of the plasma tube would be expected to give rise to instabilities growing from that point.

Slow variations in the containing magnetic field give rise to corresponding variations in the length of the plasma column. GREEN found his experimental results lie close to the curve for $a\gamma = (5/3)$ high β plasma. In this work he ignored length variations of the plasma and according to our theory his results should therefore have laid close to $a\gamma = 4$ curve if he had a high β. There would seem to be three possible explanations of this difference.

(a) Our theory is invalid because GREEN's experiment involved a plasma where length was small compared with the diameter.

(b) GREEN's rate of compression was so high that our quasi-equilibrium theory does not apply. Under these circumstances one would expect to see shocks propagating longitudinally along the plasma. We have worked out the Rankine-Hugoniot relations appropriate to thin plasmas confined by magnetic fields and this will be published shortly.

(c) GREEN had a low β plasma—i.e. the particles had become inter-mixed with some of the magnetic field.

At the moment there is insufficient evidence to decide between the three reasons for the discrepancy.

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L'HYPOTHESE DE PRESSION SCALAIRE DANS LA DYNAMIQUE DES PLASMAS SANS COLLISION

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Sommaire—On étudie la validité de l'hypothèse de pression scalaire pour un plasma sans collisions en supposant pour chaque espèce une fonction de distribution isotrope autour d'une vitesse moyenne. La solution exacte du système d'équations formé par l'équation de Boltzmann sans collisions et les équations de Maxwell est recherchée dans une approximation non relativiste cohérente. Ce système s'intègre mais les conditions imposées sont incompatibles avec des solutions dynamiques du problème. Le pinch hélicoïdal stationnaire étudié dans un travail précédent est la seule solution possible. Des hypothèses magnétohydrodynamiques simples (pressions scalaires, équations d'état en l'absence de transport de chaleur) conduisent aux mêmes résultats.

Abstract—The validity of the assumption of a scalar pressure is studied for a collisionless plasma, assuming the isotropy of the velocity distribution functions around a mean velocity. The exact solution of the system of equations formed by the collisionless Boltzmann's equation and the Maxwell's equations is investigated in a consistent non-relativistic approximation. Dynamical solutions are found to be impossible. The helical static pinch considered in a previous work is the only case. The magnetohydrodynamical assumptions of scalar pressure, state equation, without heat transport lead to the same result.

INTRODUCTION

ON décrit souvent le comportement d'un gaz complètement ionisé par des équations analogues à celles qu'on utilise en hydrodynamique: équations de continuité et équations sur les vitesses macroscopiques. Pour compléter le système d'équations ainsi obtenu on fait en général l'hypothèse de pressions scalaires satisfaisant à des équations d'état (SPITZER, 1956).

De plus les collisions sont souvent négligées pour des plasmas chauds et raréfiés (conductivité infinie). Moyennant cette série d'approximations et d'hypothèses on utilise de telles équations magnétohydrodynamiques pour résoudre de nombreux problèmes statiques ou dynamiques: équilibre (KRUSKAL *et al.*, 1958), stabilité (BERNSTEIN *et al.*, 1958; JOHNSON *et al.*, 1958), ondes (GARDNER *et al.*, 1958), etc. . . . . La justification de ces hypothèses, faites pour des raisons de simplicité est une question encore en suspens.

L'hypothèse de pression scalaire a été empruntée à la théorie des gaz habituels; on admet généralement qu'elle est justifiée s'il y a suffisamment de collisions. D'une façon plus précise, si les gradients des diverses grandeurs sont faibles à l'échelle du libre parcours moyen et si l'évolution macroscopique du fluide est lente par rapport aux temps de relaxation des anisotropies de vitesses, on peut admettre pratiquemment que la fonction de distribution des vitesses est isotrope autour d'une vitesse moyenne et donc que la pression est scalaire. Ces temps de relaxation ont été calculés dans des cas particuliers (SPITZER, 1956; DELCROIX, 1959). Ils sont de l'ordre du temps de collision caractéristique de la conductivité, du moins pour l'espèce électronique. La justification d'une pression scalaire par les collisions apparait donc difficilement compatible avec l'hypothèse d'une conductivité infinie.

Mais le plus grave est que les théories à pression scalaire sont couramment utilisées pour décrire des phénomènes comme les instabilités magnétohydrodynamiques dont la durée est beaucoup plus petite qu'un temps de collision. Dans un tel cas il est manifestement impossible de faire appel aux collisions pour justifier une telle hypothèse. La question se pose donc de savoir si l'existence d'une pression scalaire peut être compatible avec un faible taux de collisions ou à la limite avec l'absence de collisions. C'est ce que l'on se propose d'étudier dans cet article.

On peut aussi décrire un plasma sans collisions par les fonctions de distribution des vitesses des particules qui satisfont à l'équation de Boltzmann sans second membre. La façon la plus simple d'obtenir alors des pressions scalaires est de supposer l'isotropie de chacune de ces fonctions autour de la vitesse macroscopique correspondante. On adopte ici cette hypothèse qui a déjà été utilisée pour l'étude du pinch hélicoïdal stationnaire; la théorie de ce pinch avait donné une relation bien confirmée par les résultats expérimentaux (REBUT, 1961). De plus on montrera que cette hypothèse d'isotropie, *a priori* plus sévère que celle des pressions scalaires équivaut, quant à cette étude, aux hypothèses purement magnétohydrodyna-

miques suivantes: pressions scalaires, équations d'état, absence de transport de chaleur et bien entendu de collisions. Celles-ci apparaissent souvent dans la littérature.

On étudie donc un plasma à deux espèces, électronique et ionique, complètement ionisé et sans collisions en supposant l'isotropie des fonctions de distribution de vitesses autour des vitesses moyennes (ce qui équivaut aux hypothèses magnétohydrodynamiques mentionnées.) L'équation de Boltzmann sans second membre est utilisée dans l'approximation non relativiste; cette approximation conduit, par l'intermédiaire de équations de Maxwell, à la quasi neutralité électrique. Aucune autre hypothèse n'est nécessaire et en particulier on ne néglige pas les termes quadratiques dans les vitesses macroscopiques comme on le fait habituellement. Le système d'équations est complet et le calcul se mène jusqu'au bout. La seule solution obtenue est stationnaire.

Dans la première partie on examine les équations qui ne font intervenir qu'un seule espèce de particules. Le mouvement particulaire est presque entièrement déterminé par la solution de l'équation de Boltzmann et par la forme très particulière du champ de vitesses: celui d'un solide dilaté uniformément. Des invariants dits temporels apparaissent: ce sont des invariants en suivant le mouvement.

Le couplage des deux expèces par l'intermédiaire de ces invariants temporels et des équations de Maxwell permet d'introduire des invariants d'espace. Ceci fait l'objet de la seconde partie. On introduit une approximation non relativiste et on montre que ces différents invariants sont incompatibles avec l'existence d'une solution dynamique du problème.

Il ne reste alors que le cas stationnaire étudié en troisième partie. C'est celui du pinch hélicoïdal et l'on retrouve la relation dejà vérifiée par l'expérience (REBUT, 1961).

On établit ensuite qu'un ensemble d'hypothèses purement magnétohydrodynamiques conduit aux mêmes résultats que l'hypothèse d'isotropie.

Ces résultats sont également les mêmes si l'on suppose que les fonctions de distribution possèdent des anisotropies d'ordre 1.

A. EQUATIONS DU MOUVEMENT D'UNE SEULE ESPECE

1. *Résolution de l'Equation de Boltzmann*

Hypothèses et notations. Soit $f(\mathbf{r}, \mathbf{w}, t)$ la fonction de distribution des particules de masse m et de charge q dans l'espace des phases à 6 dimensions $\mathbf{r}$ et $\mathbf{w}$; on désignera par $\mathbf{E}(\mathbf{r})$ et $\mathbf{B}(\mathbf{r})$ les champs électriques et magnétiques appliqués au point $\mathbf{r}$. La fonction $f(\mathbf{r}, \mathbf{w}, t)$ satisfait à l'équation de Boltzmann sans collisions:

$$\frac{\partial f}{\partial t} + \mathbf{w} \,.\, \nabla_r f + \frac{q}{m}(\mathbf{E} + \mathbf{w} \times \mathbf{B}) \,.\, \nabla_w f = 0 \quad (1)$$

En prenant comme nouvelle variable de vitesse:

$$\mathbf{u} = \mathbf{w} - \mathbf{V}(\mathbf{r}, t). \quad (2)$$

où $\mathbf{V}(\mathbf{r}, t)$ représente la vitesse macroscopique moyenne des particules de l'espèce considérée, on fait l'hypothèse d'isotropie:

$$f(\mathbf{r}, \mathbf{w}, t) \equiv \mathbf{g}(\mathbf{r}, u^2, t) \quad (3)$$

La fonction $g(\mathbf{r}, u^2, t)$ est la fonction de distribution des vitesses autour de la vitesse moyenne $\mathbf{V}(\mathbf{r}, t)$; nous la supposons donc isotrope.

Equations générales. L'équation de Boltzmann (1) relativement à la nouvelle fonction g définie par (3) et la nouvelle variable de vitesse $\mathbf{u}$ s'écrit:

$$\frac{dg}{dt} + \left\{2\frac{\partial g}{\partial(u^2)}\left[-\frac{d\mathbf{V}}{dt} + \frac{q}{m}(\mathbf{E} + \mathbf{V} \times \mathbf{B})\right] + \nabla_r g\right\} \,.\, \mathbf{u} - 2\frac{\partial g}{\partial(u^2)}\,\mathbf{u} \,.\, \nabla_r \mathbf{V} \,.\, \mathbf{u} = 0 \quad (4)$$

avec

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + (\mathbf{V} \,.\, \nabla)$$

$\frac{d}{dt}$ représente classiquement la dérivée d'une grandeur en suivant le mouvement fluide.

$\mathbf{V}(\mathbf{r}, t)$ est une fonction *a priori* quelconque de la position et du temps. Les variables indépendantes sont $\mathbf{r}$, $\mathbf{u}$, t.

D'après la condition (3) $\mathbf{u}$ ne doit apparaitre que sous la forme u^2 dans l'équation (4), ce qui impose deux conditions:

–les termes quadratiques sont proportionnels à u^2:

$$\mathbf{u} \,.\, \nabla_r \mathbf{V} \,.\, \mathbf{u} \;\propto u^2$$

soit encore en décomposant le tenseur $\nabla_r \mathbf{V}$ en parties symétrique et antisymétrique:

$$\nabla_r \mathbf{V} \equiv \lambda(\mathbf{r}, t)\mathbf{I} + \Omega(\mathbf{r}, t) \times .$$

Ceci se traduit par les cinq relations:

$$\nabla_x V_x = \nabla_y V_y = \nabla_z V_z = \lambda$$
$$\nabla_i V_j + \nabla_j V_i = 0 \; (i \neq j) \quad (5)$$

et il reste alors:

$$\mathbf{u} \,.\, \nabla_r \mathbf{V} \,.\, \mathbf{u} = \lambda(\mathbf{r}, t)u^2 = \tfrac{1}{3}u^2 \,\nabla \,.\, \mathbf{V}$$

–les termes linéaires sont nuls

$$2\frac{\partial g}{\partial(u^2)}\left[-\frac{d\mathbf{V}}{dt} + \frac{q}{m}(\mathbf{E} + \mathbf{V} \times \mathbf{B})\right] + \nabla_r g = 0 \quad (6)$$

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L'équation (4) se réduit à:

$$\frac{dg}{dt} - \frac{2}{3}\frac{\partial g}{\partial(u^2)}\, u^2\, \nabla \cdot \mathbf{V} = 0. \tag{7}$$

Ces trois équations (5), (6), (7), sont équivalentes à l'équation de Boltzmann dans l'hypothèse considérée; elles s'intègrent toutes.

Intégration des équations (5). En différenciant le système (5) on obtient un système de 15 équations et en particulier

$$\frac{\partial^2 V_i}{\partial x_j \partial x_k} = 0 \quad i \neq j \neq k.$$
$$\frac{\partial^2 V_i}{\partial x_i^2} = -\frac{\partial^2 V_i}{\partial x_k^2} = -\frac{\partial^2 V_i}{\partial x_j^2}. \tag{8}$$

Le système (8) se résout aisément. En poursuivant l'intégration de proche en proche on trouve le champ de vitesses:

$$\mathbf{V} = \mathbf{b}r^2 - 2\mathbf{r}(\mathbf{b} \cdot \mathbf{r} + c) + \boldsymbol{\Omega} \times \mathbf{r} + \mathbf{d} \tag{9}$$

dans lequel les vecteurs $\mathbf{b}, \boldsymbol{\Omega}, \mathbf{d}$ et le scalaire c ne dépendent que du temps.

D'après cette équation

$$\nabla \cdot \mathbf{V} = -6c - 6\mathbf{b} \cdot \mathbf{r} = 3\lambda.$$

Un peu plus loin on démontrera que λ ne dépend que du temps. Dans ce cas $\mathbf{b} = 0$ et le champ de vitesses se réduit à

$$\mathbf{V} = \lambda\mathbf{r} + \boldsymbol{\Omega} \times \mathbf{r} + \mathbf{V}_0 \tag{10}$$

C'est celui du mouvement général d'un solide dilaté uniformément.

On remarquera:

$$\nabla \cdot \mathbf{V} = 3\lambda(t)$$
$$\nabla \times \mathbf{V} = 2\boldsymbol{\Omega}(t). \tag{10a}$$

Intégration de l'équation (6). L'équation de transport de la quantité de mouvement, obtenue à partir de l'équation de Boltzmann s'écrit:

$$\frac{\nabla p}{mn} = -\frac{d\mathbf{V}}{dt} + \frac{q}{m}(\mathbf{E} + \mathbf{V} \times \mathbf{B})$$

où $n(\mathbf{r}, t)$ est la densité de particules et p la pression; p est scalaire d'après l'hypothèse d'isotropie (3). On peut alors mettre (6) sous la forme

$$2\frac{\partial g}{\partial(u^2)}\frac{\nabla_r p(\mathbf{r}, t)}{m\ n(\mathbf{r}, t)} + \nabla_r g = 0 \tag{11}$$

Il suit nécessairement de la proportionnalité des gradients que:

$$g \equiv g(p, u^2, t)$$
$$n \equiv n(p, t).$$

La 2ème équation est plus générale qu'une équation d'état; elle dépend du temps.

(11) s'écrit alors:

$$2\frac{\partial g}{\partial(u^2)} + nm\frac{\partial g}{\partial p} = 0.$$

L'équation aux caractéristiques est:

$$d\frac{u^2}{2} = \frac{dp}{nm}.$$

En posant:

$$h = \int \frac{dp}{nm} \qquad \text{(Fonction de Helmholtz)},$$

soit:

$$\nabla h = -\frac{d\mathbf{V}}{dt} + \frac{q}{m}(\mathbf{E} + \mathbf{V} \times \mathbf{B}) \tag{12}$$

on obtient la solution générale de (11):

$$g \equiv g\left[\frac{u^2}{2} - h(\mathbf{r}, t), t\right]. \tag{13}$$

Intégration de l'équation (7). D'après (13) g n'est plus fonction que de deux variables *indépendantes* $x \equiv \frac{u^2}{2} - h(\mathbf{r}, t)$ et t. En effectuant ce changement de variables (7) devient:

$$\frac{\partial g(x, t)}{\partial t} + \frac{\partial g(x, t)}{\partial x}\left[-\frac{dh}{dt} - \frac{2}{3}(x + h)\nabla \cdot \mathbf{V}\right] = 0. \tag{14}$$

La quantité entre crochets ne doit dépendre que des deux variables x et t; elle est donc de la forme $\alpha(t)x + \beta(t)$; ceci impose que $\nabla \cdot \mathbf{V}$ ne dépende que du temps, résultat annoncé précédemment; les termes restant donnent:

$$\frac{dh}{dt} + 2\lambda h = 0 \quad \text{en posant} \quad \nabla \cdot \mathbf{V} = 3\lambda(t). \tag{15}$$

L'équation (14) se réduit à

$$\frac{\partial g}{\partial t} - 2\lambda x\frac{\partial g}{\partial x} = 0$$

qui a pour équations aux caractéristiques

$$\frac{dx}{x} = -2\lambda\, dt.$$

La solution générale est donc:

$$g \equiv g\left[\left(\frac{u^2}{2} - h(\mathbf{r}, t)\right)\exp + 2\mu(t)\right] \tag{16}$$

avec

$$\mu(t) = \int_{t_0}^{t} \lambda(t')\, dt'. \tag{17}$$

En résumé, l'hypothèse d'isotropie permet de résoudre complètement le problème du mouvement particulaire, si l'on connait l'évolution du champ de vitesses au cours du temps et les conditions initiales:

D'une part (équation 10), ce champ de vitesses n'est pas quelconque: c'est le champ le plus général dont le rotationnel et la divergence ne dépendent que du temps (champ d'un solide uniformément dilaté). Son évolution au cours du temps dépend des paramètres (équation 10) $\mathbf{V}_0(t), \boldsymbol{\Omega}(t)$ et $\lambda(t)$ (ou $\mu(t)$).

D'autre part la fonction de distribution g ne dépend plus que d'une seule variable (équation 16). Elle dépend de l'espace par l'intermédiaire de la seule fonction de Helmholtz h dont l'évolution est connue. En effet (15) permet d'écrire l'invariance de h en suivant le mouvement macroscopique:

$$h = h_0 \exp -2\mu(t). \tag{18}$$

La connaissance de h_0 et g_0, valeurs de h et g à l'instant initial, nous déterminera ainsi h et g en tout temps (équations 16 et 18) si on connait le champ $\mathbf{V}(\mathbf{r}, t)$ à tout instant. Toutes les propriétés microscopiques du gaz sont ainsi connues.

En particulier on obtient l'évolution des moments d'ordre pair de g, les seuls moments non nuls.

Posant $$M_{2n} = \int u^{2n} g \, 4\pi u^2 \, du$$

on a aisèment les invariants:

$$M_{2n} = M_{2n(0)} \exp -(2n + 3)\mu(t)$$

dont

$$M_0 \equiv n = n_0 \exp -3\mu(t)$$

$$M_2 \equiv p = p_0 \exp -5\mu(t)$$

et

$$\frac{d}{dt}(pn^{-(5/3)}) = 0.$$

Le mouvement particulaire étant maintenant connu, il reste à examiner le problème de l'existence des champs qui interviennent à la fois dans h (équation 12) et dans les équations de Maxwell. Les équations macroscopiques de couplage entre les deux espèces donneront les conditions de compatibilité. Il est d'abord utile de considérer les équations macroscopiques relatives à une espèce seule.

2. *Equations macroscopiques*

On s'attache aux équations macroscopiques suivantes qui ne font intervenir qu'une espèce:

$$\nabla h = -\frac{d\mathbf{V}}{dt} + \frac{q}{m}(\mathbf{E} + \mathbf{V} \times \mathbf{B}) \quad \text{(Fonctionde Helmholtz)} \tag{12}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{19}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{(Equations de Maxwell)} \tag{20}$$

En faisant agir l'opérateur rotationnel $\nabla \times$ sur (12) et en tenant compte de (20) nous obtenons l'équation classique pour tous les fluides à pression scalaire fonction de la densité:

$$\frac{\partial \mathbf{K}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{K}) \tag{21}$$

où

$$\mathbf{K} = m\nabla \times \mathbf{V} + q\mathbf{B} \tag{22}$$

qui exprime l'invariance du flux de $\mathbf{K}$, tourbillon du moment généralisé, au travers d'un contour lié au fluide.

Dans le problème qui nous occupe on peut écrire (21) en tenant compte de (19) sous la forme

$$\frac{d\mathbf{K}}{dt} - \boldsymbol{\Omega} \times \mathbf{K} + 2\lambda\mathbf{K} = 0$$

ou encore:

$$\frac{D\mathbf{K}}{Dt} + 2\lambda\mathbf{K} = 0 \tag{23}$$

si l'on pose:

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + (\mathbf{V} \cdot \nabla) - \boldsymbol{\Omega} \times . \tag{24}$$

Le mouvement fluide est celui d'un solide dilaté uniformément. On peut définir un repère solide S lié au fluide à la dilation près; c'est un repère dont l'origine est un point fluide et dont les axes sont définis par 3 autres points fluides. Dans un tel repère le mouvement fluide se rèduit à une dilatation. L'opérateur $\frac{D}{Dt}$ représente la dérivée en suivant le mouvement fluide d'un vecteur exprimé dans un repère S. L'interprétation de (23) est alors évidente; dans S et en suivant le mouvement de dilatation:

$$\mathbf{K} = \mathbf{K}_0 \exp -2\mu(t). \tag{25}$$

L'uniformité de $\boldsymbol{\Omega}$, ($\nabla \times \mathbf{K} = q\nabla \times \mathbf{B}$), permet d'écrire par une dérivation géométriquement évidente dans le repère S:

$$\nabla \times \mathbf{B} = \nabla \times \mathbf{B}_0 \exp -3\mu(t). \tag{26}$$

Ainsi à partir des équations on a obtenu des propriétés d'invariance. Celles-ci s'interprètent facilement dans le repère S lié au fluide; elles portent sur des scalaires $\left(\frac{d}{dt}\right)$ ou sur des vecteurs $\left(\frac{D}{Dt}\right)$. Nous convenons de les représenter par un opérateur unique $\frac{D}{Dt}$ et nous dirons qu'une grandeur A, scalaire ou

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vectorielle possède une invariance dite 'temporelle' d'ordre ν (ν entier positif, négatif ou nul) si

$$\frac{DA}{Dt} - \nu\lambda A = 0 \rightleftarrows A = A_0 \exp \nu\mu(t) \quad \text{dans } S. \tag{27}$$

On a obtenu les invariances temporelles de:

$$\begin{array}{ll} h & -2 \\ n(h) & -3 \\ \mathbf{K} & -2 \\ \nabla \times \mathbf{B} & -3 \end{array} \tag{28}$$

Pour aller plus loin il devient nécessaire de coupler les espèces.

B. COUPLAGE DES IONS ET DES ELECTRONS

Equations du problème dans l'approximation non relativiste

On considère un plasma complètement ionisé formé par les deux espéces électrons et ions d'indices respectifs 1 et 2 et de charges q_1 et q_2:

On posera: $\quad q_2 = -q_1 = e.$

Le traitement du cas d'ions multiplement chargés aboutirait aux mêmes conclusions.

Les équations de Maxwell s'écrivent alors en unités electromagnétiques:

$$\begin{aligned} &\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + 4\pi\mathbf{I} = \nabla \times \mathbf{B} \\ &\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \text{(u.e.m.)} \\ &\frac{1}{c^2}\nabla \cdot \mathbf{E} = 4\pi e\,(n_2 - n_1) \end{aligned} \tag{29}$$

On supposera de plus que les diverses grandeurs envisagées sont d'un ordre comparable et que la vitesse de la lumière est infinie en unités électromagnétiques, ce qui néglige les courants de déplacements. La cohérence mathématique du problème est ainsi réalisée par cette approximation non relativiste puisqu'on a déjà envisagé une équation de Boltzmann non relativiste.

Ces hypothèses conduisent à la 'quasi-neutralité'.

Les équations de Maxwell deviennent:

$$\begin{aligned} &\nabla \times \mathbf{B} = 4\pi\mathbf{I} \qquad \nabla \cdot \mathbf{B} = 0 \\ &\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ &n_1 = n_2 = n. \end{aligned} \tag{30}$$

L'ensemble des équations du problème, équations de Maxwell et équations fluides sont alors:

$$\mathbf{V}_i = \mathbf{V}_i^0 + \lambda_i \mathbf{r} + \boldsymbol{\Omega}_i \times \mathbf{r} \tag{31}$$

$$\nabla h_i = -\frac{d_i \mathbf{V}_i}{dt} + \frac{q_i}{m_i}(\mathbf{E} + \mathbf{V}_i \times \mathbf{B}) \tag{32}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \begin{cases} i = 1 & \text{électrons} \\ i = 2 & \text{ions} \end{cases} \tag{33}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{34}$$

$$\nabla \times \mathbf{B} = 4\pi n e\, \mathbf{V}_r. \tag{35}$$

On joint à ce système les invariances temporelles (28), avec d'autre part:

$$n_1 = n_2$$

$$\mathbf{V}_r = \mathbf{V}_2 - \mathbf{V}_1.$$

Une équation (32) nous donnera le champ électrique et l'on peut supprimer (34), compatible avec (32) grâce à (28).

Le couplage entre les deux expèces est donné à la fois par l'équation (35) et par les invariants temporels (28) qui font intervenir les quantités communes n et $\mathbf{B}$. On va voir que ce couplage ne permet pas de satisfaire à l'équation (32) dans le cas général.

Conséquences de l'équation (35)

$$\nabla \times \mathbf{B} = 4\pi e n \mathbf{V}_r \tag{35}$$

Détermination du champ des vitesses relatives. On connait déjà les invariances temporelles sur $\nabla \times \mathbf{B}$ et n toutes deux d'ordre 3. Appliquant à l'équation (35) l'opérateur $\frac{D_i}{Dt}$ on obtient pour $\mathbf{V}_r$ l'invariance d'ordre 0:

$$\frac{D_i \mathbf{V}_r}{Dt} = 0 \qquad i = 1 \text{ ou } 2. \tag{36}$$

La soustraction des deux équations (36) donne simplement

$$(\lambda_1 - \lambda_2)\,\mathbf{V}_r = 0 \quad \text{soit} \quad \lambda_1 = \lambda_2 \tag{37}$$

si l'on remarque que pour un champ de la forme (31) on a l'identité:

$$(\mathbf{V} \cdot \nabla)\mathbf{V} - \boldsymbol{\Omega} \times \mathbf{V} = \lambda \mathbf{V}. \tag{38}$$

D'après (37) les dilatations des deux espèces sont donc les mêmes et le champ de vitesse relatif est purement hélicoïdal:

$$\mathbf{V}_r = \mathbf{V}_r^0 + \boldsymbol{\Omega}_r \times \mathbf{r}. \tag{39}$$

Les équations (36) montrent de plus que ce champ est lié aux deux fluides. En le rapportant à son axe

central qui possède évidemment cette dernière propriété, on voit facilement que dans un repère S_i lié au fluide i:

$$\mathbf{V}_r^0 = C^{te} \qquad \mathbf{V}_r^0 \times \boldsymbol{\Omega}_r = 0 \tag{40}$$

$$\boldsymbol{\Omega}_r = \boldsymbol{\Omega}_{r(0)} \exp -\mu(t) \qquad \text{(invariance } -1) \tag{41}$$

Les lignes (C) du champ $\mathbf{V}_r$ sont à un instant donné le système d'hélices de pas constant $2\pi \mathbf{V}_r^0/\Omega_r$; elles sont liées aux fluides; les transformées par les deux mouvements fluides d'une ligne originale coïncident donc en tout instant.

Invariance de ligne (C). Posant

$$\frac{D_r}{Dt} \equiv \frac{D_2}{Dt} - \frac{D_1}{Dt} \equiv \begin{cases} (\mathbf{V}_r \cdot \nabla) & \text{pour un scalaire} \\ (\mathbf{V}_r \cdot \nabla) - \boldsymbol{\Omega}_r \times & \text{pour un vecteur} \end{cases} \tag{42}$$

on dira qu'une grandeur A est invariante sur les lignes (C) si:

$$\frac{D_r A}{Dt} = 0. \tag{43}$$

En effet (43) exprime bien l'invariance de la grandeur A dans les déplacements hélicoïdaux définis par les lignes (C). Toute invariance de ligne (C) est entendue à un instant déterminé.

Des invariances temporelles (28) on peut déduire des invariances de ligne lorsque les quantités temporellement invariantes sont les mêmes pour les deux espèces. Par exemple de

$$\left(\frac{D_1}{Dt} + 3\lambda_1\right) \nabla \times \mathbf{B} = 0$$

et

$$\left(\frac{D_2}{Dt} + 3\lambda_2\right) \nabla \times \mathbf{B} = 0$$

on déduit immédiatement puisque $\lambda_1 = \lambda_2$:

$$\frac{D_r}{Dt} \nabla \times \mathbf{B} = 0.$$

De cette façon, n, $\nabla \times \mathbf{B}$ et bien entendu $\mathbf{V}_r$ sont invariants sur les lignes (C).

D'après ce qui précède (équation 16) il en est de même pour $h_i(n, t)$, fonction de la densité seule à un instant donné (n_i s'obtient par une intégration de $g_i(u^2, h_i, t)$ sur la variable u).

Conséquences des invariances quant à l'équation des quantités de mouvement

L'addition des 2 équations (32) relatives aux 2 espèces donne:

$$\nabla \tilde{h} = -\frac{\widetilde{d\mathbf{V}}}{dt} + e\mathbf{V}_r \times \mathbf{B} \tag{44}$$

avec $\tilde{A} = m_1 A_1 + m_2 A_2$ par définition.

Invariances de lignes. Le scalaire $\tilde{h}$ invariant sur une ligne (C) est donc constant sur cette ligne. On a:

$$\mathbf{V}_r \cdot \nabla \tilde{h} = 0 \qquad \text{d'où}$$

$$\mathbf{V}_r \cdot \frac{\widetilde{d\mathbf{V}}}{dt} = 0. \tag{45}$$

Cette équation doit être vraie quel que soit $\mathbf{r}$. En explicitant $\dfrac{\widetilde{d\mathbf{V}}}{dt}$ on a:

$$\frac{\widetilde{d\mathbf{V}}}{dt} = \frac{\partial \tilde{\mathbf{V}}}{\partial t} + \sum_i m_i (\mathbf{V}_i \cdot \nabla)\mathbf{V}_i$$

$$= \frac{\partial \tilde{\mathbf{V}}}{\partial t} + \sum_i m_i (\lambda \mathbf{V}_i + \boldsymbol{\Omega}_i \times \mathbf{V}_i) \tag{46}$$

$\dfrac{d\mathbf{V}}{dt}$ s'obtient d'après la forme (31) des champs de vitesse par l'action d'un opérateur linéaire sur $\mathbf{r}$. On se convaincra aisément au vu de la forme du champ $\mathbf{V}_r$ et de l'équation (45) que l'opérateur a necessairement la forme $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \quad)$, $\boldsymbol{\omega}$ étant un vecteur porté par l'axe central du champ $\mathbf{V}_r$. On a donc en désignant par $\boldsymbol{\rho}$ le rayon vecteur distance à cet axe central:

$$\frac{\widetilde{d\mathbf{V}}}{dt} = \alpha \boldsymbol{\rho}. \tag{47}$$

Le calcul de $\dfrac{\widetilde{d\mathbf{V}}}{dt}$ se développe aisément et l'on trouve

$$\frac{\widetilde{d\mathbf{V}}}{dt} = \frac{\partial \tilde{\mathbf{V}}^0}{\partial t} + \lambda \tilde{\mathbf{V}}^0 + \overbrace{\tilde{\boldsymbol{\Omega}} \times \mathbf{V}}$$

$$+ (m_1 + m_2)\left(\frac{\partial \lambda}{\partial t} + \lambda^2\right)\mathbf{r} + \left(\frac{\partial \tilde{\boldsymbol{\Omega}}}{\partial t} + 2\lambda \tilde{\boldsymbol{\Omega}}\right) \times \mathbf{r} \tag{48}$$

$$+ \overbrace{\tilde{\boldsymbol{\Omega}} \times (\boldsymbol{\Omega} \times \mathbf{r})}.$$

Une des conséquences immédiates est que

$$\frac{\partial \tilde{\boldsymbol{\Omega}}}{\partial t} = -2\lambda \tilde{\boldsymbol{\Omega}} \tag{49a}$$

avec $\tilde{\boldsymbol{\Omega}} = m_1 \boldsymbol{\Omega}_1 + m_2 \boldsymbol{\Omega}_2$.

Il s'ensuit que le vecteur $\tilde{\boldsymbol{\Omega}}$ est fixe en direction. D'autre part le terme $\sum_i m_i \boldsymbol{\Omega}_i \times (\boldsymbol{\Omega}_i \times \mathbf{r})$ de l'équation (48) ne se réduira à la forme requise $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ que si les vecteurs $\boldsymbol{\Omega}_1$ et $\boldsymbol{\Omega}_2$ sont colinéaires.

Les vecteurs $\boldsymbol{\Omega}_1$ et $\boldsymbol{\Omega}_2$ sont donc colinéaires et conservent une direction fixe; d'après (41) et (49a), $\boldsymbol{\Omega}_r$ et $\tilde{\boldsymbol{\Omega}}$ possèdent respectivement les invariances temporelles d'ordre -1 et -2.

On n'insistera pas sur les conséquences de l'annulation des termes constants ou proportionnels à $\mathbf{r}$ dans

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l'équation (48). Il nous suffira pour la suite de prendre:

$$\frac{\widetilde{d\mathbf{V}}}{dt} = \sum_i m_i \boldsymbol{\Omega}_i \times (\boldsymbol{\Omega}_i \times \mathbf{r}) = -\tilde{\Omega}^2 \boldsymbol{\rho}. \qquad (49b)$$

Invariances temporelles. Nous connaissons celles de $h(-2), \boldsymbol{\Omega}_r(-1), \tilde{\boldsymbol{\Omega}}(-2)$ et bien entendu de $\boldsymbol{\rho}(+1)$. S'ensuivront celles de $\tilde{h}(-2)$ de $\nabla\tilde{h}(-3)$ et de $\dfrac{\widetilde{d\mathbf{V}}}{dt}$ (par (49)). Mais l'équation (44) fait intervenir $\mathbf{V}_r \times \mathbf{B}$ qui n'a pas d'invariance *a priori*. Nous allons éliminer cette quantité.

Pour cela nous remarquons les invariances suivantes

$\mathbf{K}_1 = -e\mathbf{B} + 2m_1\boldsymbol{\Omega}_1$ ordre -2 pour les électrons

$\mathbf{K}_2 = e\mathbf{B} + 2m_2\boldsymbol{\Omega}_2$ ordre -2 pour les ions

$\tilde{\boldsymbol{\Omega}} = m_1\boldsymbol{\Omega}_1 + m_2\boldsymbol{\Omega}_2$ ordre -2 pour les deux espéces.

Le vecteur:

$$\mathbf{M} = e\mathbf{B} + m_2\boldsymbol{\Omega}_2 - m_1\boldsymbol{\Omega}_1$$
$$= -\mathbf{K}_1 + \tilde{\boldsymbol{\Omega}} = \mathbf{K}_2 - \tilde{\boldsymbol{\Omega}} \qquad (50)$$

possède une invariance d'ordre -2 quant aux deux espèces. Il en sera de même pour $\mathbf{V}_r \times \mathbf{M}$, $\mathbf{V}_r$ invariant d'ordre 0 n'affectant pas les invariances. (44) se récrit alors

$$\nabla\tilde{h} = -\frac{\widetilde{d\mathbf{V}}}{dt} + \mathbf{V}_r \times \mathbf{M} - \mathbf{V}_r \times (m_2\boldsymbol{\Omega}_2 - m_1\boldsymbol{\Omega}_1)$$

soit avec (49b)

$$\nabla\tilde{h} = (m_1 + m_2)\Omega_1\Omega_2\boldsymbol{\rho} + \mathbf{V}_r \times \mathbf{M}$$

ou encore en exprimant Ω_1 et Ω_2 en fonction des invariants connus Ω_r et $\tilde{\Omega}$:

$$\nabla\tilde{h} = -\frac{m_1m_2}{m_1+m_2}\Omega_r^2\boldsymbol{\rho} + \frac{\tilde{\Omega}^2\boldsymbol{\rho}}{m_1+m_2}$$
$$-3 \qquad -1 \qquad -3$$
$$+ \frac{m_1-m_2}{m_1+m_2}\Omega_r\tilde{\Omega}\boldsymbol{\rho} + \mathbf{V}_r \times \mathbf{M}$$
$$-2 \qquad -2 \qquad (51)$$

Chaque vecteur $\mathbf{A}_\nu$ intervenant dans l'équation (51) est un invariant temporel d'ordre ν indiqué. Dans un système S_i lié à l'espèce i, $\mathbf{A}_\nu(t)$ se déduit en suivant le mouvement purement de dilation de sa valeur $\mathbf{A}_\nu(0)$ par:

$$\mathbf{A}_\nu(t) = \mathbf{A}_\nu(0) \exp \nu\mu(t).$$

L'équation $\sum_\nu \mathbf{A}_\nu = 0$ n'est satisfaite que pour $\mathbf{A}_\nu = 0$. Ceci nous donne un cas trivial à éliminer (en particulier $\Omega_r = 0$). Une solution dynamique est donc impossible et il ne reste que le cas stationnaire:

$$\lambda = 0. \qquad (52)$$

C. ETUDE DU CAS STATIONNAIRE. RESULTATS

1. *Etude théorique*

Généralités. C'est le cas d'un pinch où les deux champs de vitesse sont purement hélicoïdaux et leurs axes centraux sont confondus. On prendra le système de coordonnées cylindriques (ρ, θ, z) autour de l'axe central commun Oz. Les lignes (C) du champ relatif $\mathbf{V}_r$ dépendent des deux paramètres ρ et s avec:

$$s = V_r^0\theta - \Omega_r z \qquad (\mathbf{V}_r^0 \times \boldsymbol{\Omega}_r = 0). \qquad (53)$$

D'après les considérations qui précèdent les grandeurs $\mathbf{V}_r, \mathbf{B}, n, h$ sont des invariants de lignes (C) c'est-à-dire s'expriment en fonction des deux seules variables ρ et s.

Les équations du cas stationnaire vont être obtenues simplement en réexprimant en fonction de ρ et s le champ magnétique et les deux équations:

$$\nabla \times \mathbf{B} = 4\pi e n \mathbf{V}_r \qquad (54)$$

$$\nabla\tilde{h} = -\frac{\widetilde{d\mathbf{V}}}{dt} + e\mathbf{V}_r \times \mathbf{B}. \qquad (55)$$

Forme générale du champ magnétique. On démontre aisèment qu'un champ $\mathbf{B}$ à divergence nulle, hélicoïdalement invariant $\dfrac{D_r\mathbf{B}}{Dt} = 0$ s'écrit sous la forme générale:

$$\mathbf{B} = k(\rho, s)\frac{\mathbf{V}_r}{V_r^2} + \frac{\mathbf{V}_r}{V_r^2} \times \nabla\phi(\rho, s) \qquad (56)$$

où k et ϕ sont deux fonctions arbitraires de ρ et s. On remarque:

$$\mathbf{V}_r \cdot \mathbf{B} = k \qquad (57)$$
$$\mathbf{V}_r \times \mathbf{B} = -\nabla\phi.$$

On calcule ensuite:

$$\nabla \times \mathbf{B} = \left[(V_r^{0^2} + \Omega_r^2\rho^2)L\phi + \frac{2V_r^0\Omega_r}{V_r^{0^2} + \Omega_r^2\rho^2}k\right]\frac{\mathbf{V}_r}{V_r^2}$$
$$- \frac{\mathbf{V}_r}{V_r^2} \times \nabla k \qquad (58)$$

avec

$$L \equiv \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\frac{\rho}{V_r^{0^2} + \Omega_r^2\rho^2}\frac{\partial}{\partial\rho}\right] + \frac{1}{\rho^2}\frac{\partial^2}{\partial s^2} \qquad (59)$$

Calcul des fonctions k et ϕ. Des deux expressions (54) et (58) de $\nabla \times \mathbf{B}$ on déduit

$$L\phi + \frac{2V_r^0\Omega_r k}{(V_r^{0^2} + \Omega_r^2\rho_e^2)^2} = 4\pi e n \qquad (60)$$

$$k = \text{cte}. \qquad (61)$$

(61) donne l'expression remarquable:

$$\mathbf{V}_r \cdot \mathbf{B} = \Omega_r \rho\, B_\theta + V_r^0 B_z = k = \text{cte.} \qquad (62)$$

En tenant compte de (49) et (57), (55) s'écrit maintenant:

$$\tilde{h} + e\phi - (m_1\Omega_1^2 + m_2\Omega_2^2)\frac{\rho^2}{2} = \text{cte.} \qquad (63)$$

Il suffit pour résoudre complètement le problème de se donner arbitrairement la dépendance fonctionnelle $\tilde{h}(n)$ (ou encore une équation d'état). Nous savons qu'elle existe d'après les considérations de la première partie.

Les équations (60) et (63) déterminent alors n et ϕ en supposant la constante k donnée.

Le champ électrique est calculable par l'équation aux impulsions dont on n'a pas encore tenu compte.

Le cas statique a fait l'objet d'une étude (REBUT, 1961).

2. *Validité des résultats et des hypothèses vis-à-vis des décharges expérimentales*

Le cas d'un plasma stationnaire en géométrie hélicoïdale est ainsi la seule solution possible. De nombreuses décharges expérimentales possèdent cette géométrie (expériences de S. A. Colgate, Tuck, Bickerton. . .).

En effet si les plasmas fortement ionisés ont été expérimentalement obtenus en régimes dynamiques, leur état instantané reste proche de l'état stationnaire: on peut voir que le terme $(\mathbf{V}_r \cdot \nabla)$ est prépondérant devant le terme $\frac{\partial}{\partial t}$ dans l'opérateur $\frac{d}{dt}$; $(\mathbf{V}_r \cdot \nabla)$ est de l'ordre de V_r/ϖ, ϖ étant le pas des lignes (C) de courant et $\frac{\partial}{\partial t}$ de l'ordre de $1/t_c$, t_c étant un temps caractéristique de la variation des grandeurs macroscopique; la condition:

$$\frac{\partial}{\partial t} \ll (\mathbf{V}_r \cdot \nabla) \text{ s'écrit alors}$$
$$t_c \gg \varpi/V_r.$$

En prenant par exemple pour t_c un temps de montée du courant de 10^{-5} sec, pour V_r une vitesse comprise entre 10^7 et 10^8 cm/sec et pour pas ϖ une longueur de 10 cm on a

$$10^{-7}\text{ sec} < \frac{\varpi}{V_r} < 10^{-6}\text{ sec} \ll t_c.$$

D'autre part on obtient une bonne vérification expérimentale de la relation (62) (REBUT, 1961) pour des décharges linéaires, ce qui est en faveur de la validité des hypothèses faites.

On peut s'attendre à trouver des résultats valables en supposant uniquement que les collisions électrons–ions sont négligeables: les collisions entre les particules d'une même espèce n'affectent pas ici les mouvements macroscopiques qui sont des mouvements de solides.

On remarquera que l'hypothèse d'isotropie est liée à la recherche d'une entropie maximum et qu'il n'est pas étonnant de retrouver un régime stationnaire et un mouvement de solides.

D. FORMULATION MAGNETOHYDRODYNAMIQUE DES HYPOTHESES

L'hypothèse d'isotropie de la fonction de distribution autour de la vitesse moyenne n'est pas fondamentale. En effet on peut obtenir les résultats de la partie A en faisant des hypothèses *a priori* moins restrictives, et plus intuitives macroscopiquement.

Les équations (5) sur le champ de vitesse s'obtiennent en supposant simplement l'isotropie du tenseur de pression $\mathbf{P}$ et celle du tenseur $\nabla_r \cdot \mathbf{Q}$, $\mathbf{Q}$ désignant le tenseur:

$$Q_{ijk} = m\int (w_i - v_i)(w_j - v_j)(w_k - v_k) f\, d^3\mathbf{w}.$$

Pour démontrer ceci il suffit de considérer l'equation de transport de la pression cinétique $\mathbf{P}$ (DELCROIX, 1959):

$$\frac{d\mathbf{P}}{dt} + (\nabla \cdot \mathbf{V})\mathbf{P} + \nabla\mathbf{V} \cdot \mathbf{P} + (\nabla\mathbf{V} \cdot \mathbf{P})^{\mathsf{T}} + \nabla_r \cdot \mathbf{Q} = \mathbf{H} + \mathbf{R} \qquad (64a)$$

où T a le sens d'une transposition, $\mathbf{R}$ désigne un terme de collision et $\mathbf{H}$ le tenseur:

$$H_{ij} = \frac{q}{m}\left[(\mathbf{B} \times \mathbf{e}_i) \cdot (\mathbf{e}_j \cdot \mathbf{P}) + (\mathbf{B} \times \mathbf{e}_j) \cdot (\mathbf{e}_i \cdot \mathbf{P})\right]$$

soit encore

$$\mathbf{H} = \frac{q}{m}\left[\mathbf{P} \cdot (\mathbf{B} \times\ \ ) - \mathbf{B} \times (\mathbf{P} \cdot\ \ )\right].$$

En l'absence de collisions et pour un tenseur de pressions isotropes:

$$\begin{cases} \mathbf{R} = 0 \\ \mathbf{H} = 0 \end{cases} \text{ et l'on a en explicitant (64a)}$$

$$\frac{dP_{ij}}{dt} + (\nabla \cdot \mathbf{V})P_{ij} + \nabla_i V_\alpha P_{\alpha j} + \nabla_j V_\alpha P_{\alpha i} \nabla_{r_\alpha} \cdot Q_{\alpha ij} = 0 \qquad (64b)$$

(sommation sur l'indice $\alpha = 1, 2, 3$).

Si l'on suppose:

$$\mathbf{P} \equiv p\mathbf{I} \quad \text{et} \quad \nabla_r \cdot \mathbf{Q} \equiv (\nabla_r \cdot \mathbf{Q})\mathbf{I}$$

le système des six équations (64b) s'explicite aisément.

Pour donner les 5 équations du champ de vitesses:

$$\begin{cases} \nabla_i V_j + \nabla_j V_i = 0 & i \neq j \\ \nabla_1 V_1 = \nabla_2 V_2 = \nabla_3 V_3 = \frac{1}{3}\nabla \cdot \mathbf{V} \end{cases} \tag{5}$$

et l'équation de l'énergie:

$$\frac{dp}{dt} + \frac{5}{3} p \nabla \cdot \mathbf{V} + \nabla \cdot \mathbf{Q} = 0. \tag{65}$$

Le système (5) a été intégré dans la partie A. On obtenait le champ de vitesse:

$$\mathbf{V} = \mathbf{b}r^2 - 2\mathbf{r}(\mathbf{b} \cdot \mathbf{r} + c) + \boldsymbol{\Omega} \times \mathbf{r} + \mathbf{d}. \tag{9}$$

Ce champ est singulier dans le cas général $b \neq 0$. Il représente un écoulement de révolution source-puits autour d'un axe central, à une rotation prés autour de cet axe. En effet (9) s'écrit en choisissant convenablement l'origine des axes ($b \neq 0$):

$$\mathbf{V} = \mathbf{b}r^2 - 2\mathbf{r}(\mathbf{b} \cdot \mathbf{r}) + \boldsymbol{\Omega} \times \mathbf{r}$$

avec $$\boldsymbol{\Omega} \times \mathbf{b} = 0.$$

Mais le cas non singulier, le plus intéressant physiquement est celui où $b = 0$; il redonne le champ d'un solide dilaté uniformément et

$$\nabla \cdot \mathbf{V} = 3\lambda(t).$$

L'équation de continuité conduit alors à l'invariant temporel:

$$n = n_0 \exp -3\mu(t). \tag{66}$$

Si de plus on suppose qu'il n'y a pas de transport de chaleur $\nabla \cdot \mathbf{Q} = 0$ (65) permet d'écrire un autre invariant temporel

$$p = p_0 \exp -5\mu(t). \tag{67}$$

La réunion des deux équations (66) et (67) $\left(\frac{d}{dt} p n^{-(5/3)} = 0\right)$ ne suffit pas pour établir une équation d'état. Ce serait le cas si l'on partait d'un état initial du gaz de pression et de densité initiale uniformes. L'état initial que l'on doit considérer ici est *a priori* quelconque: c'est celui qui existe à la fin du processus d'ionisation. Pour obtenir une fonction de Helmholtz il faut donc faire l'hypothèse supplémentaire d'une équation d'état de forme très générale:

$$p \equiv p(n, t).$$

Des trois hypothèses

$$\begin{cases} \mathbf{P} = p\mathbf{I} & \text{pression scalaire} \\ \nabla_r \cdot \mathbf{Q} = 0 & \text{pas de transport de chaleur} \\ p \equiv p(n, t) & \text{équation d'état} \end{cases}$$

on déduit ainsi tous les résultats de la 1ère partie qu ont été utilisés dans la suite du problème. On doit également ajouter une hypothèse de non singularité du champ de vitesses ($b = 0$). La conclusion sera alors la même: la seule solution du problème dynamique est le pinch hélicoïdal stationnaire.

Remarque. L'approximation (CHEW *et al.*, 1956) donne les deux invariants

$$\frac{p_\perp}{nB} \qquad \text{et} \qquad \frac{p/|B^2}{n^3}.$$

Elle suppose aussi explicitement (DELCROIX, 1959) $\nabla_r \cdot \mathbf{Q} = 0$. Avec l'hypothèse d'une pression scalaire ces deux invariants se ramènent à

$$pn^{-(5/3)} \qquad \text{et} \qquad \frac{B^3}{n^2}.$$

La théorie exposée ici fait apparaitre les invariants

$$pn^{-(5/3)} \qquad \text{et} \qquad \frac{K^3}{n^2}.$$

Le second invariant est bien l'invariant de CHEW *et al.* (1956) pour un champ magnétique fort.

E. DEVELOPPEMENT DES FONCTIONS DE DISTRIBUTION DES VITESSES EN HARMONIQUES SPHERIQUES
(SOMON et VUILLEMIN, 1960)

L'hypothèse d'isotropie de la fonction de distribution des vitesses autour de la vitesse moyenne, adoptée dans la partie A, peut paraitre trop restrictive puisque l'on cherche seulement à assurer une pression scalaire. Dans le cas général on developpera cette fonction en harmoniques sphériques:

$$(3') \quad f(\mathbf{r}, \mathbf{u} + \mathbf{v}, t) \equiv g(\mathbf{r}, \mathbf{u}, t) \text{ et}$$

$$g(\mathbf{r}, \mathbf{u}, t) = g_0(\mathbf{r}, u, t) + \Sigma\, \alpha_{lm}(\mathbf{r}, u, t) C_{lm} + \beta_{lm}(\mathbf{r}, u, t) S_{lm}$$

avec $$\begin{cases} C_{lm} = u^l P_{lm}(\theta) \cos m\phi \\ S_{lm} = u^l P_{lm}(\theta) \sin m\phi. \end{cases}$$

La fonction g est telle que, par construction, la vitesse moyenne $\langle \mathbf{u} \rangle$ relative à $\mathbf{u}$ soit nulle. On suppose de plus que la pression est scalaire. Ceci impose des conditions sur les moments de g par rapport à $\mathbf{u}$. Par l'intermédiaire de (3') on écrit que g satisfait à l'équation de Boltzmann. Le calcul a été effectué en supposant simplement des anisotropies d'ordre 1 ce qui revient à prendre g de la forme:

$$g(\mathbf{r}, \mathbf{u}, t) = g_0(\mathbf{r}, u, t) + \mathbf{A}(\mathbf{r}, u, t) \cdot \mathbf{u}.$$

La pression est alors scalaire. L'anisotropie $\mathbf{A}$ doit être bien entendu telle que $\langle \mathbf{u} \rangle = 0$ mais elle introduit *a priori* un terme supplémentaire de transport de chaleur. $\nabla \cdot \mathbf{Q}$. Le calcul, un peu long, montre que $\nabla \cdot \mathbf{Q} = 0$.

On retrouve encore le meme champ de vitesses macroscopiques (équation 10) et une équation d'état. Les conclusions sont donc les mêmes.

F. ASPECT MATHEMATIQUE DU PROBLEME. CONCLUSIONS

Cette théorie repose sur la résolution exacte d'un systéme d'équations et n'est donc valable que dans la mesure où les hypothèses faites sont rigoureusement exactes. La solution trouvée est singulière; c'est un état permanent bien particulier: le pinch hélicoïdal.

Lorsqu'on adopte des hypothèses voisines des hypothèses faites ici (faible anisotropie de pression, léger taux de collision, etc. . .) il est difficile de se prononcer sur la convergence mathématique des solutions du problème vers cette solution singulière.

La vérification expérimentale de la relation (62) pour des décharges voisines du pinch hélicoïdal serait plutôt en faveur de la convergence.

Si l'on suppose la convergence assurée il y a donc lieu de reviser notablement les hypothèses faites pour des cas expérimentaux très différents de la configuration hélicoïdale trouvée.

Il est alors indispensable de tenir compte d'un ou plusieurs des facteurs tels que: anisotropie des pressions, absence d'équation d'état, collisions, transport de chaleur, ou phénomènes à fréquence élevée. La magnétohydrodynamique usuelle doit être modifiée en conséquence.

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ENERGY ABSORPTION IN A LONGITUDINAL WAVE IN A COLLISIONLESS, MAGNETIZED PLASMA

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Abstract—The present paper is devoted to calculations of the energy absorption in a longitudinal plasma wave. The direction of propagation is arbitrary with respect to an external magnetic field $\mathbf{H}_0$.

Electrons experiencing a constant electric field are responsible for the absorption. In the non-magnetic case these electrons have a velocity equal to the wave's phase velocity. On the assumption that the absorbed energy is extracted from the wavefield, a damping factor is calculated. When this factor is evaluated for the same cases as studied previously by other authors, complete agreement is found. This damping is called 'Landau damping.'

1. INTRODUCTION

IN recent years the kinetic theory of longitudinal oscillations in an electron plasma with or without an external magnetic field has been extensively studied by different authors.

The first important contribution came from VLASOV (1945) who assumed wave solutions of the form $\exp i(kx - \omega t)$ in Boltzmann's equation. He obtained a dispersion relation in the form of a singular integral equation.

LANDAU (1946) subjected the basic equations to a Laplace transformation and was able to improve the results of VLASOV. He showed that the waves experienced a damping, now called 'LANDAU damping', and by his treatment the singularity disappeared. GROSS (1951) extended the theory of VLASOV to the magnetic case, studying waves propagated transverse to an external magnetic field $\mathbf{H}_0$. From a study of the dispersion equation GROSS showed that the frequency spectrum contained bands of forbidden frequencies around multiples of the gyration frequency ω_c.

Suspecting that the singularities giving rise to the gaps were a consequence of the substitution method used, SEN (1952) treated the same case as GROSS by the help of Laplace transformations. He too found the gaps, thus supporting the result of GROSS.

Later SITENKO and STEPHANOV (1957) and BERNSTEIN (1958) extended the theory of the above authors to the case of arbitrary direction of propagation.

Among other results derived from a study of the dispersion equation, both papers contain calculations of the LANDAU damping for different cases, dependent on the values of the parameters of the problem.

The frequency-gaps, appearing in the limit of transverse propagation are also discussed in both papers.

Since these calculations are based on the dispersion equation, the physical causes of the phenomena are obscure. Especially the real cause of the LANDAU damping has occupied many authors. For a discussion see JACKSON (1960).

In the present paper the LANDAU damping and the frequency gaps are investigated from the point of view of energy absorption. A longitudinal wave with arbitrary direction of propagation is assumed. We find that the main contribution to the energy absorption comes from the resonance electrons. These are electrons experiencing an electric field which is constant in space in a linear theory. The damping of the wave which this absorption must give rise to if the absorbed energy is taken from the wave field, is calculated for different values of the parameters of the problem. Agreement with the paper of SITENKO-STEPHANOV and that of BERNSTEIN is found. This indicates that the LANDAU damping is due to a resonance absorption.

As for the frequency-gaps it seems as if these are due to an extreme energy absorption. We shall 'show' that, when an undamped wave is postulated with a frequency $\omega = n\omega_c$, the absorption tends to infinity.

Since in the calculation to follow we are using results obtained in a previous paper by the present writer, KILDAL (1961), hereafter referred to as Paper I, we shall briefly recapitulate the latter in order to make the present paper more self-consistent.

2. RESUME OF PAPER I

In this paper a theory of plasma-oscillations based on a particle description is given. The physical conditions are an unbounded, quasi-neutral and collision-free plasma, consisting of electrons and single-charged ions. The plasma is embedded in an

external magnetic field $\mathbf{H}_0$, and the ions are considered motionless. We assume the existence of a wave of the form $\mathbf{E} = \mathbf{E}_0 \exp i(\mathbf{k} \cdot \mathbf{r} - \omega t)$ with complex ω, propagated in a direction $\mathbf{k}$. The wave vector $\mathbf{k}$ lies in the x–z plane and the angle between $\mathbf{k}$ and $\mathbf{H}_0$ is θ. Lagrangian co-ordinates for the electrons are introduced.

Of the results, we shall here make use of the perturbation f_1 of the equilibrium distribution function $f_0(v)$, given by equation (43), (equation (6) of the present paper), and the dispersion relation equation (52) which we quote mainly in order to introduce notation,

$$-1 = \frac{{\omega_p}^2}{\sqrt{2\pi}\, k^2} \left(\frac{m}{\kappa T}\right)^{5/2} \sum_{-\infty}^{\infty} \int_0^{\infty} d\rho \rho\, {J_n}^2(k_1 r)\, \mathrm{e}^{-\frac{m}{2\kappa T}\rho^2} \times \left[\int_{\mathscr{C}} (n\omega_c + k_2 v_z)\, \mathrm{e}^{-\frac{m}{2\kappa T} v_z^2} \frac{dv_z}{[n_0]}\right],$$

where $\mathscr{C}$ is the same integration path as first used by Landau.

The notation used is:

$-e$, m = charge and mass of an electron, κ = Boltzmann's constant, T = temperature, ρ, ϕ, v_z = cylindrical co-ordinates in velocity space, J_n = Bessel function of the first kind and nth order, k = wave number, $k_1 = k \sin\theta$, $k_2 = k\cos\theta$, $r = \rho/\omega_c$ is the gyration radius, ω_p is the plasma frequency,

$$[n_0] = n\omega_c + k_2 v_z - \omega, \qquad (n = \pm 1, \pm 2, \pm \ldots).$$

3. A GENERAL EXPRESSION

An expression for the mean value in space and time of the energy absorption is the following

$$A = -e \iiint \langle \overline{\mathbf{E} \cdot \mathbf{v} f(\mathbf{v})} \rangle\, d^3v, \tag{1}$$

where $\langle\ \rangle$ indicates mean value in time and the bar indicates mean value in space. $\mathbf{E}$ is the electric field, $\mathbf{v}$ = electron velocity and $f(\mathbf{v})$ is the electron distribution function which in equilibrium will be taken to be a Maxwell-Boltzmann distribution.

The terms of the integrand up to first order may be written

$$\begin{aligned} \mathbf{E} &= \mathbf{O} + \mathbf{E} \\ f &= f_0 + f_1 \end{aligned} \tag{2}$$

hence

$$\overline{\mathbf{E} \cdot \mathbf{v} f(\mathbf{v})} = \overline{\mathbf{E} \cdot \mathbf{v} f_1(\langle \mathbf{r}, \mathbf{v}, t)} \tag{3}$$

where f_1 indicates the surplus of electrons.

Since $\mathbf{E}$ which in our case is parallel to $\mathbf{k}$, lies in the xz–plane we may write

$$E_x = \frac{k_1}{k} E, \qquad E_z = \frac{k_2}{k} E, \tag{4}$$

hence

$$\overline{\mathbf{E} \cdot \mathbf{v} f(\mathbf{v})} = \overline{\frac{E}{k} f_1 (k_1 v_x + k_2 v_z)}. \tag{5}$$

From Paper I, equation (43), we find (except for the minus in front of f_1 which is missing there)

$$-f_1 = \frac{e}{2m} \frac{\partial f_0}{\partial \rho} \mathrm{e}^{-ik_1 r\cos\phi} \sum_{-\infty}^{\infty} i^n J_n(k_1 r) \left[\frac{F_1 \mathrm{e}^{i(n-1)\phi}}{[n_-]} - \frac{F_2 \mathrm{e}^{i(n+1)\phi}}{[n_+]}\right] + i \frac{e}{m} \frac{\partial f_0}{\partial v_z} \mathrm{e}^{-ik_1 r\cos\phi} \sum_{-\infty}^{\infty} i^n J_n(k_1 r) \frac{E_z \mathrm{e}^{in\phi}}{[n_0]} \tag{6}$$

Here

$$[n_\pm] = (n \pm 1)\omega_c + k_2 v_z - \omega. \tag{7}$$

The meaning of the symbols is given in the introductory sections.

The f_1 given by (6) can be found by solving the linearized VLASOV equation, for instance by the substitution method.

In the pure longitudinal case, $E_y = 0$, consequently

$$\begin{aligned} F_1 &= E_x + iE_y = E_x \\ F_2 &= E_x - iE_y = E_x. \end{aligned} \tag{8}$$

We insert the expressions (4) and (8) into equation (6) and use the relations

$$\frac{\partial f_0}{\partial v_z} = -\frac{m}{\kappa T} v_z f_0, \qquad \frac{\partial f_0}{\partial \rho} = -\frac{m}{\kappa T} \rho f_0. \tag{9}$$

Equation (6) then becomes

$$f_1 = \frac{e}{2\kappa T} \rho f_0 \mathrm{e}^{-ik_1 r\cos\phi} \frac{k_1}{k} E \sum_{-\infty}^{\infty} i^n J_n \left[\frac{\mathrm{e}^{i(n-1)\phi}}{[n_-]} - \frac{\mathrm{e}^{i(n+1)\phi}}{[n_+]}\right] + i \frac{e}{\kappa T} v_z f_0 \mathrm{e}^{-ik_1 r\cos\phi} \frac{k_2}{k} E \times \sum_{-\infty}^{\infty} i^n J_n \frac{\mathrm{e}^{in\phi}}{[n_0]}. \tag{10}$$

Further

$$\begin{aligned} \sum_{-\infty}^{\infty} i^n J_n \left[\frac{\mathrm{e}^{i(n-1)\phi}}{[n_-]} - \frac{\mathrm{e}^{i(n+1)\phi}}{[n_+]}\right] &= \sum_{-\infty}^{\infty} (i^{n+1} J_{n+1} - i^{n-1} J_{n-1}) \frac{\mathrm{e}^{in\phi}}{[n_0]} \\ &= \sum_{-\infty}^{\infty} i^{n+1} \frac{2n \cdot J_n(k_1 r)}{k_1 r} \frac{\mathrm{e}^{in\phi}}{[n_0]} \end{aligned} \tag{11}$$

where the recurrence relation

$$J_{n+1}(k_1 r) + J_{n-1}(k_1 r) = \frac{2n}{k_1 r} J_n(k_1 r) \tag{12}$$

is employed.

We now insert (11) into (10) and use the relation

$r = \rho/\omega_c$. Equation (10) then takes the form

$$f_1 = \frac{eE}{\kappa T k} f_0 \, e^{-ik_1 r\cos\phi} \sum_{-\infty}^{\infty} i^{n+1} J_n(k_1 r) \frac{n\omega_c + k_2 v_z}{[n_0]} e^{in\phi} \tag{13}$$

This last expression is to be inserted into (5), but we must be careful not to multiply two complex terms together, but only the real parts of each or the real part of one and then let the other be complex. Since we have introduced cylindrical co-ordinates, the integration in (1) is to be performed over ϕ, ρ and v_z. Expressions for v_x and v_z in cylindrical co-ordinates are

$$v_x = -\rho \sin\phi = -\rho \frac{e^{i\phi} - e^{-i\phi}}{2i}$$
$$v_z = v_z. \tag{14}$$

When these expressions are inserted into (5) and the integration is performed over ϕ, we arrive at the result

$$\int_0^{2\pi} (k_1 v_x + k_2 v_z) f_1 \, d\phi = \frac{2\pi e i}{\kappa T} f_0 E \times \sum_{-\infty}^{\infty} J_n^2(k_1 r) \frac{(n\omega_c + k_2 v_z)^2}{[n_0]}, \tag{15}$$

E being independent of ϕ.

Now we take the real part of (15) and multiply it with the real part of $\mathbf{E} = \mathbf{E}_0 \exp i(\mathbf{k} \cdot \mathbf{r} - \omega t)$. The product is inserted into (1) and the mean value in space is taken. The result is

$$A = -\frac{\pi e^2}{\kappa T k^2} \langle E_0^2 \, e^{-2\gamma t} \rangle \sum_{-\infty}^{\infty} \times \int_{-\infty}^{\infty} \int_0^{\infty} f_0 \frac{(n\omega_c + k_2 v_z)^2 J_n^2 \gamma \rho \, d\rho \, dv_z}{(n\omega_c + k_2 v_z - \omega)^2 + \gamma^2}. \tag{16}$$

Here ω has been replaced by $\omega' = \omega - i\gamma$ where ω is now real.

We also observe that we may write the integrand of (16) as the imaginary part of a function of a complex variable, thus writing

$$A = \frac{\pi e^2}{\kappa T k^2 k_2} \langle E_0^2 \, e^{-2\gamma t} \rangle \operatorname{Im} \sum_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} f_0 \times \frac{(n\omega_c + k_2 v_z)^2 J_n^2}{v_z - \dfrac{\omega' - n\omega_c}{k_2}} \rho \, d\rho \, dv_z. \tag{17}$$

From this expression or from equation (16) we see that A considered as a function of γ is discontinuous for $\gamma = 0$, changing from a positive value $\neq 0$ when $\gamma \to -0$ to the same negative value when $\gamma \to +0$. From a physical point of view, this is unrealistic. The present case is analogous to that of LANDAU (1945). Guided by his results, we shall perform an analytic continuation of A similar to that performed by him. We take equation (17) to be the correct expression for A when $\gamma < 0$, and when $\gamma > 0$, we write

$$A = \frac{\pi e^2}{\kappa T k^2 k_2} \langle E_0^2 \, e^{-2\gamma t} \rangle \operatorname{Im} \sum_{-\infty}^{\infty} \int_0^{\infty} \rho \, J_n^2(k_1 r) \times \left\{ \int_{-\infty}^{\infty} \frac{f_0(\rho, v_z)(n\omega_c + k_2 v_z)^2}{v_z - u_n} dv_z + 2\pi i \omega'^2 f_0(\rho, u_n) \right\} d\rho, \tag{18}$$

where $u_n = (\omega' - n\omega_c)/k_2$. The value of A when $\gamma = 0$, we define to be the limit of A when $|\gamma| \to 0$.

By this procedure A has become a continuous function of γ. We may note that if the absorbed energy is to be taken from the energy density of the wave field, no solutions with $\gamma < 0$ can exist since the electric wave then has to be damped in time, i.e. $\gamma > 0$. So we shall be concerned with equation (18).

4. THE CASE $ka \ll 1$ LONG WAVELENGTH

We now let $ka \ll 1$, ($a = \sqrt{\dfrac{\kappa T}{4\pi e^2 N_0}}$ is the Debye length) and assume *a priori* that in the limit $k \to 0$, $\gamma/k_2 \to +0$ and $\omega/k \to \infty$.

The absorption, equation (18) is then given by

$$A = \frac{\pi^2 e^2}{\kappa T k^2 k_2} \omega^2 \langle E_0^2 \, e^{-2\gamma t} \rangle \sum_{-\infty}^{\infty} \times \int_0^{\infty} \rho J_n^2 f_0 \left(\rho, \frac{\omega - n\omega_c}{k_2} \right) d\rho. \tag{19}$$

If this energy is taken from the wave, the latter must be damped except it be supplied with energy from a reservoir. For the moment we assume that this is not the case and must then have the following relation

$$2\gamma = \frac{A}{\langle E_0^2/8\pi \, e^{-2\gamma t} \rangle} = -\frac{\left\langle \dfrac{\partial}{\partial t} (E_0^2/8\pi \, . \, e^{-2t\gamma}) \right\rangle}{\langle E_0^2/8\pi \, . \, e^{-2\gamma t} \rangle}, \tag{20}$$

where $\langle E_0^2/8\pi \, e^{-2\gamma t} \rangle$ is the energy of the field in a longitudinal wave. We insert (19) into (20) and let f_0 be given as

$$f_0 = N_0 \left(\frac{m}{2\pi\kappa T} \right)^{3/2} \exp\left(-\frac{m}{2\kappa T} v^2 \right).$$

We then obtain

$$\gamma = \sqrt{\frac{\pi}{8}} \frac{\omega_p^2 \omega^2}{k^2 k_2} \left(\frac{m}{\kappa T} \right)^{5/2} \sum_{-\infty}^{\infty} \times \int_0^{\infty} \rho J_n^2 \, e^{-(m/2\kappa T)\rho^2 - (m/2\kappa T)((\omega - n\omega_c)^2/k_2^2)} d\rho \tag{21a}$$

By using the result

$$\int_0^\infty J_n^2\left(\frac{k_1}{\omega_c}\rho\right) e^{-(m/2\kappa T)\rho^2}\rho\, d\rho = \frac{\kappa T}{m} e^{-\nu} I_n(\nu)$$

where $\nu = \frac{k_1^2 \kappa T}{m\omega_c^2}$ and $I_n(\nu)$ is the modified Bessel function of the first kind, equation (21a) can also be written

$$\gamma = \sqrt{\frac{\pi}{8}}\frac{\omega_p^2\omega^2}{k^2 k_2}\left(\frac{m}{\kappa T}\right)^{3/2} \times \sum_{-\infty}^{\infty} e^{-\nu} I_n(\nu) e^{-(m/2\kappa T)(\omega - n\omega_c)^2/k_2^2)} \qquad (21b)$$

The formula given by equation (21a) or (21b) is fairly general and can be used whenever the assumptions made are satisfied.

We shall now justify the assumption that $\gamma/k_2 \to 0$ when $k \to 0$.

Since $e^{-\nu} I_n(\nu) \leq 1$ we can replace equation (21b) by an inequality. Let us choose an $n = n_1$ so large that

$$2n\omega\omega_c < \tfrac{1}{2}n^2\omega_c^2, \qquad \text{i.e.} \qquad n_1 > \frac{4\omega}{\omega_c}.$$

After division by k_2, equation (21b) can then be replaced by the following inequality

$$\gamma/k_2 \leq \frac{\delta}{k^2 k_2^2}\left[\sum_{n=-n_1}^{n_1} e^{-(\beta/k_2^2)(\omega - n\omega_c)^2} + 2e^{-\beta(\omega^2/k_2^2)} \sum_{n=n_1}^{\infty} e^{-(\beta/2k_2^2)n^2\omega_c^2}\right], \qquad (22)$$

where the abbreviations $\beta = \frac{m}{2\kappa T}$ and $\delta = \sqrt{\pi}\,\omega_p^2\omega^2\beta^{3/2}$ are introduced.

The last, infinite series of equation (22) obviously converges and the better the smaller k_2 is.

Let us now assume $\omega \neq n\omega_c$ for all n. Then we see that the right-hand side of equation (22) goes to zero when $k \to 0$, or equivalently when $k_1 = k \sin\theta$ and $k_2 = k\cos\theta$ both tend to zero. This is true because of the exponential factor which is contained in every term of the parenthesis.

We also learn from equation (22) that when we let $\theta \to \pi/2$ ($k_2 = k\cos\theta \to 0$) and let k be finite, then $\gamma \to 0$ for the same reason as above.

From the last result we conclude that waves propagating transverse to an external magnetic field are not damped when the frequency ω is not a multiplum of the gyration frequency ω_c.

If on the other hand $\omega = n\omega_c$ for some n, we see from equation (21a) or equation (21b) that $\gamma/k_2 \to \infty$, consequently these equations cannot be employed in that case.

We may, however, conceive an undamped wave $\gamma \to 0$, extracting energy from a reservoir if necessary, being propagated with a frequency ω. From this wave, energy is absorbed according to formula (18) which in the limit $\gamma \to 0$ yields formula (19). We now put $\omega = n\omega_c$ and let $k_2 \to 0$. We then find that $A \to \infty$, a tentative conclusion from this is, that waves with frequencies $\omega = n\omega_c$ cannot be propagated in the plasma since the energy of the wave would be absorbed by the electrons.

5. SOME SPECIAL CASES OF FORMULA (21a)

We shall now assume $\omega_c \gg \omega$ and $\frac{\kappa T}{m}\frac{k^2}{\omega_c^2} \ll 1$, the case of strong magnetic field and low temperature.

When these assumptions are satisfied the main contribution to the damping factor comes from the term with $n = 0$. Neglect of all other terms yields for the damping

$$\gamma = \sqrt{\frac{\pi}{8}}\frac{\omega_p^2}{k^2}\frac{\omega^2}{k_2}\left(\frac{m}{\kappa T}\right)^{3/2} e^{-(m/2\kappa T)(\omega^2/k_2^2)} \qquad (23)$$

In order to compare this expression with the result of BERNSTEIN, we must find an expression for ω. Since we are investigating the case of low temperature, we may use the dispersion equation with $T = 0$ which for our purpose is a good approximation. The same equation is also obtained by the hydrodynamic description. It reads

$$\frac{\omega_p^2}{\omega^2 - \omega_c^2}\sin^2\theta + \frac{\omega_p^2}{\omega^2}\cos^2\theta = 1. \qquad (24)$$

A solution of this in the case $\omega \ll \omega_c$ is $\omega = \omega_p\cos\theta$, which, when inserted into (23) yields

$$\gamma = \sqrt{\frac{\pi}{8}}\frac{\omega_p^2\cos\theta}{(ka)^3} e^{-(\omega_p^2/2(ak)^2)}. \qquad (25)$$

This γ agrees completely with the result of BERNSTEIN (1958). If we put $\theta = 0$, the result of LANDAU is recovered.

Next we study the case $\omega = n\omega_c$ and $\frac{\kappa T}{m}\frac{k^2}{\omega_c^2} \ll 1$, $\left(\theta \neq \frac{\pi}{2}\right)$ i.e. the case of resonance and low temperature.

For the n_1 satisfying the relation $\omega = n\omega_c$ we assume

$$\left|\frac{\omega' - n\omega_c}{k_2}\frac{m}{2\kappa T}\right| \ll 1. \qquad (26)$$

For other n the same expression is assumed $\gg 1$. This means that $\gamma/k_2 \to 0$ when $k \to 0$ and formula (21a) is valid.

In this case the main contribution to γ comes from

the term with $n = n_1$. When we take this into account and substitute in equation (21a) the approximate expression

$$J_n\left(k_1\frac{\rho}{\omega_c}\right) = \frac{k_1{}^n\rho^n}{2^n \cdot \omega_c{}^n n!},$$

valid for small arguments, we obtain

$$\gamma = \sqrt{\frac{\pi}{8}}\frac{n^2 \sin^{2n}\theta\, \alpha^{2n-3}\omega_p{}^2}{\cos\theta\, 2^n \cdot n!\, \omega_c{}^{2n-2}}. \quad (n = 1, 2, 3, \ldots) \tag{27}$$

Here $$\alpha^2 = \frac{\kappa T}{m}k^2.$$

From equation (24) we find that when $\omega = n\omega_c$,

$$\omega_p{}^2 = \frac{n^2\omega_c{}^2}{\cos^2\theta\,[1 + n^2(n^2-1)^{-1}\operatorname{tg}^2\theta]}, \tag{28}$$

which on introduction in equation (27) gives the following result

$$\gamma = \sqrt{\frac{\pi}{8}}\frac{n^4 \sin^{2n}\theta\, \alpha^{2n-3}}{\cos^3\theta\, 2^n \cdot n!\, \omega_c{}^{2n-4}[1 + n^2(n^2-1)^{-1}\operatorname{tg}^2\theta]} \tag{29}$$

Except for the last term in the parenthesis which in the paper of SITENKO and STEPHANOV is $n^4(n^2-1)^{-2}\operatorname{tg}^2\theta$, the agreement is perfect. The above expression (29) cannot be used for $n = 1$.

When $\omega \approx \omega_c$ it is found from the exact dispersion equation that $\omega \approx \omega_c \approx \omega_p$. Equation (27) then yields

$$\gamma = \frac{1}{4}\sqrt{\frac{\pi}{2}}\frac{\omega_c{}^2}{\alpha}\frac{\sin^2\theta}{\cos\theta}. \tag{30}$$

In order that the assumption (26) be satisfied, we find from (30) that the condition $\dfrac{\sin^2\theta}{\cos\theta} \ll ak$ must be fulfilled. This means that θ must be very small and we may put $\sin\theta \approx \theta$ in equation (30). The result then agrees with that of SITENKO and STEPHANOV.

6. THE CASE $ak \gg 1$, SHORT WAVELENGTH

We shall now turn our interest to the case of short wavelength. We let $k \to \infty$ and assume *a priori* that $\beta = \gamma/k \to \infty$ and $\omega/k \to 0$, assumptions which are justified by the result. In this case equation (19) is not valid. We use instead equation (18).

Using the result

$$\int_0^\infty \frac{e^{-b^2v^2}dv}{\gamma^2 + v^2} = [1 - \phi(b\gamma)]\frac{\pi}{2\gamma}e^{b^2\gamma^2}, \tag{31}$$

where ϕ is the error function, we can show that the contribution to A from the integral term in the bracket is negligible when γ is appreciable. The main contribution comes from the second term. When we demand that equation (20) be fulfilled, we obtain the relation

$$\frac{\gamma}{k} = \frac{\sqrt{2\pi}\left(\dfrac{m}{\kappa T}\right)^{3/2}}{(ak)^2 k^2\cos\theta}\sum_{-\infty}^{\infty}\int_0^\infty d\rho\rho J_n{}^2 \times e^{-(m/2\kappa T)\rho^2}\operatorname{Im}\{i\omega'^2\, e^{-(m/2\kappa T)u_n{}^2}\}. \tag{32}$$

In particular when $H_0 = 0$ or what amounts to the same, $k_1 = 0$, we obtain

$$\gamma/k = \frac{\sqrt{2\pi}\left(\dfrac{m}{\kappa T}\right)^{3/2}}{(ak)^2 k^2}\int_0^\infty d\rho\rho\, e^{-(m/2\kappa T)\rho^2} \times\left\{-\gamma^2\cos\frac{\gamma\omega}{k^2\bar{u}^2} + \frac{2\gamma\omega}{\bar{u}^2}\sin\frac{\gamma\omega}{k^2\bar{u}^2}\right\}. \tag{33}$$

The smallest damping is found by choosing $\dfrac{\gamma\omega}{k^2\bar{u}^2} = \pi$, a choice which is in agreement with the paper of LANDAU (1946). Equation (33) then becomes

$$1 = \frac{\sqrt{2\pi}\,\beta}{(ka)^2(\bar{u}^2)^{1/2}}e^{\beta^2/2\bar{u}^2} \tag{34}$$

where $\bar{u}^2 = \dfrac{\kappa T}{m}$ and $\beta = \dfrac{\gamma}{k}$.

From this relation we see that as $k \to \infty$, $\beta \to \infty$, which means that waves with wavelength much less than the Debye length cannot exist. Except for notation, equation (34) is the result of LANDAU which he obtained from a study of the dispersion relation.

The same consideration applies also in the general case of arbitrary propagation direction not too close to $\theta = \pi/2$, which may be seen from a study of formula (32).

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STABILITY OF THE HARD-CORE LIQUID MODEL OF THE PINCH

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Abstract—A theoretical investigation for the sausage instability of hard-core mercury jet is made. It is found that a parallel current system in the core and the jet has a destabilizing influence and antiparallel current improves the stability of the configuration. For a configuration characterized by the same current density in the jet and the core, the maximum growth rate is smaller than the corresponding growth rate for a homogeneous liquid jet without a core.

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1. INTRODUCTION

RECENTLY LEHNERT and SJÖGREN (1960) with a view to investigating the effect of a hard core in pinch stabilization performed an experiment with a hollow mercury jet which may be regarded as a liquid conductor model of the hollow pinch. We present here a theoretical investigation of a sausage instability for such a configuration, assuming the liquid jet to have vanishingly small electrical conductivity. However, it should be understood that the present investigation does not strictly apply to a hollow pinch since the fluid is taken to be incompressible and the magnetic field lines are not 'frozen' in the fluid. We find that a current in the core in the same direction as that in the liquid has in general a destabilizing influence, whereas an antiparallel current improves the stability of the configuration. In the case of parallel currents it is shown that there exists a mode of maximum instability and the maximum growth rate increases with the increase in the ratio of the total current in the core to that in the liquid. Further, it is found that for a uniform current density both in the jet and the core the maximum growth is smaller than the corresponding growth for a homogeneous liquid jet without a core.

2. FORMULATION OF THE PROBLEM

We picture to ourselves an infinitely long cylindrical jet of inviscid, incompressible fluid of small electrical conductivity confined to the region R_0 to R_1 where R_0 represents the radius of a hard metallic core insulated from the liquid by a thin insulating sheath. The core as well as the liquid are assumed to carry total axial currents I_c and I respectively so that the corresponding current densities are denoted by $j_0 (= I_c/\pi R_0^2)$ and $j_1 (= I/\pi(R_1^2 - R_0^2))$. We shall investigate the stability of this equilibrium configuration for an axially-symmetric disturbance. We suppose, then, that the cylinder is subject to a sausage-type disturbance, the result of which is to deform the free boundary of the liquid into the form,

$$r = R + a \cos kz. \tag{1}$$

Since the fluid is taken to be incompressible, it follows from mass conservation per unit-length of the cylinder that,

$$R_1^2 = R^2 + \frac{a^2}{2}. \tag{2}$$

We shall be calculating the change, $\Delta\mathscr{M}$ in the magnetic energy and the change, ΔS_T in the surface-tension energy resulting from the disturbance defined by equation (1). Then we shall have stability or instability depending upon whether the total change in potential energy namely, $\Delta\mathscr{M} + \Delta S_T$ is positive or negative.

3. MAGNETIC FIELD IN THE EQUILIBRIUM CONFIGURATION

Since currents in the core and the liquid are assumed to flow in the positive z-direction the equilibrium magnetic field will be only toroidal (θ-direction) and can be easily written down from the equations,

$$\operatorname{div} \mathbf{H} = 0, \operatorname{curl} \mathbf{H} = 4\pi\mathbf{j}. \tag{3}$$

The requirement of the continuity of tangential components of field at the core and the outer boundary gives the following components of the equilibrium magnetic field in the three regions of core, liquid and vacuum respectively,

$$\mathbf{H}_0 = (0, 2\pi j_0 r, 0) \qquad r < R_0 \tag{4a}$$

$$\mathbf{H}_1 = \left(0, \frac{2\pi}{r}[j_1 r^2 + R_0^2(j_0 - j_1)], 0\right) \quad R_0 < r < R \tag{4b}$$

$$\mathbf{H}_2 = \left(0, \frac{2\pi}{r}[j_1 R^2 + R_0^2 (j_0 - j_1)], 0\right) \quad r > R. \tag{4c}$$

4(a). DISPLACEMENT COMPONENTS

An arbitrary deformation of an incompressible fluid can be regarded as arising from a displacement $\boldsymbol{\xi}$ applied to each point of the fluid. Assuming the displacement to be irrotational, it follows from the condition of incompressibility div $\boldsymbol{\xi} = 0$ that

$$\boldsymbol{\xi} = \text{grad } \psi \tag{5}$$

so that

$$\nabla^2\psi = 0. \tag{6}$$

The solution of this equation is,

$$\psi = [C_1 I_0(kr) + C_2 K_0(kr)] \cos kz \tag{7}$$

where C_1, C_2 are constants to be determined from boundary conditions. Employing the boundary conditions that the normal displacement vanishes at the core $r = R_0$ and is equal to $a \cos kz$ (see equation (1)) at $r = R$, we get the following components of displacement,

$$\begin{aligned} \xi_r &= [p_1 I_1(kr) - p_2 K_1(kr)] a \cos kz \\ \xi_z &= -[p_1 I_0(kr) + p_2 K_0(kr)] a \sin kz \end{aligned} \tag{8}$$

where

$$p_1 = \frac{K_1(kR_0)}{I_1(kR)K_1(kR_0) - K_1(kR)I_1(kR_0)}$$

and (9)

$$p_2 = \frac{I_1(kR_0)}{I_1(kR)K_1(kR_0) - K_1(kR)I_1(kR_0)}.$$

4(b). PERTURBATIONS IN CURRENT DENSITY AND MAGNETIC FIELD

The expression for the change in magnetic field which can be easily derived from the Maxwell's equation with the usual hydromagnetic approximation, can be written down as,

$$\nabla^2\mathbf{h} = 4\pi\sigma\left[\frac{\partial \mathbf{h}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{H}_1)\right] \tag{10}$$

where $\mathbf{h}$, $\mathbf{v}$ denote the perturbation vectors of field and velocity, $\mathbf{H}_1$ is the initial field in the incompressible medium of electrical conductivity σ. Since we have assumed the electrical conductivity σ to be vanishingly small, we can write equation (10) (to the first order) as

$$\nabla^2\mathbf{h} = 0. \tag{11}$$

From the equation

$$\mathbf{j} = \sigma[\mathbf{E} + (\mathbf{v} \times \mathbf{H}_1)]$$

it follows that $j_\theta = 0$ since $\mathbf{H}_1$ is toroidal and $\mathbf{v}$ is in the meridian plane. This implies that $h_r = h_z = 0$ so that it follows from (11) that

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} - \frac{1}{r^2}\right) h_\theta = 0. \tag{12}$$

The equation (12) suggests that,

$$h_\theta = [AI_1(kr) + BK_1(kr)] \cos kz. \tag{13}$$

The perturbation in current density in the liquid is evaluated from,

$$4\pi\delta\mathbf{j} = \text{curl } \mathbf{h} \tag{14}$$

so that

$$\begin{aligned} 4\pi\delta j_r &= [AI_1(kr) + BK_1(kr)]k \sin kz \\ 4\pi\delta j_z &= [AI_0(kr) - BK_0(kr)]k \cos kz. \end{aligned} \tag{15}$$

and

The constants A and B in the above equations will be determined by appropriate boundary conditions. Since the core is insulated from the liquid, and the medium external to the liquid is vacuum, an appropriate boundary condition is that the normal component of current vanishes at the boundary surfaces i.e.

$$(\mathbf{j} \,.\, \mathbf{n}) = 0 \text{ at } r = R_0 \text{ and at } r = R + a \cos kz. \tag{16}$$

From (15) and (16) it follows that

$$\begin{aligned} A &= -4\pi j_1 a p_1 \\ B &= 4\pi j_1 a p_2. \end{aligned} \tag{17}$$

We can, therefore, rewrite the equations (13) and (15) for the perturbation in field and current density as,

$$\mathbf{h} = [0, -4\pi j_1(p_1 I_1(kr) - p_2 K_1(kr))\, a \cos kz, 0] \tag{18}$$

and

$$\delta j_r = -j_1 k[p_1 I_1(kr) - p_2 K_1(kr)]\, a \sin kz \tag{19}$$

$$\delta j_z = -j_1 k(p_1 I_0(kr) + p_2 K_0(kr))\, a \cos kz \tag{20}$$

where p_1, p_2 are given by the equations (9).

5. THE CHANGE, $\Delta\mathscr{M}$ IN MAGNETIC ENERGY PER UNIT LENGTH

The change in the magnetic energy, $\delta\Delta\mathscr{M}$ per unit length involved in increasing the perturbation from $\boldsymbol{\xi}$ to $\boldsymbol{\xi} + \delta\boldsymbol{\xi}$ can be evaluated by integrating over the whole cylinder the work done by the displacement $\delta\boldsymbol{\xi}$ against the Lorentz force $\mathbf{j} \times \mathbf{H}$ in the fluid. It is therefore given by,

$$\delta\Delta\mathscr{M} = -\int_{R_0}^{R + a\cos kz} \delta\boldsymbol{\xi} \,.\, [\mathbf{j} \times \mathbf{H}]\, 2\pi r\, dr \Big|_{\text{av}} \tag{21}$$

where the averaging is to be done with respect to z.

The above equation can be rewritten as,

$$\delta\Delta\mathscr{M} = -\int_{R_0}^{R+a\cos kz} \delta\boldsymbol{\xi} \,.\, [\mathbf{j}_1 \times \mathbf{H}_1]\, 2\pi r\, dr \Big|_{\text{av}} - \int_{R_0}^{R+a\cos kz} \delta\boldsymbol{\xi} \,.\, [\mathbf{j}_1 \times \mathbf{h} + \delta\mathbf{j}_1\mathbf{H}_1] \times 2\pi r\, dr \Big|_{\text{av}} = L_1 + L_2. \tag{22}$$

With $\mathbf{j}_1 = j_1{}^z$ and H_1 given by equation (4b) and the disturbance components by equations (8) and (9) we can readily evaluate the first integral in the equation (22). We get,

$$L_1 = 2\pi^2 a\delta a[\, j_1{}^2R^2 + j_1(j_0 - j_1)R_0{}^2]. \tag{23}$$

For evaluating the second integral in equation (22) we make use of the equations (4b), (8), (13), (19) and (20). We get

$$L_2 = -4\pi^2 a\, \delta a \int_{R_0}^{R} \{k[j_1{}^2r^2 + j_1(j_0 - j_1)R_0{}^2] \times [(p_1I_0(kr) + p_2K_0(kr)) \times (p_1I_1(kr) - p_2K_1(kr)] + j_1{}^2r(p_1I_1(kr) - p_2K_1(kr))^2\}\, dr. \tag{24}$$

With the help of standard integrals involving Bessel's Functions we can evaluate the above integrals and the result after much algebra can be written as,

$$L_2 = 2\pi^2 a\delta a \left[\left\{p_1{}^2(I_0I_2 - 2I_1{}^2) + p_2{}^2(K_0K_2 - 2K_1{}^2) - 2p_1p_2\left(I_0K_1{}' - I_1K_1 + \frac{I_1K_0}{kr}\right)\right\} j_1{}^2r^2 - (p_1{}^2I_0{}^2 + p_2{}^2K_0{}^2)j_1(j_0 - j_1)R_0{}^2\right]_{R_0}^{R}. \tag{25}$$

Combining the equations (23) and (25) and integrating over a from 0 to a, we obtain for the change in magnetic energy, the following expression,

$$\Delta\mathscr{M} = \pi^2a^2j_1{}^2R^2\left[\left(1 - \frac{j_0}{j_1}\right)\{p_1{}^2(I_0{}^2(x) - I_0{}^2(x_0) + p_2{}^2(K_0{}^2(x) - K_0{}^2(x_0)) - 1\}\left(\frac{R_0}{R}\right)^2 + \left\{1 + p_1{}^2\left(\{I_0(x)I_2(x) - 2I_1{}^2(x)\} - \{I_0(x_0)I_2(x_0) - 2I_1{}^2(x_0)\}\left(\frac{R_0}{R}\right)^2\right) + p_2{}^2\left(\{K_0(x)K_2(x) - 2K_1{}^2(x)\} - \{K_0(x_0)K_2(x_0) - 2K_1{}^2(x_0)\}\left(\frac{R_0}{R}\right)^2\right) + p_1p_2\left(\left\{2I_1(x)K_1(x) + I_0(x)(K_0(x) + K_2(x)) - \frac{2I_1(x)K_0(x)}{x}\right\} - \left\{2I_1(x_0)K_1(x_0) + I_0(x_0) \times (K_0(x_0) + K_2(x_0)) - \frac{2I_1(x_0)K_0(x_0)}{x_0}\right\}\left(\frac{R_0}{R}\right)^2\right)\right\}\right], \tag{26}$$

where $x = kR$ and $x_0 = kR_0$.

6. THE CHANGE IN SURFACE TENSION ENERGY

The change in surface tension energy per unit length of the cylinder resulting from the deformation of the type given by equation (1) is,

$$\Delta S_T = \frac{\pi T}{2R}(x^2 - 1)a^2 \tag{27}$$

where T is the surface-tension of the liquid. The equation (27) shows that the change in surface tension energy is of second order in a, positive for $x > 1$ and negative for $x < 1$.

7. DISPERSION RELATION

To obtain the dispersion relation, we have to evaluate the kinetic energy of motion resulting from the varying amplitude. The expression for kinetic energy τ is

$$\tau = \tfrac{1}{2}\rho \int_{R_0}^{R+a\cos kz} v^2\, 2\pi r\, dr \Big|_{\text{av}} \tag{28}$$

which on evaluation yields,

$$\tau = \tfrac{1}{2}\pi\rho R^2\left(\frac{da}{dt}\right)^2\left[p_1{}^2\left\{\frac{I_0(x)I_1(x)}{x} - \frac{I_0(x_0)I_1(x_0)}{x_0}(R_0/R)^2\right\} - p_2{}^2\left\{\frac{K_0(x)K_1(x)}{x} - \frac{K_0(x_0)K_1(x_0)}{x_0}(R_0/R)^2\right\} + p_1p_2\left\{\frac{I_1(x)K_0(x) - I_0(x)K_1(x)}{x} - \frac{I_1(x_0)K_0(x_0) - I_0(x_0)K_1(x_0)}{x_0}\left(\frac{R_0}{R}\right)^2\right\}\right]. \tag{29}$$

The Lagrangian equation of motion for a can then easily be written as,

$$\frac{d^2a}{dt^2} = -\frac{2\pi I^2 a}{\pi^2(R^2 - R_0{}^2)^2\rho} \cdot \frac{Q}{S}. \tag{30}$$

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Here

$$Q = W^2(x^2-1) + \left\{\frac{R_0^2}{R^2} - \frac{I_c}{I}\left(1-\frac{R_0^2}{R^2}\right)\right\} \times \{p_1^2(I_0^2(x) - I_0^2(x_0)) + p_2^2(K_0^2(x) - K_0^2(x_0)) - 1\} + \Big[1 + p_1^2[\{I_0(x)I_2(x) - 2I_1^2(x)\} - \{I_0(x_0)I_2(x_0) - 2I_1^2(x_0)\}(R_0/R)^2] + p_2^2[\{K_2(x)K_0(x) - 2K_1^2(x)] - \{K_2(x_0)K_0(x_0) - 2K_1^2(x_0)\}(R_0/R^2) + p_1p_2\left(\left\{2I_1(x)K_1(x) + I_0(x)K_2(x) + I_0(x)K_0(x) - \frac{2I_1(x)K_0(x)}{x}\right\} - \left\{2I_1(x_0)K_1(x_0) + I_0(x_0)K_2(x_0) + I_0(x_0)K_0(x_0) - \frac{2I_1(x_0)K_0(x_0)}{x_0}\right\}\left(\frac{R_0}{R}\right)^2\right)\Big] \tag{31}$$

in which

$$W^2 = \frac{\pi TR}{2I^2}\left(1-\frac{R_0^2}{R^2}\right)^2 \tag{32}$$

and

$$S = p_1^2\left\{\frac{I_0(x)I_1(x)}{x} - \frac{I_0(x_0)I_1(x_0)}{x_0}(R_0/R)^2\right\} - p_2^2\left\{\frac{K_0(x)K_1(x)}{x} - \frac{K_0(x_0)K_1(x_0)}{x_0}(R_0/R)^2\right\} + p_1p_2\left\{\frac{I_1(x)K_0(x) - I_0(x)K_1(x)}{x} - \frac{I_1(x_0)K_0(x_0) - I_0(x_0)K_1(x_0)}{x_0}(R_0/R)^2\right\}. \tag{33}$$

The solution of equation (30) is of the form

$$a = a_0 e^{int} \tag{34}$$

where

$$n^2 = \frac{2I^2}{\pi\rho R^4\left(1-\frac{R_0^2}{R^2}\right)^2}\cdot\frac{Q}{S}. \tag{35}$$

For no core i.e. $R_0 \to 0$ the equation (35) simplifies to,

$$n_0^2 = \frac{2I^2}{\pi\rho R^4}\left[\frac{\pi TR}{2I^2}(x^2-1) + \frac{I_2(x)I_0(x)}{I_1^2(x)} - 1\right]\frac{xI_1(x)}{I_0(x)}. \tag{36}$$

This equation should apply to the configuration adopted by DATTNER *et al.* (1958) in their experiment with homogeneous mercury jet subject to an axial current. Further, the expression (36) is the same as the one obtained by MURTY (1960) by alternative considerations.

8. DISCUSSION OF RESULTS

To ascertain the role which the current system envisaged here would play in determining the stability of the configuration, we have to look to the sign of the change in magnetic energy which is of second order in displacement parameter a. In Fig. 1, we plot the variation of $\Delta\mathcal{M}/\pi^2R^2j_1^2a^2$ with $x(=kR)$ for various values of the ratio I_c/I (I_c and I respectively denote the total current in the core and the liquid) taking $R_0/R = 0{\cdot}5$. From the figure we conclude that when I_c and I are in the same direction, the change in magnetic energy is negative and so a parallel current system has a destabilizing influence (curves $|d|$ and $|e|$ in Fig. 1). On the other hand the curves $|a|$ and $|b|$ for negative values of I_c/I exhibit the stabilizing influence of an antiparallel current system on the configuration since the change in magnetic energy is positive. However, even for antiparallel currents there may be instability produced for some values of I_c/I lying between 0 and -1. In the absence of any current in the core, the change in magnetic energy is, however, negative showing the destabilizing nature of the current in the jet (see curve $|c|$ in Fig. 1).

In Fig. 2 we show that the variation of n^2 (the square of the growth rate) with x for the unstable situation corresponding to the parallel currents in the core and the liquid. The values adopted for calculation are $R = 2{\cdot}5$ mm, $T = 490$ dyne/cm and $I = 150$A. The curves $|a|$, $|b|$ and $|c|$ correspond to $I_c/I = 2, 1, 0$ respectively. It follows from the figure that for every positive value, of the ratio I_c/I the situation is unstable below a critical value of x and that there exists a mode of maximum instability for each I_c/I. Further, it is clear from the figure that the maximum growth rate and the extent of the unstable region increases with the increase in the core current. For antiparallel currents with $|I_c/I| > 1$, the frequency of oscillation is, however, seen to be increasing with x.

It should be noted that LEHNERT and SJÖGREN reported that the instability as produced by current in the liquid jet is almost suppressed by a sufficiently high parallel current in the core; and that the jet breaks up violently under an antiparallel current system. The disparity in our results and those of LEHNERT and SJÖGREN for the antiparallel current system may be arising due to our restricted analysis of sausage instabilities. For parallel currents the configuration is unstable (see Fig. 2). This instability has, however,

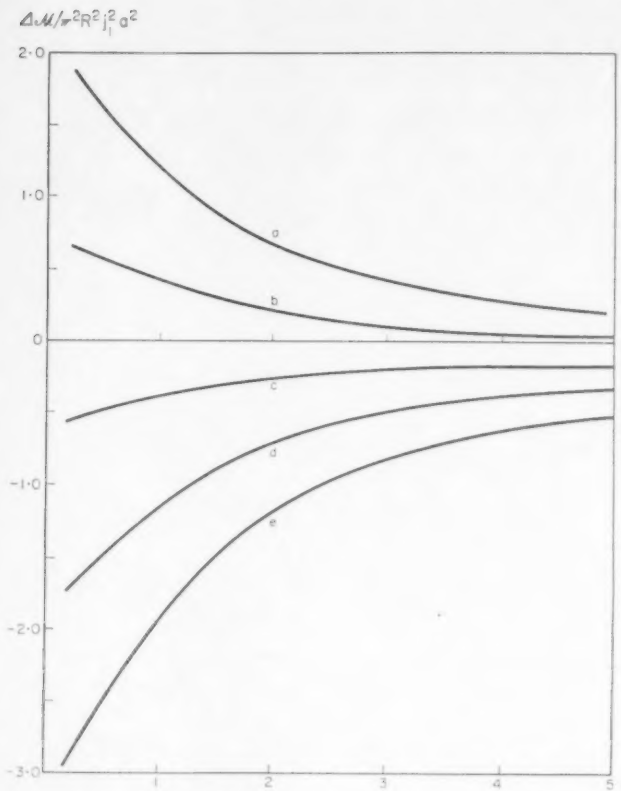

FIG. 1.—Plot of $\Delta\mathcal{M}/\pi^2R^2j_1^2a^2$ against $x\,(=kR)$ for various values of the ratio I_c/I (I_c and I denote respectively the total current in the core and in the liquid). Curves a, b, c, d and e correspond respectively to values -2, -1, 0, $+1$, $+2$ of I_c/I.

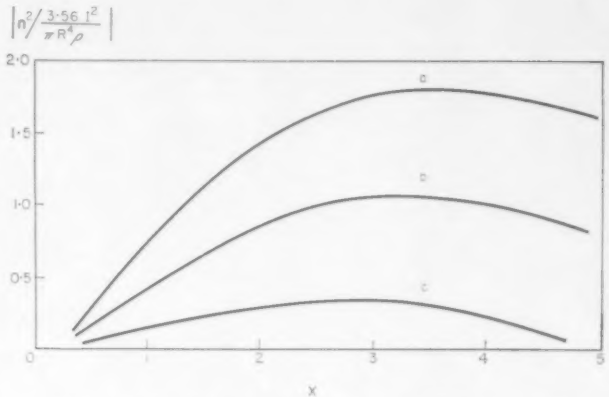

FIG. 2.—Curves a, b and c exhibit the dependence of growth rate on x for values $+2$, $+1$, 0 of I_c/I respectively.

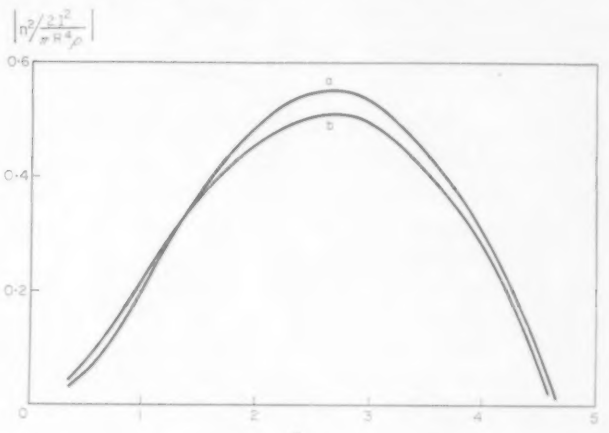

FIG. 3.—Growth rate as a function of x. Curve a is for a homogeneous model and the curve b for a core—model characterized by homogeneous current density.

not been observed by LEHNERT and SJÖGREN in their experiment. They believe that the presence of a core should suppress the growth rate of a sausage instability.

To elucidate this point further we considered two configurations of liquid jet—one homogeneous (without core) and the other with core, both characterized by equal current densities throughout. The results for $R_0/R = 0{\cdot}5$ are presented in Fig. 3 from which we conclude that the *maximum growth rate is diminished* due to the presence of the core. However, this question, it should be admitted, requires further elaboration.

We hope to extend the present analysis in the near future to include non-axisymmetric disturbances and high electrical conductivity of the fluid so as to make it directly applicable to the hard-core pinch.

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NECESSARY AND SUFFICIENT CONDITIONS FOR THE HYDROMAGNETIC RAYLEIGH-TAYLOR STABILITY OF A CYLINDRICAL PLASMA

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Abstract—It is shown that necessary and sufficient conditions for the stability of a cylindrical discharge can be extended to the case in which there is an external gravitational potential present. In particular explicit forms are obtained for the analogues of three well-known stability criteria: the sufficient condition for stability of the hard-core pinch, SUYDAM's necessary condition for stability and the necessary and sufficient condition for stability against interchange perturbations of a system with a magnetic field of constant pitch.

1. INTRODUCTION

THE purpose of this paper is to show how the many necessary or sufficient conditions for the stability of a cylindrical pinched discharge can be modified so as to apply to the problem in which there is also an external gravitational potential present.

There is interest in the hydromagnetic Rayleigh-Taylor stability problem not because it is thought that gravitational forces play an important role in plasma stability but because it is hoped that acceleration forces which act on many plasma configurations can be simulated by a gravitational-type term in the equations. Thus consider a cylindrical discharge executing radial bounces. At the moment at which the system is instantaneously at rest in its position of maximum radius, its inward acceleration plays a similar role in the equation of motion to that of an outward gravitational force. The simplest theory of hydromagnetic Rayleigh-Taylor stability in which a plasma is supported against gravity by a magnetic field predicts instability (KRUSKAL and SCHWARZSCHILD, 1954). The introduction of a magnetic field in the plasma in a different direction to that supporting it reduces the instability but does not remove it completely (MEYER, 1958). Many experiments have been performed in which observed instabilities have been interpreted as Rayleigh-Taylor instabilities (COLGATE, 1957; CURZON *et al.*, 1960; GREEN and NIBLETT, 1960).

It is clear that the substitution of an equivalent gravitational force for the acceleration acting on the plasma will not lead to a complete description of the plasma stability problem. Instabilities whose growth time is comparable to the time in which the plasma configuration changes significantly cannot be adequately represented. An attempt to study these instabilities, also in a very simple configuration, has been given in TAYLER, (1959a).

In the present paper the origin of the gravitational potential is not discussed. It is shown that stability criteria already known for a plasma without a gravitational field can easily be extended to the case in which such a field is present. The reason for this is as follows. It can be shown that in both problems the investigation of the stability of the system is reduced to determining whether an expression of the form

$$\delta W = \int (A\xi^2 + B\xi'^2)\, dr \qquad (1.1)$$

can ever be negative for functions $\xi(r)$ satisfying various boundary conditions, where A and B are determined by the equilibrium configuration and the type of perturbation considered and the prime denotes differentiation with respect to r. The introduction of the gravitational potential only alters the form of A and, as the stability criteria can be obtained without specifying the precise form of A, they can easily be extended to include the gravitational term.

Three particular criteria which are obtained in the present paper are:

(i) A sufficient condition for stability

$$-\frac{2B_0}{r^2}(rB_0)' - \rho'\phi' - \frac{\rho^2\phi'^2}{\gamma p} > 0. \qquad (1.2)$$

(ii) A necessary condition for stability

$$\left(\frac{r\mu'}{\mu}\right)^2 + \frac{8r(p' + \rho\phi')}{B_z^2} > \frac{4r^2B^2}{B_0^2B_z^2}\left[\rho'\phi' + \frac{\rho^2\phi'^2}{\gamma p}\right]. \qquad (1.3)$$

(iii) A necessary and sufficient condition for stability

against interchange perturbations when the magnetic field has constant pitch

$$2B_0(B_0/r)' + \rho'\phi' + (2B_0^2/r + \rho\phi')^2/(B_0^2 + B_z^2 + \gamma p) \leqslant 0. \quad (1.4)$$

In these equations B_0 and B_z are components of the magnetic field, p, ρ and γ are the plasma pressure, density and ratio of specific heats and ϕ is the gravitational potential. In (1.3) μ is B_0/rB_z.

The remainder of the paper is arranged as follows. In Section 2 the change of potential energy of the system upon a perturbation $\boldsymbol{\xi}$ is reduced to the form (1.1) and the stability criteria are deduced in Section 3.

2. DERIVATION OF EXPRESSION FOR CHANGE OF POTENTIAL ENERGY

A cylindrical ideally conducting plasma of density $\rho(r)$ and pressure $p(r)$ carries a magnetic field $(0, B_0(r), B_z(r))$ and a current $(0, j_\theta(r), j_z(r))$. It is acted on by an external gravitational field $(-d\phi(r)/dr, 0, 0)$. The plasma is surrounded by an ideally conducting wall at radius R_0 and it may have a conducting rod of radius r_0 through its centre. The magnetic field and the current are related by the equation

$$\mathbf{j} = \operatorname{curl} \mathbf{B}$$

and the system is in equilibrium provided that

$$B_0\, dB_0/dr + B_0^2/r + B_z\, dB_z/dr + dp/dr + \rho\, d\phi/dr = 0. \quad (2.1)$$

In BERNSTEIN *et al.* (1957a, 1958) it is shown that, if a perturbation of the equilibrium is considered in which any plasma element moves a distance $\boldsymbol{\xi}$ from its initial position, the change in potential energy of the system is given by

$$\delta W = \tfrac{1}{2}\int d\tau[\mathbf{Q}^2 - \mathbf{j}\cdot\mathbf{Q}\times\boldsymbol{\xi} + \gamma p\,(\operatorname{div}\boldsymbol{\xi})^2 + \boldsymbol{\xi}\cdot\operatorname{grad} p \operatorname{div}\boldsymbol{\xi} - \boldsymbol{\xi}\cdot\operatorname{grad}\phi \operatorname{div}\rho\boldsymbol{\xi}], \quad (2.2)$$

where $\mathbf{Q} = \operatorname{curl}(\boldsymbol{\xi}\times\mathbf{B})$. Using the hydrostatic equation (2.1), (2.2) can be rewritten

$$\begin{aligned}\delta W &= \tfrac{1}{2}\int d\tau[\mathbf{Q}^2 - \mathbf{j}\cdot\mathbf{Q}\times\boldsymbol{\xi} + \gamma p\,(\operatorname{div}\boldsymbol{\xi})^2 + \boldsymbol{\xi}\cdot\mathbf{j}\times\mathbf{B}\operatorname{div}\boldsymbol{\xi} - 2\boldsymbol{\xi}\cdot\operatorname{grad}\phi\,\rho\operatorname{div}\boldsymbol{\xi} - \boldsymbol{\xi}\cdot\operatorname{grad}\phi\,\boldsymbol{\xi}\cdot\operatorname{grad}\rho] \\ &= \delta W_1 - \tfrac{1}{2}\int d\tau[2\boldsymbol{\xi}\cdot\operatorname{grad}\phi\,\rho\operatorname{div}\boldsymbol{\xi} + \boldsymbol{\xi}\cdot\operatorname{grad}\phi\,\boldsymbol{\xi}\cdot\operatorname{grad}\rho], \end{aligned} \quad (2.3)$$

where δW_1 is the usual expression for the change of potential energy when there is no gravitational field present.

The system is unstable against the perturbation $\boldsymbol{\xi}$ if δW is negative; if the system is stable δW must be positive for all perturbations $\boldsymbol{\xi}$ which satisfy certain boundary conditions. Because of the symmetry of the present problem, the perturbation $\boldsymbol{\xi}$ can be Fourier analysed into non-interacting normal modes. Thus

$$\boldsymbol{\xi} = \tfrac{1}{2}\sum_{m,k}\{\boldsymbol{\xi}_{mk}(r)\,e^{i(m\theta+kz)}) + \boldsymbol{\xi}^*_{mk}\,e^{-i(m\theta+kz)}\}, \quad (2.4)$$

where the asterisk denotes the complex conjugate. If this form for $\boldsymbol{\xi}$ and the corresponding form for $\mathbf{Q}$ are substituted into expression (2.3) and the θ and z integrations are performed, it can be seen that there are no non-vanishing contributions to the energy from products of $\boldsymbol{\xi}$'s belonging to distinct modes. In addition (except in the special case $m = k = 0$) only terms of the type $\boldsymbol{\xi}\cdot\boldsymbol{\xi}^*$, as opposed to $\boldsymbol{\xi}^2$, $\boldsymbol{\xi}^{*2}$, contribute.

The change in potential energy must be positive for every single normal mode for stability and for such a mode

$$\begin{aligned}\delta W \propto \int r\,dr[&\mathbf{Q}\cdot\mathbf{Q}^* - \tfrac{1}{2}\mathbf{j}\cdot\mathbf{Q}\times\boldsymbol{\xi}^* - \tfrac{1}{2}\mathbf{j}\cdot\mathbf{Q}^*\times\boldsymbol{\xi} \\ &+ \gamma p\operatorname{div}\boldsymbol{\xi}\operatorname{div}\boldsymbol{\xi}^* + \tfrac{1}{2}\boldsymbol{\xi}\cdot\mathbf{j}\times\mathbf{B}\operatorname{div}\boldsymbol{\xi}^* \\ &+ \tfrac{1}{2}\boldsymbol{\xi}^*\cdot\mathbf{j}\times\mathbf{B}\operatorname{div}\boldsymbol{\xi} - \boldsymbol{\xi}\cdot\operatorname{grad}\phi\,\rho\operatorname{div}\boldsymbol{\xi}^* \\ &- \boldsymbol{\xi}^*\cdot\operatorname{grad}\phi\,\rho\operatorname{div}\boldsymbol{\xi} - \boldsymbol{\xi}\cdot\operatorname{grad}\phi\,\boldsymbol{\xi}^*\cdot\operatorname{grad}\rho], \end{aligned} \quad (2.5)$$

where the suffixes m and k have been suppressed and in the last term it has been noted that grad ϕ and grad ρ both have only radial components. In TAYLER (1959b) an explicit expression has been given for δW_1 in terms of m and k and the equilibrium quantities. Thus

$$\begin{aligned}\delta W_1 = \int r\,dr\Big[&\Big(\frac{mB_0}{r} + kB_z\Big)^2\boldsymbol{\xi}\cdot\boldsymbol{\xi}^* \\ &+ (B_0^2 + B_z^2 + \gamma p)\operatorname{div}\boldsymbol{\xi}\operatorname{div}\boldsymbol{\xi}^* \\ &- 2B_0(B_0/r)'\xi_r\xi_r^* - (2B_0^2/r)(\xi_r\operatorname{div}\boldsymbol{\xi}^* \\ &+ \xi_r^*\operatorname{div}\boldsymbol{\xi}) + i\Big(\frac{mB_0}{r} + kB_z\Big) \\ &\times\{2B_0/r(\xi_r^*\,\xi_\theta - \xi_\theta^*\,\xi_r) - B_0(\xi_\theta\operatorname{div}\boldsymbol{\xi}^* \\ &- \xi_\theta^*\operatorname{div}\boldsymbol{\xi}) - B_z(\xi_z\operatorname{div}\boldsymbol{\xi}^* - \xi_z^*\operatorname{div}\boldsymbol{\xi})\}\Big]. \end{aligned} \quad (2.6)$$

Clearly, from equation (2.5)

$$\delta W = \delta W_1 - \int r\,dr\,[\xi_r\xi_r^*\,\phi'\rho' + \rho\phi'(\xi_r\operatorname{div}\boldsymbol{\xi}^* + \xi_r^*\operatorname{div}\boldsymbol{\xi})]. \quad (2.7)$$

In equations (2.6) and (2.7) the prime denotes differentiation with respect to r.

The expression (2.7) must be minimized with respect to all possible perturbations. Stationary values of δW are obtained if $\boldsymbol{\xi}$ is a solution of the Euler-Lagrange equations of δW but these stationary values are not necessarily minima. If δW is written

$$\delta W = \int F(\boldsymbol{\xi}, \boldsymbol{\xi}^*)\, dr,$$

the Euler-Lagrange equations are

$$\frac{\partial F}{\partial \xi_i^*} = \frac{\partial}{\partial r}\frac{\partial F}{\partial(\partial \xi_i^*/\partial r)}, \tag{2.8}$$

where ξ_i^* is any component of $\boldsymbol{\xi}^*$. In TAYLER (1959b) it is shown that, in the absence of gravitational forces, the θ and z Euler-Lagrange equations are minimizing equations and it is easy to see that the proof is still valid for the present problem. Use of these two equations enables an expression to be found for δW in terms of ξ_r alone.

For the present problem the θ and z Euler-Lagrange equations have the form

$$B_z\left(\frac{mB_\theta}{r} + kB_z\right)\left(k\xi_\theta - \frac{m\xi_z}{r}\right) + \operatorname{div}\boldsymbol{\xi}\left[-\frac{im\gamma p}{r} - \frac{imB_z^2}{r} + ikB_\theta B_z\right] + \xi_r\left[\frac{im\rho\phi'}{r} - \frac{2ikB_\theta B_z}{r}\right] = 0 \tag{2.9}$$

and

$$-B_\theta\left(\frac{mB_\theta}{r} + kB_z\right)\left(k\xi_\theta - \frac{m\xi_z}{r}\right) + \operatorname{div}\boldsymbol{\xi}\left[-ik\gamma p - ikB_\theta^2 + \frac{imB_\theta B_z}{r}\right] + \xi_r\left[\frac{2ik}{r}B_\theta^2 + ik\rho\phi'\right] = 0. \tag{2.10}$$

If equation (2.9) is multiplied by B_θ and (2.10) by B_z and they are added, it can be seen that

$$\left(\frac{mB_\theta}{r} + kB_z\right)[\rho\phi'\xi_r - \gamma p \operatorname{div}\boldsymbol{\xi}] = 0. \tag{2.11}$$

Thus, unless $\dfrac{mB_\theta}{r} + kB_z = 0$,

$$\operatorname{div}\boldsymbol{\xi} = (\rho\phi'/\gamma p)\xi_r. \tag{2.12}$$

As in the previous problem, it can be shown that if $mB_\theta/r + kB_z$ only vanishes at a discrete set of points no error is made in assuming that equation (2.12) holds everywhere. If, however, there is a finite region in which the magnetic field has a constant pitch, there will be pairs of values of m and k for which $mB_\theta/r + kB_z$ vanishes throughout the region. For these interchange perturbations, the expression for δW takes the form

$$\delta W = \int r\, dr\, [(B_\theta^2 + B_z^2 + \gamma p)\operatorname{div}\boldsymbol{\xi}\operatorname{div}\boldsymbol{\xi}^* - (2B_\theta(B_\theta/r)' + \rho'\phi')\xi_r\xi_r^* - (2B_\theta^2/r + \rho\phi')(\xi_r \operatorname{div}\boldsymbol{\xi}^* + \xi_r^* \operatorname{div}\boldsymbol{\xi})]. \tag{2.13}$$

When the fields are not of constant pitch, equations (2.9) and (2.10) can also be combined to give

$$\left(k\xi_\theta - \frac{m\xi_z}{r}\right) = i\xi_r\left[\frac{2kB_\theta}{r} + \frac{\rho\phi'}{\gamma p} \times \left(\frac{mB_z}{r} - kB_\theta\right)\right]\Big/\left(\frac{mB_\theta}{r} + kB_z\right). \tag{2.14}$$

If equations (2.12) and (2.14) are substituted into (2.7), an expression for δW in terms of ξ_r alone is obtained. Thus

$$\delta W = \int r\, dr\left[\xi^2\left\{\left(\frac{mB_\theta}{r} + kB_z\right)^2 - \frac{2B_\theta}{r^2}(rB_\theta)' - \rho'\phi' - \frac{\rho^2\phi'^2}{\gamma p}\right\} + \left\{\left(\frac{mB_\theta}{r} + kB_z\right)r\xi' - \left(\frac{mB_\theta}{r} - kB_z\right)\xi\right\}^2\Big/(m^2 + k^2r^2)\right], \tag{2.15}$$

where ξ is $|\xi_r|$.

BERNSTEIN *et al.* (1957b) give an alternative expression for δW_1 which is sometimes very useful. Thus

$$\delta W_1 = \tfrac{1}{2}\int d\tau[(\mathbf{Q} + \mathbf{n}\,.\,\boldsymbol{\xi}\,\mathbf{j}\times\mathbf{n})^2 + \gamma p(\operatorname{div}\boldsymbol{\xi})^2 - 2(\mathbf{n}\,.\,\boldsymbol{\xi})^2\,\mathbf{j}\times\mathbf{n}\,.\,(\mathbf{B}\,.\,\nabla\mathbf{n})] \tag{2.16}$$

where $\mathbf{n}$ is a unit normal vector to the magnetic surfaces and with their definition it is in the inward radial direction in cylindrical geometry. The additional terms in equation (2.3) can easily be written in terms of $\mathbf{n}$ so that

$$\delta W = \delta W_1 - \tfrac{1}{2}\int d\tau\, \mathbf{n}\,.\,\boldsymbol{\xi}\phi'(\mathbf{n}\,.\,\boldsymbol{\xi}\rho' - 2\rho\operatorname{div}\boldsymbol{\xi}). \tag{2.17}$$

It is easily seen that (2.17) is an expression involving only ξ_r, $m\xi_\theta/r + k\xi_z$ and $\xi_\theta B_z - \xi_z B_\theta$. As long as $mB_\theta/r + kB_z \neq 0$, $m\xi_\theta/r + k\xi_z$ and $\xi_\theta B_z - \xi_z B_\theta$ are distinct quantities and they can be taken as new independent perturbations instead of ξ_θ and ξ_z. In fact it is convenient to use ξ_r, $\operatorname{div}\boldsymbol{\xi}$ and $\xi_\theta B_z - \xi_z B_\theta$ as the three independent quantities. It can then be seen at once that the $\operatorname{div}\boldsymbol{\xi}$ Euler equation is

$$\operatorname{div}\boldsymbol{\xi} = \rho\phi'\xi_r/\gamma p$$

and the second Euler equation can be obtained quite easily. The expression for δW in terms of the above

variables has been used by NEWCOMB (1960) and similar variables have been used by MERCIER (1960) in his discussion of necessary conditions for toroidal stability.

3. THE STABILITY CRITERIA

(a) *Sufficient conditions for stability*

From equation (2.15) it can be seen that the only term in δW which can possibly be negative is

$$\int r\,dr\,\xi^2\left[-\frac{2B_0}{r^2}(rB_0)'-\rho'\phi'-\frac{\rho^2\phi'^2}{\gamma p}\right] \tag{3.1}$$

and therefore a sufficient condition for stability is that

$$-\frac{2B_0}{r^2}(rB_0)'-\rho'\phi'-\frac{\rho^2\phi'^2}{\gamma p}>0 \tag{3.2}$$

throughout the plasma. It may be noted that this condition does not depend on m and k. This criterion is the analogue of the hard core or unpinch criterion in the absence of gravity. The last term in (3.2) is always negative; thus one at least of the first two must be definitely positive. If the first term is to be positive there must be a rod down the centre of the plasma carrying a current; if the second term is positive the density must be increasing in the direction in which gravity is acting. There will also be configurations in which one term is positive in one part of the plasma and the other is positive in the other part.

A second sufficient condition for stability can be obtained by integrating by parts the term in $\xi\xi'$ in equation (2.15). This condition has been given in the absence of gravity by LAING (1958). As ξ must vanish at both $r=r_0$ and $r=R_0$ (and if there is no central conductor $r\xi$ vanishes at $r=0$) the integrated part vanishes and δW takes the form

$$\delta W=\int(A\xi^2+B\xi'^2)\,dr, \tag{3.3}$$

where

$$\begin{aligned}A=&\,r\left(\frac{mB_\theta}{r}+kB_z\right)^2-\frac{2B_\theta}{r}(rB_\theta)'-r\rho'\phi'\\&-\frac{r\rho^2\phi'^2}{\gamma p}+r\left(\frac{mB_\theta}{r}-kB_z\right)^2\Big/(m^2+k^2r^2)\\&-\left\{r^2\left(k^2B_z{}^2-\frac{m^2B_\theta{}^2}{r^2}\right)\Big/(m^2+k^2r^2)\right\}'\end{aligned} \tag{3.4}$$

and

$$B=r^3\left(\frac{mB_\theta}{r}+kB_z\right)^2\Big/(m^2+k^2r^2). \tag{3.5}$$

A sufficient condition for stability is that

$$A>0. \tag{3.6}$$

This is a much more difficult criterion to apply than (3.2) as it depends in such a complicated fashion on m and k and for complete stability it must be satisfied for all m and k. LAING (1958) has shown that, in the absence of gravity, the criterion can only be satisfied if the pressure increases outwards everywhere; no such general discussion of criterion (3.6) will be attempted at the moment.

SUYDAM (1960) has shown how to obtain a more general class of sufficient conditions. He introduces a function Φ which is arbitrary at the moment and notes that

$$\int_{r_0}^{R_0}2\Phi\xi\xi'\,dr=-\int_{r_0}^{R_0}\Phi'\xi^2\,dr, \tag{3.7}$$

since the integrated part vanishes at both limits. (if $r_0=0$ he applies the additional constraint that Φ vanishes at $r=0$ for $m=1$ perturbations). Using equation (3.7), it is easy to see that (3.3) can be rewritten

$$\delta W=\int\left[\left(A-\Phi'-\frac{\Phi^2}{B}\right)\xi^2+B\left(\xi'-\frac{\Phi}{B}\xi\right)^2\right]dr. \tag{3.8}$$

A sufficient condition for stability is now that

$$A>\Phi'+\frac{\Phi^2}{B}. \tag{3.9}$$

If a function Φ can be found so that, for some pairs of values of m and k, $\Phi'+\dfrac{\Phi^2}{B}$ is negative throughout the plasma, for these values of m and k (3.9) is a more convenient criterion than (3.6).

As all the above conditions are sufficient conditions for stability, there is no necessity for them to be satisfied for the plasma to be stable, However it is often convenient to use criterion (3.9) to show that the system is certainly stable for some pairs of values of m and k. This reduces the range of wave numbers to which more sophisticated arguments have to be applied. LAING (1958) has also shown that it is sometimes convenient to perform an integration by parts for only part of the volume of the plasma; thus for example a different functional form for Φ could be used in different regions. However in this case the integrated part $[\Phi\xi^2]$ does not vanish at such an interface and this introduces an additional term which must be positive. LAING (1958) has used this technique to find stable configurations of the cylindrical pinch.

(b) *Necessary condition for stability*

As has been mentioned previously, stationary values of (2.15) can be found by making ξ a solution of the Euler-Lagrange equation of this expression. This may

or may not give a minimum value for δW but for stability δW must certainly be positive for such perturbations. The Euler-Lagrange equation, most readily obtained from the expression (3.3) for δW, is

$$(B\xi')' = A\xi. \tag{3.10}$$

From the expressions (3.4) and (3.5) for A and B it can be seen that the solutions of this equation have singularities at points at which $\left(\frac{mB_\theta}{r} + kB_z\right)$ vanishes. If such a point is $r = a$, ξ is proportional to $(r - a)^\nu$ in the neighbourhood of the singularity, where ν satisfies the equation

$$\nu^2 + \nu + M^2 = 0 \tag{3.11}$$

and M^2 can be shown to take the form

$$M^2 = \frac{1}{B_z^2}\left(\frac{\mu}{\mu'}\right)^2\left[\frac{B^2}{B_\theta^2}\left(\rho'\phi' + \frac{\rho^2\phi'^2}{\gamma p}\right) - \frac{2(p' + \rho\phi')}{r}\right], \tag{3.12}$$

where $\mu = B_\theta/rB_z$.

A singular ξ is not physically realistic and, in the case in which gravity is absent, SUYDAM (1958) considers perturbations which follow the Euler-Lagrange solution to within a distance ε on either side of the singularity and are constant within that region. He shows that the value of δW is determined by the behaviour of ξ near to the singularity and that, if δW is to be non-negative for all perturbations, the roots of equation (3.11) must be real. This proof, which is discussed in detail in TAYLER (1959c) clearly extends to the present problem and a necessary condition for stability is thus that

$$1 \geqslant 4M^2$$

$$\text{or} \quad \left(\frac{r\mu'}{\mu}\right)^2 + \frac{8r(p' + \rho\phi')}{B_z^2} \geqslant \frac{4r^2B^2}{B_\theta^2 B_z^2}\left[\rho'\phi' + \frac{\rho^2\phi'^2}{\gamma p}\right]. \tag{3.13}$$

Since at every radius there are some pairs of values of m and k for which $mB_\theta/r + kB_z$ vanishes, it is necessary for stability that criterion (3.13) is satisfied at all points in the plasma.

It is perhaps worthwhile to write (3.13) in the slightly different form

$$\frac{(rB_\theta'B_z - B_\theta B_z - rB_z'B_\theta)^2}{B_\theta^2 + B_z^2} \geqslant -\frac{8rB_\theta^2(p' + \rho\phi')}{B^2} + 4r^2\left[\rho'\phi' + \frac{\rho^2\phi'^2}{\gamma p}\right]. \tag{3.14}$$

In this form it can be readily seen that the first term expresses the stabilizing influence of field shear, the second expresses the destabilizing influence of the curved B_θ lines in a region in which the combined pressure and gravity forces act outwards and the third term is the influence of gravity which as in criterion (3.2) is destabilizing if $\rho'\phi' + \frac{\rho^2\phi'^2}{\gamma p} > 0$.

(c) *Necessary and sufficient conditions for stability*

In the absence of gravitational forces, NEWCOMB (1960) and SUYDAM (1960) have both shown how the true minimization of δW can be performed so that necessary and sufficient conditions for stability can be obtained. The conditions are expressible entirely in terms of the behaviour of the solution of the Euler-Lagrange equation (3.10) and the derivation of the criteria does not depend critically on the forms of the functions A and B; in particular the modification of A produced by the gravitational field does not affect the following expression of the necessary and sufficient conditions for stability.

Suppose that, for a given pair of values of m and k, $mB_\theta/r + kB_z$ vanishes at the radii $r_1, r_2, \ldots\ldots r_n$. Then a necessary and sufficient condition for the stability of this mode is that:

(i) The Euler-Lagrange solution which vanishes at $r = r_0$ should not have another zero before $r = r_1$.

(ii) No Euler-Lagrange solution should have more than one zero between any two singular points r_i, r_j.

(iii) The Euler-Lagrange solution which vanishes at $r = R_0$ should not have previously vanished between $r = r_n$ and $r = R_0$.

NEWCOMB (1960) has given many alternative expressions for these criteria which can be applied more conveniently to the solution of specific problems. The introduction of gravity does not alter the stability criteria above but only the form of the equation to be solved. It should be noted that these NEWCOMB conditions, unlike the conditions obtained previously, do not depend only on the values of the equilibrium quantities and their derivatives at one particular point; for this reason they are much more difficult to apply. It may also be noted that, if criterion (3.6) is satisfied, it is obvious that no solution can have more than one zero between successive singularities whereas, if the SUYDAM criterion (3.13) is violated, there is an infinite number of zeros in such a region.

(d) *Interchange instabilities in a region of constant field pitch*

The expression (2.13) for δW is a function of the two variables div $\boldsymbol{\xi}$ and ξ_r and it is minimized with respect to the tangential components of $\boldsymbol{\xi}$ if

$$\text{div}\,\boldsymbol{\xi} = (2B_\theta/r + \rho\phi')\,\xi_r/(B_\theta^2 + B_z^2 + \gamma p). \tag{3.15}$$

The expression for δW then becomes

$$\delta W = -\int r\,dr\,\xi_r\xi_r^*[2B_\theta(B_\theta/r)' + \rho'\phi' + (2B_\theta^2/r + \rho\phi')^2/(B_\theta^2 + B_z^2 + \gamma p)]. \quad (3.16)$$

It is clearly sufficient for stability against this type of perturbation that

$$2B_\theta(B_\theta/r)' + \rho'\phi' + (2B_\theta^2/r + \rho\phi')^2/(B_\theta^2 + B_z^2 + \gamma p) < 0. \quad (3.17)$$

This condition is also necessary for stability; for suppose it is violated in some region. Since the expression (3.16) does not contain the derivative of ξ_r, a perturbation can be chosen for which ξ_r is only non-zero in the region in which condition (3.17) is violated and for such a perturbation the system is unstable. Condition (3.17) can be rewritten in various ways by making use of the equations

$$B_\theta/rB_z = \text{const.}$$

and $B_\theta B_\theta' + B_z B_z' + B_\theta^2/r + p' + \rho\phi' = 0.$

(e) *Special cases*

Special cases of fields of constant pitch are fields which are either purely axial or purely azimuthal. For such fields criterion (3.17) gives the necessary and sufficient condition for the stability of perturbations which are parallel to the field lines. Perturbations which are not parallel to the field can be studied directly from the energy integral (2.15).

When the field is purely axial, this takes the form

$$\delta W = \int r\,dr\left[\xi^2\left(k^2B_z^2 - \rho'\phi' - \frac{\rho^2\phi'^2}{\gamma p}\right) + k^2B_z^2(r\xi' + \xi)^2/(m^2 + k^2r^2)\right]. \quad (3.18)$$

This expression is valid for all m and $k \neq 0$. It is clearly sufficient for stability that

$$\rho'\phi' + \frac{\rho^2\phi'^2}{\gamma p} < 0. \quad (3.19)$$

This condition is also necessary; for suppose it is violated in a certain neighbourhood. Choose a given non-zero m and a ξ which is non-zero only in the neighbourhood where the criterion is violated. Further choose k^2 to satisfy the inequality

$$k^2 < \int r\,dr\xi^2\left(\rho'\phi' + \frac{\rho^2\phi'^2}{\gamma p}\right)\Big/ \int r\,drB_z^2[\xi^2 + \frac{1}{m^2}(r\xi' + \xi)^2]. \quad (3.20)$$

For such a perturbation δW is negative and the system is unstable. This necessary and sufficient condition for the stability of perturbations not parallel to the field lines is more stringent than the criterion for interchange perturbations, which in this case takes the form

$$\rho'\phi' + \frac{\rho^2\phi'^2}{B_z^2 + \gamma p} < 0. \quad (3.21)$$

Thus criterion (3.19) is the necessary and sufficient condition for Rayleigh-Taylor stability of Thetatron type configurations. It should be noted that the condition is certainly violated if ρ' and ϕ' have the same sign.

When the field is purely azimuthal, the expression for δW becomes

$$\delta W = \int r\,dr\left[\xi^2\left\{\frac{m^2B_\theta^2}{r^2} - \frac{2B_\theta}{r^2}(rB_\theta)' - \rho'\phi' - \frac{\rho^2\phi'^2}{\gamma p}\right\} + \frac{m^2B_\theta^2}{r^2}(r\xi' - \xi)^2\Big/(m^2 + k^2r^2)\right]. \quad (3.22)$$

This expression is valid for all k and $m \neq 0$. In this case it is sufficient for stability that the coefficient of ξ^2 is positive. This has its smallest value for $m = 1$ and gives the criterion

$$\rho'\phi' + \frac{\rho^2\phi'^2}{\gamma p} + \frac{2B_\theta}{r}(rB_\theta)' - \frac{B_\theta^2}{r^2} < 0. \quad (3.23)$$

This criterion is also necessary for stability. For the perturbation can again be confined to the region in which the criterion is violated and then, if k is chosen large enough, the positive term in (3.22) can be made smaller than the negative one. In this case the necessary and sufficient condition for interchange instabilities is

$$\rho'\phi' + 2B_\theta(B_\theta/r)' + (2B_\theta^2/r + \rho\phi')^2/(B_\theta^2 + \gamma p) < 0. \quad (3.24)$$

Both of criteria (3.23) and (3.24) must be satisfied but neither is always more stringent than the other.

(f) *Further problems*

As mentioned previously the derivation of the stability criteria given in the paper depend only on δW having the form (3.3). Thus for example NEWCOMB's derivation of necessary and sufficient conditions for stability will apply to any hydromagnetic stability problem of sufficient symmetry to enable δW to be put in the above form. Recently comparable conditions to those given here have been found by COWLEY (1961) for the plane Rayleigh-Taylor problem in which gravity acts in the z direction. Thus he finds

(i) A sufficient condition for stability

$$\rho'\phi' + \rho^2\phi'^2/\gamma p < 0. \quad (3.25)$$

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(ii) A necessary condition for stability

$$\frac{(B_x'B_y - B_y'B_x)^2}{B_x^2 + B_y^2} > 4(\rho'\phi' + \rho^2\phi'^2/\gamma p). \quad (3.26)$$

(iii) A necessary and sufficient condition for stability against interchange perturbations in a field without shear

$$\rho'\phi' + \rho^2\phi'^2/(\gamma p + B_x^2 + B_y^2) < 0. \quad (3.27)$$

In these criteria the prime denotes differentiation with respect to z. It can be seen that criteria (3.2), (3.13) and (3.17) reduce to the above if all the terms arising from field curvature are omitted. COWLEY (1961) has also applied NEWCOMB-type criteria to a particular configuration.

Since this paper was written, a paper by NEWCOMB (1961) has appeared in which he discusses the Rayleigh-Taylor stability problem involving plane fields and he derives values for the growth rates of the instabilities.

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IMPURITY RADIATION LOSSES FROM A HIGH TEMPERATURE PLASMA*†

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Abstract—This discussion will be concerned with the general problem of enhanced radiation losses from a hydrogen plasma containing a small percentage, particle-wise, of high Z impurities. Because the intrinsic radiation rate per atom from impurities can be many orders of magnitude greater than the Bremsstrahlung radiation from ionized hydrogen, even a small fractional impurity content can greatly increase the total escaping radiation flux.

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INTRODUCTION

IN the laboratory, the study of any high temperature plasma is a study of a system not in equilibrium with its surroundings. Thus it is a system in which energy must be supplied to keep the kinetic temperature high, and confinement must be provided to prevent the plasma particles from merely escaping to the walls of the experimental equipment. For this reason any energy loss process which is substantial can have a corresponding large influence on the behaviour of the plasma.

Since the mass density of a laboratory hydrogen plasma might typically lie in the range of a few milli-micrograms per cm^3, the addition of only a microgram or so per litre of an impurity such as oxygen would result in a fractional impurity content of several atoms per cent. This is such a small amount of material that it is virtually impossible to produce a hydrogen plasma with a percentage purity comparable to that which can be achieved with solid materials. Some degree of impurity must therefore be accepted in any plasma experiment.

Radiation losses can be produced by impurity atoms and ions in two important ways. First, if the impurity atom or ion possesses a nuclear charge Z, collisions with fast moving electrons of the plasma will give rise to the well-known Bremsstrahlung losses, the intensity of which is proportional to Z^2, and is given approximately by the relation

$$p_b = 0{\cdot}5 \times 10^{-30}\, n_e n_i Z^2 T_e^{1/2} \text{ watt/cm}^3 \qquad (1)$$

where T_e is the kinetic temperature of the electrons in kilo-electronvolts, n_e is the electron particle density and n_i the impurity particle density.

Setting $Z = 1$ in (1), one obtains a lower limit to the radiation which can be emitted from any plasma. The radiation losses from all laboratory plasmas must therefore lie between this limit and the upper limit defined by considerations of radiation equilibrium.

In a slightly impure hydrogen plasma, where the particle density of each type of impurity present is a fraction q_α of the electron particle density, with $\sum_\alpha q_\alpha < 1$, the extra radiation loss produced by the impurities through Bremsstrahlung is approximately given by

$$p_b = 0{\cdot}5 \times 10^{-30}\left[\sum_\alpha q_\alpha Z_\alpha^2\right] n_e^2 T_e^{1/2}. \qquad (2)$$

As will be discussed, however, Bremsstrahlung losses are not the most important radiation losses in a plasma containing impurities.

The second, and usually the most important way by which the presence of impurities can give rise to radiation losses is through the process of excitation radiation, i.e. the emission of line spectra by excitation and subsequent radiative transitions of electrons which remain bound to the impurity nuclei. Excited states which decay to produce such line spectra will be produced by inelastic collisions of the ions and the electrons of the plasma. To calculate the magnitude of this emission it will be necessary to consider several processes in the plasma in detail.

LOSSES IN RADIATION EQUILIBRIUM

If the plasma existed in a state of equilibrium between particles and the radiation field, the mean free path of the radiation photons would be small compared to the plasma dimensions. In this case the problem of calculating plasma radiation losses would be much easier, albeit somewhat discouraging to the experimentalist. In such a case the *kinetic* temperature and the *radiation* temperature are equal, and the radiation losses would depend only on the surface area and the

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temperature and would be given by the Stefan-Boltzmann law

$$S_p = \sigma T^4 \text{ erg.cm}^{-2}.\text{sec}^{-1}$$
$$= 1{\cdot}0 \times 10^{17}\, T_{\text{keV}}^4 \text{ watt/cm}^2. \quad (3)$$

For all temperatures of interest in high temperature plasma research this, of course, represents an enormous radiation loss, too large ever to be sustained. If it were not true that laboratory plasmas exist far from a state of radiation equilibrium, no hope would exist for laboratory experimentation in this field.

The Stefan-Boltzmann value is, however, important in plasma physics for the following reasons: in any plasma experiment, it will be true (disregarding small corrections) that the radiation losses must lie between a lower bound given by the Bremsstrahlung losses, summed over the volume of the plasma, and an upper bound, given by the Planck value, summed over the surface area of the plasma. Furthermore, at all emission frequencies large compared to the plasma frequency, it is not possible for the plasma to radiate at a rate greater than the local spectral intensity of the Planck radiation law, which gives for the radiation loss per unit interval:

$$r_\nu\, d\nu = \frac{2\pi h\nu^3}{c^2} \frac{d\nu}{\epsilon^{h\nu/kT} - 1} \text{ erg.cm}^{-2}.\text{sec}^{-1}. \quad (4)$$

In this expression we must now assume 'T' to be an excitation temperature, which can be usually taken to be the same as the electron kinetic temperature. In the optical, ultraviolet and soft X-ray part of the radiation spectrum, where (4) may become effective for high density plasmas, $(h\nu/kT) \ll 1$ except at quite low plasma temperatures. In this case

$$r_\nu\, d\nu \approx \frac{2\pi\nu^3}{c^2}(kT)\left(\frac{d\nu}{\nu}\right) \text{ erg. cm}^{-2}.\text{sec}^{-1} \quad (5)$$

$$\approx 10^{-36}\nu^3\, T_{\text{keV}} \left(\frac{d\nu}{\nu}\right) \text{ watt/cm}^2$$

thus, for example $r_\nu\, d\nu \approx 25$ MW/cm^2 if $\nu = 3 \times 10^{15}$ sec^{-1}, ($\lambda = 1000$ Å), $(d\nu/\nu) = 10^{-2}$ and $T_e = 0{\cdot}1$ keV. Such values might be encountered in dense plasmas in pinch discharges, for example. On the other hand, if $\nu = 3 \times 10^{11}$ sec^{-1} ($\lambda = 0{\cdot}1$ cm), $r_\nu\, d\nu$ is only about 25 μW/cm^2. One sees immediately, therefore, that the most important regions for intense radiation losses from impure plasmas are likely to be those in the vacuum ultra-violet or soft X-ray wavelength range. This fact will be made more clear in the discussion to follow.

DEGREE OF IONIZATION OF A PLASMA—GENERAL

In a system of ordinary densities which is in radiation equilibrium at, say, 10^6 °K or higher, the radiation energy density within the system will be many orders of magnitude larger than the total binding energy of all the electrons of the atoms of the system. In this case, as one would expect, the photo-ionization rates are so large that all atoms but those of very high atomic number would be completely ionized even including the innermost electrons. The degree of ionization reached in such a case is predicted by Saha's equation (for example ALLEN, 1955). But since laboratory plasmas are, fortunately, not in a state of radiation equilibrium, the degree of ionization reached by impurity atoms will almost always be much *less* than that predicted by the Saha equation, and must be calculated by other means.

The correct method of calculating the degree of ionization of a tenuous plasma has been pointed out by the astrophysicists (WOOLEY and STIBBS, 1953) in connexion with the problem of the solar corona. In radiation equilibrium, the two pairs of inverse processes: (1) photo-ionization and (1*) radiative recombination; and (2) collisional ionization and (2*) threebody recombination, exist in a state of detailed balance with each other. In a confined plasma of laboratory density and dimensions, however, (1) and (2*) are unimportant, and the state of ionization is determined solely by the competition between (1*) and (2). Since the pairs (1) and (1*) and (2) and (2*) depend differently on the particle density, the degree of ionization in the case of radiation equilibrium depends on the particle density as well as upon the temperature. However, (1*) and (2) depend in the same way on plasma density, so that the degree of ionization in a tenuous confined plasma is independent of density, and can be calculated as follows.

Let the particle density of incompletely stripped ions of a given type and charge state j be n_j, and let the number of these same type of ions from which an additional electron has been stripped be n_{j+1}, and so on. Since the probability of *two* electrons being stripped in a single ionizing collision is small, we can write

$$\frac{dn_j}{dt} = \dot{n}_j = n_e\, n_{j-1}\langle\sigma v\rangle_{j-1}^i$$
$$- n_e n_j[\langle\sigma v\rangle_j^i + \langle\sigma v\rangle_j^r] + n_e n_{j+1}\langle\sigma v\rangle_{j+1}^r \quad (6)$$

where $\langle\sigma v\rangle_j^i$ is the mean rate parameter for ionization of the j^{th} state, $\langle\sigma v\rangle_j^r$ is the mean rate parameter for radiative recombination on to the j^{th} state, and so on. In terms of mean ionization frequencies $\nu^i = n_e\langle\sigma v\rangle^i$

and mean recombination frequencies $\nu^r = n_e\langle\sigma v\rangle^r$, equation (6) becomes

$$n_j = n_{j-1}\nu^i_{j-1} - n_j(\nu^i_j + \nu_j) + n_{j+1}\nu^r_{j+1}. \qquad (7)$$

In steady state, this defines a set of algebraic equations which can in principle be solved for the various n_j. Solving for the ratio n_{j+1}/n_j we find

$$\frac{n_{j+1}}{n_j} = \frac{\nu^i_j}{\nu^r_{j+1}}\left[1 + \frac{\nu^r_j}{\nu^i_j}\right] - \frac{n_{j-1}}{n_j}\frac{\nu^i_{j-1}}{\nu^r_{j+1}}. \qquad (8)$$

However, when $n_{j+1} \approx n_j$ the second term may be expected to be small. Also, because of the rapid variation of ν^r_j with increasing j, one has some justification in dropping the term ν^r_j/ν^i_j, thereby committing an error less than a factor two. Making these approximations one ends up with a stripping equation of the form used by WOOLEY and STIBBS (1953) and by ELWERT (1952)

$$\frac{n_{j+1}}{n_j} \approx \frac{\nu^i_j}{\nu^r_{j+1}} = \frac{\langle\sigma v\rangle^i_j}{\langle\sigma v\rangle^r_{j+1}}. \qquad (9)$$

The answer is seen to be independent of the absolute particle density, as stated earlier.

In the above case, where we are assuming that the j^{th} and the $(j+1)^{\text{th}}$ state are the dominant ones present, we can write the equation in a simple way in terms of the fractional abundances $q_j = (n_j/n_0)$, where n_0 is the total particle density of the class (oxygen, nitrogen, etc.) of impurity ion in question. In this case $q_j + q_{j+1} = 1$, so that

$$\frac{q_{j+1}}{q_j} = \frac{q_{j+1}}{1-q_{j+1}} = \frac{1-q_j}{q_j} = \frac{\langle\sigma v\rangle^i_j}{\langle\sigma v\rangle^r_{j+1}}. \qquad (10)$$

The point where $q_{j+1} = q_j$ then corresponds to the temperature at which the fractional abundance of each ion type is 50 per cent in this approximation.

IONIZATION RATE PARAMETERS

Since accurate cross section data are not available for highly stripped ions, it is necessary to use approximate expressions. In this part of the calculations an expression will be used for the ionization cross sections which fits the theoretical (Born approximation) variation of σ^i at high energies, and agrees reasonably well with available experimental data near threshold. We take all ionization cross sections to be of the form: (the 'universal ionization function'):

$$\sigma^i_j = \frac{\pi e^4}{\chi^2_j} b_j\, g(u)\ \text{cm}^2$$

$$g(u) = \frac{\epsilon}{u}\ln u \qquad (11)$$

$$g(\epsilon) = 1, \quad u = \frac{w}{\chi_j}.$$

Here χ_j is the ionization potential of the j^{th} state, w is the electron energy, and b_j is a constant appropriate to the j^{th} state. By comparison with available experimental and theoretical data, a value of 0·2 will be taken for b_j in all numerical examples. This will lead to errors estimated to be about a factor two. This sort of accuracy is adequate for the estimation of gross energy losses, where often an order of magnitude is sufficient. It is not claimed that the energy loss calculations to be presented are more accurate than this.

Written in terms of the ionization potential (kV)

$$\sigma^i_j = \frac{0{\cdot}7 \times 10^{-19}}{\chi^2_j} b_j\, g(u)\ \text{cm}^2. \qquad (12)$$

The value of $\langle\sigma v\rangle^i_j$ may now be obtained simply by integration over the electron energy distribution, assumed to be Maxwellian. The result of this integration is

$$\langle\sigma v\rangle^i_j = \frac{4\pi e^4}{(2\pi mkT)^{1/2}kT} b_j\, h(x)\ \text{cm}^3.\ \text{sec}^{-1}$$

$$= \frac{1{\cdot}4 \times 10^{-10}}{T_e^{3/2}} b_j\, h(x), \qquad (13)$$

with T_e in keV, and where $x = \dfrac{\chi}{kT}$ and

$$h(x) = \frac{-\epsilon\ \text{Ei}(-x)}{x}, \text{ with}$$

$$-\text{Ei}(-x) = \int_x^\infty \frac{\epsilon^{-z}dz}{z}, \text{ the 'exponential integral'.}$$

The function $h(x)$ is given in Table 1.

The ionization frequency is therefore given by

$$\nu^i_j = \frac{1{\cdot}4 \times 10^{-10}}{T_e^{3/2}} b_j\, n_e\, h(x)\ \text{sec}^{-1}. \qquad (14)$$

If $n_e = 10^{14}$, for example, $\nu^i_j \approx 4 \times 10^6$ at $x = 1$, $T_e = 10$ eV.

TABLE 1

x	$h(x)$
0·01	$1{\cdot}2 \times 10^3$
0·02	$4{\cdot}6 \times 10^2$
0·05	$1{\cdot}3 \times 10^2$
0·1	$5{\cdot}0 \times 10^1$
0·2	$1{\cdot}7 \times 10^1$
0·5	3·0
1·0	0·60
2·0	0·067
5	$6{\cdot}3 \times 10^{-4}$
10	$1{\cdot}1 \times 10^{-6}$

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RECOMBINATION RATES

The calculation of recombination rates is more difficult than that of ionization probabilities, and there is less experimental evidence against which to check the theory. However, in most situations of interest in high temperature plasma research radiative recombination only becomes important for the highly stripped states of impurity ions and for these it is possible to make reasonable predictions.

In this discussion only two general types of ions will be specifically treated. However, as will be seen these two types are among the most important in the excitation radiation losses. The first ion class is the 'hydrogenic' ion—i.e. any ion which has been stripped down to a *single* bound electron. The second class is the Lithium-like ion—i.e. any ion which has been stripped down to *three* electrons. This latter class is a type of ion which happens to be particularly effective in producing excitation radiation at intermediate temperatures. The first type, the hydrogenic ion, defines the boundary (on a Z vs. T_e plot) between the region where excitation radiation is important and that where only Bremsstrahlung losses are present.

The recombination rate for one-electron ions has been calculated by KRAMERS (1923) by SPITZER (1956) and by others. The cross section for recombination to the n^{th} state is

$$\sigma_n^r = A_r \frac{v_0}{v} \frac{h\nu_0}{w} \cdot \frac{g}{n^3} \text{ cm}^2 \tag{15}$$

where $h\nu = h\nu_0/n^2 + w$, $A_r = 2{\cdot}11 \times 10^{-22}$ cm^2 and $g \approx 1$.

Integrating over a Maxwellian distribution and summing over all states an expression for $\langle \sigma v \rangle_j^r$ is obtained:

$$\langle \sigma v \rangle_Z^r = \frac{0{\cdot}8 \times 10^{-16}}{T_e^{3/2}} Z^4 k_1(y_0) \tag{16}$$

where y_0 is the ratio of the quantum energy of the ground state, to the electron temperature (keV)

$$y_0 = \frac{h\nu_0}{kT_e} = \frac{1{\cdot}35 \times 10^{-2} Z^2}{T_e} \tag{17}$$

and

$$k_1(y) = \sum_{n=1}^{\infty} \frac{\epsilon^{y/n^2}}{n^3} [-\text{Ei}(-y/n^2)]. \tag{18}$$

The function $k_1(y)$ is tabulated in Table 2.

It is now possible to write down an explicit relationship for the stripping equation for one-electron ions. From equations (10), (13) and (16) we have:

$$\frac{q_Z}{q_{Z-1}} = \frac{1 - q_{Z-1}}{q_{Z-1}} = \frac{q_Z}{1 - q_Z} = \frac{0{\cdot}8 \times 10^6}{Z^4} b \frac{h(y_0)}{k_1(y_0)}. \tag{19}$$

TABLE 2

y	$k_1(y)$	$k_3(y)$
0·01	5·3	1·2
0·02	4·5	1·2
0·05	3·6	1·0
0·1	2·8	0·80
0·2	2·1	0·65
0·5	1·4	0·48
1·0	0·96	0·36
2·0	0·63	0·27
5	0·34	0·17
10	0·20	0·11
100	0·032	—
1000	0·0043	—

In this expression $q_Z = q_j$ = fractional abundance of completely stripped ions, and q_{Z-1} is the fractional abundance of one-electron ions. It can already be seen from the Z^{-4} dependence of (19) that impurity atoms of high atomic number will be difficult to strip completely.

In an entirely analogous way, an approximate expression can be derived for the case of the 3-electron ions. By taking such an ion to be approximated by a hydrogen-like ion possessing a ground state of quantum number $n = 2$, and an effective charge $Z_{\text{eff}} = Z - 1{\cdot}6$ (obtained from examination of the level diagram), there is obtained

$$\langle \sigma v \rangle_{Z-2}^r = \frac{0{\cdot}8 \times 10^{-16} Z_{\text{eff}}^4}{T_e^{3/2}} k_3(y_2) \tag{20}$$

where

$$k_3(y) = \sum_{n=2}^{\infty} \frac{\epsilon^{y/n^2}}{n^3} [-\text{Ei}(-y/n^2)]$$

$$= k_1(y) - \epsilon^y[-\text{Ei}(-y)] \tag{21}$$

and

$$y_2 \approx \frac{y_0}{4} = \frac{1{\cdot}35 \times 10^{-2} Z_{\text{eff}}^2}{4T_e}. \tag{22}$$

The function $k_3(y)$ is tabulated in Table 2.

Thus the stripping equation for three-electron ions becomes (in cases where the only ion states which are important are $j = Z - 3$ and $Z - 2$) from (10), (13) and (20),

$$\frac{q_{Z-2}}{q_{Z-3}} = \frac{1 - q_{Z-3}}{q_{Z-3}} \approx \frac{0{\cdot}8 \times 10^6}{Z_{\text{eff}}^4} b \frac{h(y_2)}{k_3(y_2)}. \tag{23}$$

Relationships (19) and (23) have been used to calculate 'stripping curves' for one-electron and three-electron ions. These curves, obtained by setting $q_Z = q_{Z-1}$ and $q_{Z-2} = q_{Z-3}$, define the locus of points (on a Z vs. T_e plot) where the abundance of the ion in question is approximately 50 per cent. Such curves are shown in Fig. 1.

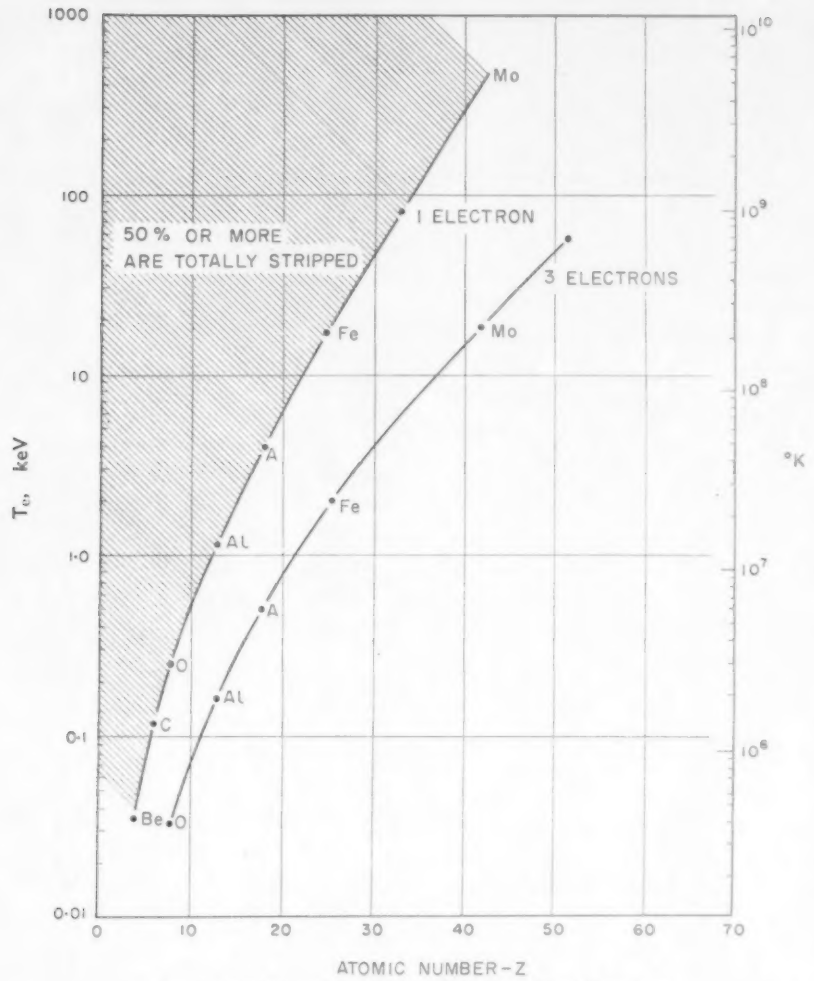

FIG. 1.—Steady-state stripping curves—1- and 3-electron ions.

Referring to Fig. 1. it is clear that common impurity ions such as carbon or oxygen will not become completely stripped until quite high temperatures are reached, and incompletely stripped ions of Al or Fe will survive up to very high temperatures indeed. Even the three-electron ions are relatively hardy and some can be expected to be present until thermonuclear temperatures are approached. It is worth remarking that the points at 30 eV and 240 eV, respectively, for three-electron and one-electron oxygen ions (Oxygen^{VI} and Oxygen^{VIII}) agree reasonably well with the calculations of KNORR (1958) for plasmas such as those in the British ZETA. Interpolating from KNORR's curves one finds a value of T_e of roughly 25 eV for the 50 per cent point in Oxygen^{VI}, and 210 eV for Oxygen^{VIII}.

EXCITATION RADIATION—STEADY STATE

It is clear from the stripping curves that incompletely stripped ions, capable of emitting excitation radiation, will persist to rather high temperatures in non-equilibrium plasmas. We shall now calculate approximate expressions for the intensity of this radiation.

For most circumstances of interest here the emission of excitation radiation can be thought of as a process in which a fast electron makes an inelastic collision with an ion, which then decays spontaneously in a time short compared to the time between collisions. Thus the electron loses a definite amount of energy in each such collision, which energy promptly escapes from the plasma. Only at high densities will it be necessary to modify this simple picture, and at such densities the radiation losses will already be very high.

The problem of calculating the probability that a colliding electron of known velocity will lift a bound electron from its ground state to an excited state is essentially the same as that of determining the cross section for collisional ionization, which is of course

but a case of excitation of a bound electron in the ground state to the continuum rather than to another bound state.

Bethe has calculated the cross section for this process for excitations to level n, in Born approximation. In the limit of high velocity impacts ($w \gg (E_n - E_0) = (h\nu)_{on}$), his result is (HEITLER, 1954)

$$\sigma_{on}^{ex} = \frac{8\pi^3 me^4}{h^2} |x_{on}|^2 \frac{1}{w} \ln \frac{4w}{h\nu_{on}} \qquad w \gg h\nu_{on}. \quad (24)$$

Here $|x_{on}|^2$ is the square of the appropriate quantum mechanical matrix element. By using a well-known relationship (HEITLER, 1954) between $|x_{on}|^2$ and the quantum mechanical 'oscillator strength' f_{on}, a simplification results, since oscillator strengths for the common transitions are tabulated.* One finds

$$\sigma_{on}^{ex} = \frac{\pi e^4}{h\nu_{on}} f_{on} \frac{1}{w} \ln \frac{4w}{h\nu_{on}} \text{ cm}^2. \quad (25)$$

This form is asymptotically almost the same as that used for calculating equation (11) except that f_{on} replaces b_j. This suggests that a 'universal excitation function' of the same form as (11) be used. Letting $\mu = w/h\nu_{on}$, and dropping the subscripts, we therefore take

$$\sigma^{ex} \approx \frac{\pi e^4}{(h\nu)^2} f \left[\frac{\epsilon}{\mu} \ln \mu\right] \text{cm}^2 \quad (26)$$

where $\frac{\epsilon}{\mu} \ln \mu = g(\mu)$ is the same function as defined in (11). With energies in keV

$$\sigma^{ex} = \frac{0{\cdot}7 \times 10^{-19}}{(h\nu)^2_{keV}} f\, g(\mu) \text{ cm}^2. \quad (27)$$

As before, the excitation rate parameter can be determined by integration over the electron distribution. With T_e in keV,

$$\langle \sigma v \rangle^{ex} = \frac{1{\cdot}4 \times 10^{-10}}{T_e^{3/2}} f\, h(v) \text{ cm}^3.\text{sec}^{-1}$$
$$v = \frac{h\nu}{kT} = \left(\frac{h\nu}{T_e}\right)_{keV} \qquad h(v) = \frac{-\epsilon\, \text{Ei}(-v)}{v} \quad (28)$$

the same function as used before in the ionization rate parameter.

We are, however, not as much interested in the rate of excitation events as we are in the rate of radiation loss represented by such a process. Since we can usually assume that each excitation to a level $h\nu$ will subsequently result in the emission of a photon of energy $h\nu$, the rate of power loss per unit volume from the plasma by excitation radiation from a state k of ion type j will be

$$p_{jk}^{ex} = n_j n_e \langle \sigma v \rangle_k^{ex} \times (h\nu)_k \text{ erg.cm}^{-3}.\text{sec}^{-1}. \quad (29)$$

In terms of a fractional abundance q_j then, with p^{ex} in watt/cm^3 and temperature in keV, substituting, from (28)

$$p_{jk}^{ex} = 0{\cdot}6 \times 10^{-26}\, q_j n_e^2 f_{jk} \frac{S(v_k)}{T_e^{1/2}} \text{ watt/cm}^3$$
$$S(v_k) = -\text{Ei}(-v_k) = \int_{v_k}^{\infty} \frac{\epsilon^{-z}}{z}\, dz. \quad (30)$$

It is first of interest to note that p_{jk}^{ex} does not depend explicitly on the level energy, which drops out, and appears only in the function $S(v_k)$. (Tabulated in Table 3.)

Second, it is interesting to compare the magnitude of the coefficient of p^{ex} with that for p_0, the hydrogenic Bremsstrahlung,

$$p_0 \approx 0{\cdot}5 \times 10^{-30}\, n_e^2\, T_e^{1/2} \text{ watt/cm}^3. \quad (1a)$$

The coefficient of (30) is about 10^5 times that of (1a), so that it can already be seen that, on an ion-for-ion basis, the rate of excitation radiation would be expected to be many orders of magnitude larger than the Bremsstrahlung losses from a hydrogen plasma, especially at temperatures below 1 keV, where the different dependence on T_e will accentuate the disparity between the two processes.

RADIATION FROM ONE-ELECTRON IONS

The excitation radiation power density from one-electron ions is easy to treat, since the oscillator strengths are well known. The dominant transition is Lyman α, for which $f = 0{\cdot}4$. To take into account, approximately, the effect of higher transitions, we shall

TABLE 3

v_k	$S(v_k) = \int_{v_k}^{\infty} \frac{\epsilon^{-z}}{z}\, dz$
0·01	4·0
0·02	3·4
0·05	2·5
0·1	1·8
0·2	1·2
0·5	0·56
1·0	0·22
2·0	$4{\cdot}9 \times 10^{-2}$
5	$1{\cdot}1 \times 10^{-3}$
10	$4{\cdot}2 \times 10^{-6}$

* See, for example, ALLEN C. W. (1955) *Astrophysical Quantities*, Athlone Press.

assume a mean f for such ions of 0·5. The level energy is $1{\cdot}0 \times 10^{-2}Z^2$ keV, thus for one-electron ions

$$p_{Z-1}^{ex} = 0{\cdot}3 \times 10^{-25} q_{Z-1} n_e^2 \frac{S(v_{Z-1})}{T_e^{1/2}} \text{ watt/cm}^3$$
$$v_{Z-1} = \frac{1{\cdot}0 \times 10^{-2}Z^2}{T_e}. \tag{31}$$

The radiation from the dominant transition will be emitted in the vacuum ultra-violet or the soft X-ray region. For example, in the case of Oxygen^{VIII} ($Z = 8$), $\hat{\lambda} = 19{\cdot}4$ Å, $h\nu = 640$ eV. Furthermore, the mean lifetime of the transition is given approximately by

$$\tau = \frac{1}{\gamma} = \frac{1{\cdot}6 \times 10^{-9}}{Z^4} \text{ sec} \tag{32}$$

so that the spontaneous decay time will be very short compared to the mean excitation rate, as earlier assumed, being about 4×10^{-13} sec for Oxygen^{VIII}.

To evaluate explicitly the radiation losses caused by one-electron ions it is only necessary to know the one-electron abundance fraction, q_{Z-1}, the electron density and the electron temperature. The stripping equations presented earlier can be used to obtain the abundance fraction. Examples of loss calculations will be presented later.

RADIATION FROM THREE-ELECTRON IONS

Three-electron ions have been discussed in the calculations on stripping. Interest in such ions arises from the fact that they represent the simplest and most durable member of an important species of ions. This species is composed of ions which resemble the alkali metals, in that they possess a single outer S-state electron added to a substrate of closed electron shells.

Examples of three-electron ions are Carbon^{IV}, Nitrogen^{V}, Oxygen^{VI}, Magnesium^{X} and Aluminium^{XI}. One of these, Magnesium^{X}, is thought to provide the dominant source of radiation loss from the solar corona (Wooley and Stibbs, 1953), despite its small abundance. That three-electron ions represent very potent sources of excitation radiation arises from the fact that all have a relatively high ionization potential, yet possess a strongly favoured $2S - 2P$ transition with an excitation energy much less than the ionization potential. The level energy for all such ions is well represented by an expression of the form:

$$(h\nu)_{Z-3} = 2{\cdot}0(Z - 2) \text{ eV}. \tag{33}$$

These lines, then, all lie in the vacuum ultra-violet (12 eV = 1032 Å for Oxygen^{VI}, for example). At the same time, the ionization potential of the ions is given closely by an expression of the form

$$\chi_{Z-3} = 3{\cdot}4\, Z_{\text{eff}}^2 \text{ eV} \tag{34}$$

where

$$Z_{\text{eff}} = (Z - 2) + 0{\cdot}4 = Z - 1{\cdot}6.$$

It is apparent from a comparison of (33) and (34) that the ratio of ionization energy to excitation energy increases with the atomic number of the ion:

$$\left(\frac{\chi}{h\nu}\right)_{Z-3} \approx 1{\cdot}7(Z - 2), \quad Z \gg 1. \tag{35}$$

Thus for Oxygen^{VI}, this ratio is 11·5.

The large ratio of ionization energy to excitation energy leads to the possibility of strong excitation of the line at a temperature where the rate of ionization is slow enough to be just balanced by the recombination, so that the ion is well represented. For high Z ions of this class the mean oscillator strength of the $2S - 2P$ transition is here taken to be 0·2, since the variations with Z are not large. Therefore we find, from equation (30), the value for the radiation power densities from these ions as:

$$p_{Z-3}^{ex} = 1{\cdot}2 \times 10^{-26} q_{Z-3} n_e^2 \frac{S(v_{Z-3})}{T_e^{1/2}} \text{ watt/cm}^3$$
$$v_{Z-3} = \frac{2 \times 10^{-3}(Z - 2)}{T_e}. \tag{36}$$

This expression can be given more substance by means of a representative example. Let us suppose that a particular plasma is composed primarily of hydrogen, but contains one per cent of Oxygen as O^{VI}. The electron temperature will be taken to be 25 eV = 0·025 keV, and the electron density is assumed to be 10^{16} cm^{-3}. In this example, then, the radiation rate predicted by (36) is $4{\cdot}4 \times 10^4$ watts/cm^3. This is almost 10^4 times the hydrogen Bremsstrahlung losses, and is an absolute loss rate which would have a substantial effect on the heat balance in the plasma. For example, since the kinetic energy density of the plasma in this example is $\frac{3}{2} n_e k T_e \approx 0{\cdot}06$ J/cm^3, the predicted initial rate of cooling by impurity radiation from this state alone would be as with an initial time constant of about 1·5 microseconds. At $n = 10^{17}$, the time constant would be 10 times shorter.

STEADY-STATE EXCITATION RADIATION

The steady-state excitation radiation losses from the plasma per unit volume may be evaluated by combining the results of the stripping calculations with

those for the rate of emission. The rates of emission which are found this way may be expected to apply under conditions where self-absorption is not important and where the steady-state stripping value is approached. The effect of deviations from these conditions will be discussed later.

Figure 2 presents the results of calculations combining the steady-state stripping calculations and excitation radiation power losses for one-electron and three-electron ions, for several different elements. Since both the excitation radiation rates and the hydrogenic Bremsstrahlung rates are volume losses both of which vary in the same way with electron density, the excitation radiation is presented as a ratio to the hydrogenic Bremsstrahlung, on an atom-for-atom basis. Thus to find the actual radiation loss it is only necessary to multiply the value of the hydrogenic Bremsstrahlung by the *total* abundance fraction of the element in question. Thus if the total fractional abundance of an impurity is 0·10, the resulting radiation will be 0·1 times the ratio indicated by the curves. Of course, at any given temperature only a fraction of the given element might exist in one or three-electron form, but this has been taken into account in obtaining the curves and is not to be separately included. Along the right margin of the figure short lines are presented, annotated with the symbols of the elements shown. These lines represent the approximate values of the Bremsstrahlung radiation rates which would be produced by completely stripped ions of the indicated elements.

The curves are plotted only for the two excitation states which have been considered. One must assume that other ionic and optical states will also be excited which will make additional contributions to the radiation. Also, because approximate cross sections have been used, one must expect quantitative errors in the results.

The curves demonstrate another important qualitative behaviour of the radiation losses. As the temperature is raised beyond a certain point the amount of impurity radiation may drop sharply, owing to the increased degree of stripping which will occur. Thus, considering, for example O^{VI} at 25 eV, the curves show that if the temperature is raised by a factor 10, the radiation losses would drop by a factor

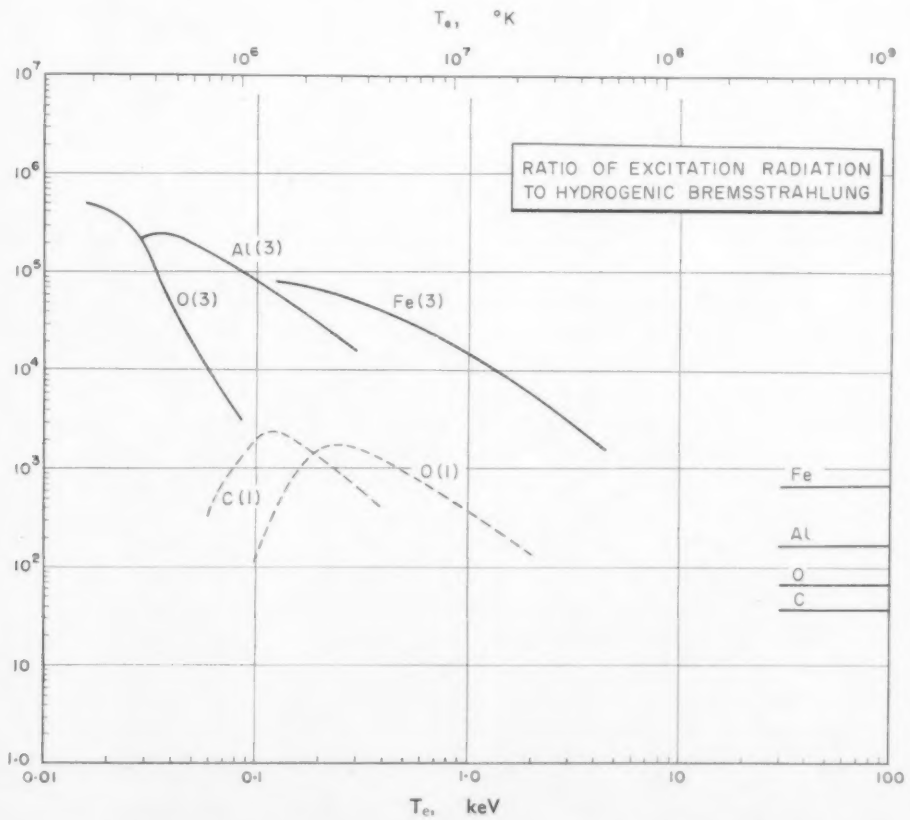

FIG. 2.—Ratio of steady-state excitation radiation to hydrogenic Bremsstrahlung for selected 1-electron and 3-electron ions. Effect of stripping is included.

150, relative to Bremsstrahlung. Thus the amount of excitation radiative loss would drop by about 50, so that the effective 'radiation temperature' would actually *fall* by a factor $(50)^{1/4}$ or 2·6.

To conclude the discussion of steady-state excitation radiation losses, it has been shown that such radiation losses can be expected to be large at relatively low temperatures, falling with increasing temperature. Owing to the neglect of other radiating states, the curve represent lower limits to the losses (aside from cross section approximations). Also, since laboratory plasmas may never attain stripping equilibrium during their transient existence, even higher radiation losses are possible, as will be discussed.

RECOMBINATION RADIATION

The recombination processes which eventually lead to the establishment of a steady-state degree of stripping also must contribute to the radiation losses, since they involve radiative transitions. Some of the expressions already derived may be used to obtain an approximate expression for the magnitude of these losses.

On the average if radiative transition times are short compared to ionization rates each recombination should lead to the radiation of energy equivalent to the ground state energy of the ion, plus the original kinetic energy of the captured electron.

Therefore, since recombinations occur at a constant rate when steady-state stripping is reached,

$$p_j^r = n_e n_j \langle \sigma v (w + \chi_j) \rangle_j^r \text{ erg.cm}^{-3}.\text{sec}^{-1} \qquad (37)$$

where the average is to be taken over the electron velocity distribution.

We can readily obtain an approximate expression for p^r by integrating over all captures that occur directly to the ground state. This will result in some underestimation, but the error is not likely to be large, because of the rapid decrease of capture probability with increasing quantum number.

From the expression for σ^r, equation (15), we therefore find for the average,

$$\langle \sigma v (w + \chi) \rangle^r = \frac{2A_r (h\nu_0)^2 g}{m} \cdot \frac{\int_0^\infty v \epsilon^{-\frac{mv^2}{2kT}} dv}{\int_0^\infty v^2 \epsilon^{-\frac{mv^2}{2kT}} dv}. \qquad (38)$$

Performing the integrations, we obtain

$$\langle \sigma v (w + \chi) \rangle^r = \frac{4}{(2\pi)^{1/2}} \frac{A_r (h\nu_0)^2}{(mkT)^{1/2}}. \qquad (39)$$

To evaluate the radiation losses by recombination to stripped nuclei to form one-electron ions we need only insert the value for $h\nu_0$, the ground state energy, finding

$$p_Z^r = 1{\cdot}3 \times 10^{-32} n_e^2 q_Z \frac{Z^4}{T_e^{1/2}} \text{ watt/cm}^3. \qquad (40)$$

A similar expression would apply for recombination to form three-electron ions.

It can already be seen from the smallness of the coefficient of (40) that recombination radiation is not likely to be as important as excitation radiation except for high Z and large T_e. However, in these cases it may be important. For example for Al at 1 keV,

$$p_{13}^r = 3{\cdot}7 \times 10^{-28} q_{13} n_e^2 \text{ watt/cm}^3$$

which is about 4·4 times the Bremsstrahlung losses calculated for Al itself.

The spectrum of recombination radiation is of course band-like, lying generally below a high frequency 'edge' at a frequency corresponding to the ground state energy.

TRANSIENT STRIPPING EQUATIONS

Most high temperature plasmas studied in the laboratory have a transient existence. Thus to calculate the radiation losses for such plasmas it is necessary to determine the rate of approach to stripping equilibrium, and to determine the radiation losses occuring during this time. Generally speaking, during the approach to equilibrium the excitation radiation from partially stripped impurity ions will be higher than it will be in steady state.

One way to visualize the radiation loss situation during a transient build-up of the plasma is first to recognize that many stages of ionization will be passed through as the final state is approached. But between each successive ionization event many collisional excitations may occur, each of which results in a loss of energy. Thus one can consider that successively to strip a given high Z atom up to a highly ionized state will require an inevitable energy 'fee' in excitation radiation losses. This fee may be much larger than the sum of the ionization energies. Moreover, after this fee has been paid, if the ion does not become completely stripped, further energetic 'operating expenses' must be borne to support the continued excitation radiation losses. If the impurity ion escapes to the wall of the vacuum chamber, is there neutralized and returns afresh, a new fee must be paid for each round trip.

The general equations which describe the transient stripping have already been given (equation(7)). This set of equations includes both ionization and recombination terms. But for all of the initial stages of the stripping recombination is quite unimportant. The

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equations thus take the simpler form (multiple ionization not included)

$$\frac{dn_j}{dt} \equiv \dot{n}_j = n_{j-1}\nu_{j-1} - n_j\nu_j, \tag{41}$$

where we have dropped the superscript, since we are dealing with only one process. These equations may be written in a convenient dimensionless form if we write then in terms of fractional abundances q_j of the initial density n_0, of atoms in the neutral state, so that $n_j = q_j n_0$. It is also convenient to measure all ionization frequencies in terms of the ionization frequency of the neutral state, ν_0 so that we write $\nu_j = a_j\nu_0$. The equations are also to be expressed in terms of the 'reduced' dimensionless time variable, $\tau = \nu_0 t$. They now take the form:

$$\begin{aligned}
\dot{q}_0 &= -a_0 q_0 \\
q_1 &= a_0 q_0 - a_1 q_1 \\
&\cdot \\
&\cdot \\
&\cdot \\
\dot{q}_j &= a_{j-1} q_{j-1} - a_j q_j
\end{aligned} \tag{42}$$

where

$$a_0 = 1,\ a_j = \frac{\nu_j}{\nu_0} = \frac{\langle\sigma v\rangle_j^i}{\langle\sigma v\rangle_0^i}\,.$$

The set of equations closely resemble the differential equations for a radioactive decay chain (the BATEMAN equations; BATEMAN, 1910). The solutions are:

$$\begin{aligned}
q_0 &= \epsilon^{-a_0\tau} \\
q_1 &= a_0\left[\frac{\epsilon^{-a_0\tau}}{a_0 - a_1} + \frac{\epsilon^{-a_1\tau}}{a_1 - a_0}\right](-1)^1 \\
&\cdot \\
&\cdot \\
&\cdot \\
q_j &= a_0 a_1 \cdots a_{j-1} \\
&\times\left[\frac{\epsilon^{-a_0\tau}}{(a_0 - a_1)(a_0 - a_2)\cdots(a_0 - a_j)}\right. \\
&+ \frac{\epsilon^{-a_1\tau}}{(a_1 - a_2)\cdots(a_1 - a_j)(a_1 - a_0)} \\
&\qquad + \cdots \\
&\left. + \cdots \frac{\epsilon^{-a_j\tau}}{(a_j - a_0)(a_j - a_1)\cdots(a_j - a_{j-1})}\right] \times (-1)^j.
\end{aligned} \tag{43}$$

In these equations the a's depend only on the element considered, and on the electron temperature. The results are, therefore, also independent of electron density, and may be used to predict the stripping rates for any electron density, even one which varies with time (with T_e fixed). This is done by simply identifying the time variable τ through

$$\tau = \int_0^t \nu_0(t')\,dt' = \langle\sigma v\rangle_0^i \int_0^t n_e(t')\,dt'. \tag{44}$$

Using the 'universal ionization rate parameter' and the known ionization energies transient stripping curves for oxygen have been calculated and are presented in Figs. 3–7, for various values of T_e. It is interesting to note that the time for growth of the highly stripped states may be of order 10^3 times the time to ionize the neutral atom. To illustrate, consider the case q_5 (O^{VI} at $T_e = 50$eV). This reaches maximum at $\tau \approx 500$. At $T_e = 50$ eV, and $n = 10^{14}$, for example, since $\chi_0 = 13{\cdot}6$ eV, we find $\nu_0 \approx 2{\cdot}7 \times 10^6$. Thus the time to reach the maximum of q_5 is 500 ν_0^{-1}, that is about 190 microseconds. Again because of the approximations used, the predicted times may be substantially in error.

TRANSIENT EXCITATION RADIATION LOSSES

To calculate the transient excitation radiation we return to the original definition of the power loss by excitation radiation. Thus if the j^{th} level possesses m states which are excited

$$p_j^{\text{ex}} = n_e n_j \sum_m \langle\sigma v\rangle_{jm}^{\text{ex}} \times (h\nu)_{jm}\ \text{erg.cm}^{-3}.\text{sec}^{-1} \tag{45}$$

Now the total energy released as excitation radiation during the build-up will be the integral of p^{ex} over time, summed over all states of ionization and excitation. Since we are concerned with the total 'fee' to reach steady-state, we may separate the additional losses occurring when recombination becomes important. Thus the integral of p^{ex} is to be carried out for all time, but only up to the final ionization states reached.

Writing $\int p^{\text{ex}}\,dt$ in terms of the abundance fractions and the dimensionless time

$$\begin{aligned}
Q^{\text{ex}} &= \int_0^\infty p^{\text{ex}}\,dt \\
&= n_0 \sum_{j=0}^{S}\sum_m \langle\sigma v\rangle_{jm}^{\text{ex}} (h\nu)_{jm} \int_0^\infty q_j(\tau')\,\frac{n_e}{\nu_0}\,d\tau'.
\end{aligned} \tag{46}$$

Again it can be seen that the result will be independent of the variation of n_e with time, since

$$\frac{n_e}{\nu_0} = [\langle\sigma v\rangle_0^i]^{-1}$$

is independent of density. Thus

$$Q^{\text{ex}} = n_0 \sum_j \sum_m \frac{\langle\sigma v\rangle_{jm}^{\text{ex}}}{\langle\sigma v\rangle_0^i} (h\nu)_{jm} \int_0^\infty q_j\,d\tau. \tag{47}$$

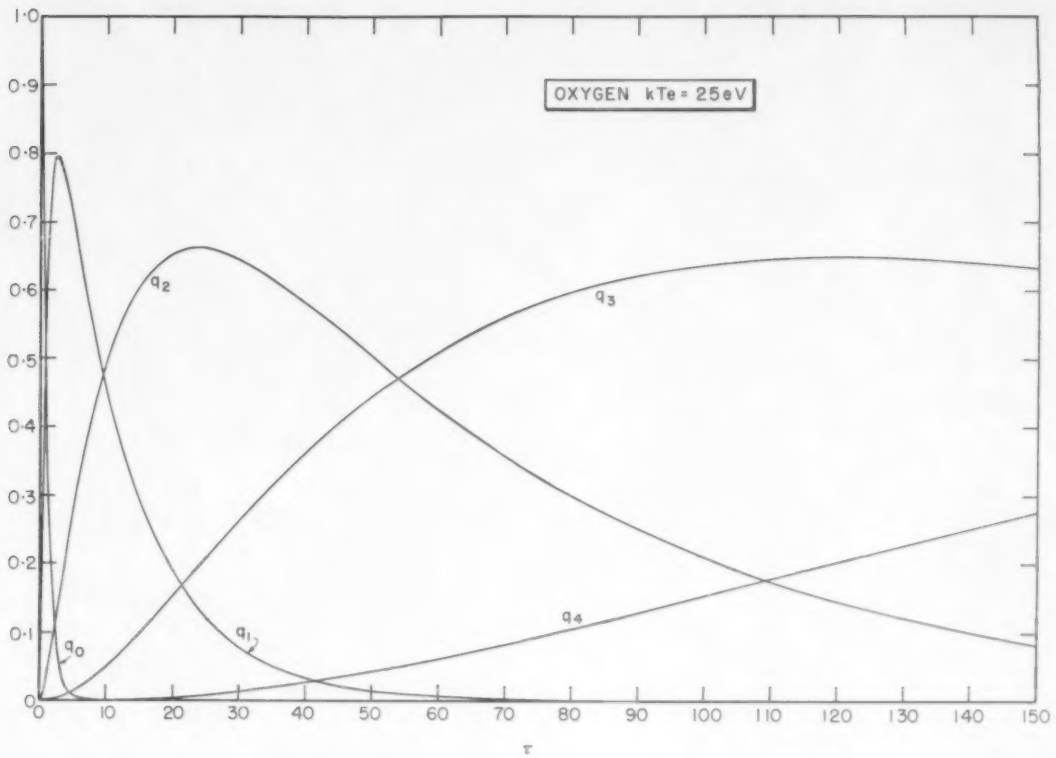

FIG. 3.—Transient stripping curves, oxygen, $T_e = 25$ eV.

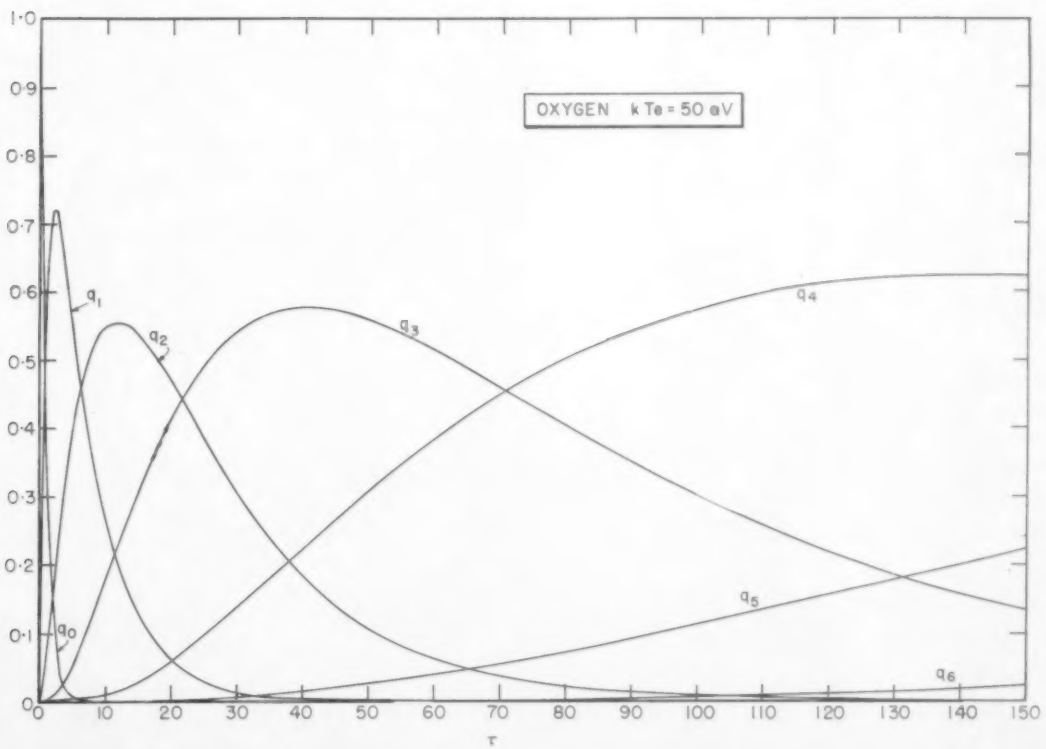

FIG. 4.—Transient stripping curves, oxygen, $T_e = 50$ eV, $\tau = 0$ to 150.

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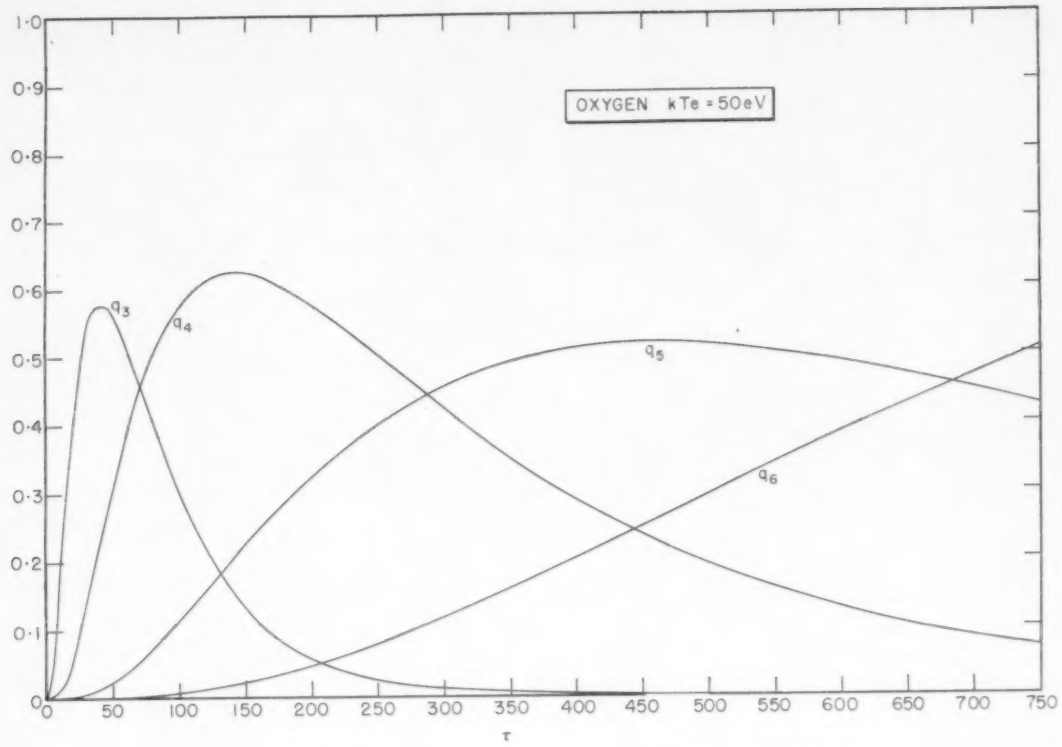

FIG. 5.—Transient stripping curves, oxygen, $T_e = 50$ eV, $\tau = 0$ to 750.

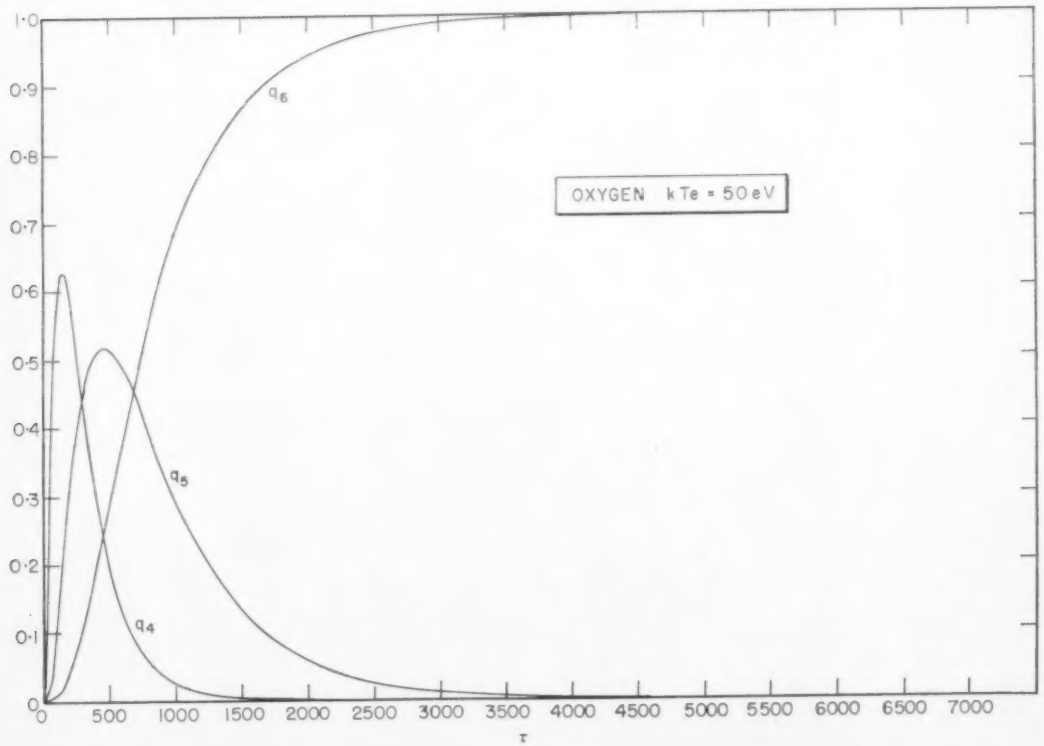

FIG. 6.—Transient stripping curves, oxygen, $T_e = 50$ eV, $\tau = 0$ to 7000.

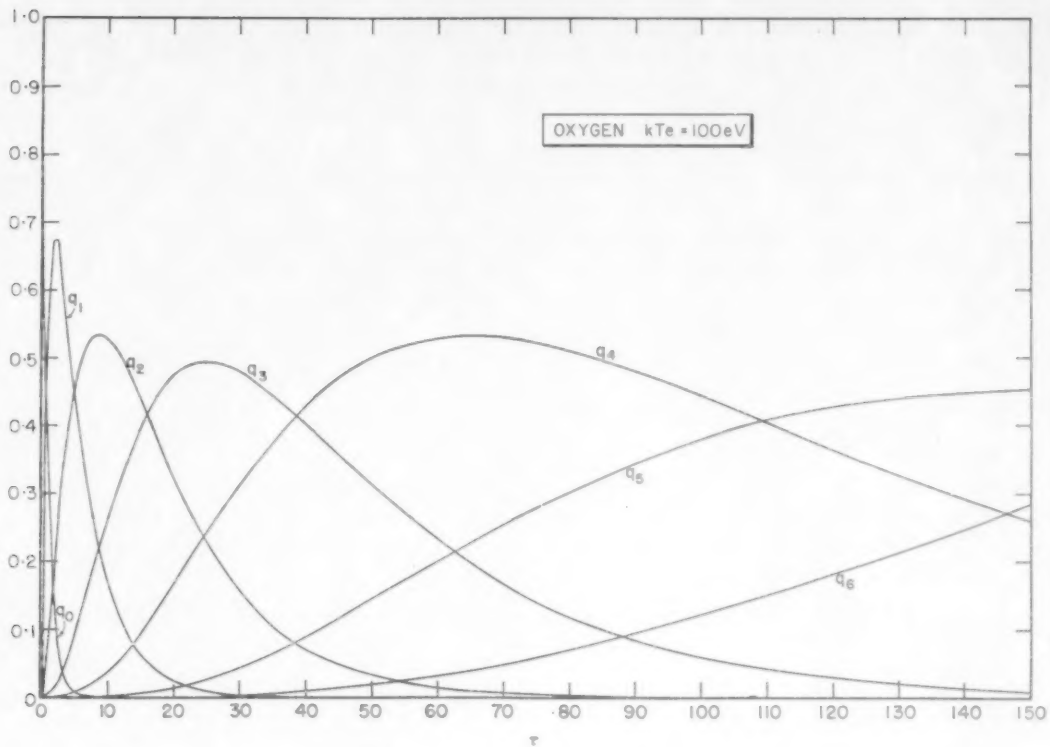

FIG. 7.—Transient stripping curves, oxygen, $T_e = 100$ eV.

Now it can be readily determined from equation (42) that

$$\int_0^\infty q_j d\tau = \frac{1}{a_j} \tag{48}$$

so that Q^{ex} takes the simple form

$$Q^{ex} = n_0 \sum_{j=0}^{S} \sum_m \frac{\langle \sigma v \rangle_{jm}^{ex}}{\langle \sigma v \rangle_0^i} \cdot \frac{(h\nu)_{jm}}{a_j} . \tag{49}$$

But the reciprocal of a_j is simply the time constant for decay of the j^{th} state, measured in units of τ. Thus equation (49) shows that during transient build-up, each successive state of ionization radiates as though it possessed the entire population of the particular impurity ions for one-time constant, after which the population then jumps discontinuously to the next higher state of ionization, and so on. Thus we can write alternatively, for (49)

$$\frac{Q^{ex}}{n_0} = \sum_{j=0}^{S} p_j^{ex} \tau_j \text{ erg/atom} \tag{50}$$

where p^{ex} represents the same steady-state excitation radiation rates, as discussed earlier, but expressed in the correct dimensionless time units.

It is clear that the most important contributions to the sum will be made by the more highly stripped, and therefore longer-lived states. In the 50 eV oxygen example given, the 'fee' for ionization up through the Oxygen^{VI} level, calculated from (50) using the approximate excitation functions, is roughly 8 keV. This is to be compared to the summation of the ionization energies up through O^{VI}, which is about 0·43 keV, and therefore almost negligible compared to the excitation radiation fee. At higher temperatures and for higher Z impurities the fee would be even larger. Consideration of losses of this kind may be helpful in explaining some of the anomalously large energy losses which occur in some unstable plasmas in magnetic bottles. In such plasmas, where turbulence is present, impurity ions might make several round trips to the chamber wall during the plasma lifetime, each time radiating away a substantial energy.

TOTAL PLASMA RADIATION—ONSET OF SELF-ABSORPTION

The calculations which have been presented in the preceding sections can be used to estimate the total radiation losses from a slightly impure hydrogen plasma by summing the radiation power loss densities over the plasma volume. For most cases of interest in laboratory high temperature plasmas, the total radiated power will be given simply by summing the power

densities over the volume. In this case the total losses will simply be proportional to the plasma volume and to the square of the electron density, (for a constant fractional impurity content). However, at a sufficiently high plasma density, self-absorption of the radiation may begin to occur for some parts of the spectrum, particularly at the position of intense line emission. The onset of self-absorption is signalled whenever the calculated spectral intensity of losses through the plasma surface approaches the Planck spectral value, which represents the upper limit of radiation power density which is possible from a system of randomly moving particles.

A simplified treatment of this problem has been given by the author (CLAUSER, 1959) which, though not rigorous, is of heuristic value and may be used to estimate the nature of self-absorption effects. In this treatment a Boltzmann-type equation for the density of excited states of a simple atom model is solved in zero order. Considering a spherical plasma, it is shown that in the approximation used the flux of escaping photons is given by

$$Q_\nu(d\nu) = \gamma' n^* P(\nu)\left\{\frac{4\pi a^3}{3} - \frac{4\pi a^2}{k}\left[\frac{1}{4} - \frac{1}{8k^2a^2} + \left(\frac{1}{4ka} + \frac{1}{8k^2a^2}\right)\epsilon^{-2ka}\right]\right\}\cdot d\nu \quad (51)$$

where γ is the total decay frequency (including stimulated emission) of the excited states (of density n^*). $P(\nu)$ is the line shape of the emitted line, a is the radius of the sphere, and k is the absorption coefficient of the plasma, to be later specified.

In the limit $ka \to 0$, this becomes

$$Q(\nu)\,d\nu = \gamma n^* P(\nu)\frac{4\pi a^3}{3}\left[1 - \frac{ka}{4} + \cdots\right]d\nu \quad (52)$$

i.e. simple volume radiation with an absorption term of order ka.

In the limit of large ka, achieved by raising the plasma density, the radiation flux takes on a value truncated at the Planck value, with a 'line' width determined by ka. One finds from (51), inserting known relationships between the variables

$$F(\nu)\,d\nu = Q(\nu)(h\nu)\,d\nu = \frac{2\pi h\nu^3}{c^2}\frac{d\nu}{\epsilon^{h\nu/kT} - 1}\left\{4\pi a^2\cdot\left[1 - \frac{1}{2k^2a^2} + \cdots\right]\right\}. \quad (53)$$

This is of course simply the Planck surface radiation loss, with a correction term small in the limit $ka \gg 1$. This expression defines a 'rectangular' emission line, the intensity of which drops to small values for frequencies on either side of the centre frequency, where $ka \approx 1$. While there are clearly many cases (for example the sun's corona) where this result is not correct in detail, it probably reproduces roughly the behaviour of many constant temperature plasmas in the laboratory.

The points in frequency where $k(\nu)a \approx 1$ are usually determined (in the limit $k(0)a \gg 1$) by Doppler broadening. In this case, k may be approximated by

$$k = \frac{n\lambda_0^2}{8\pi}\left(\frac{g_2}{g_1}\right)\frac{1}{\pi^{1/2}}\frac{\gamma}{\delta\nu}\ \text{cm}^{-1} \quad (54)$$

where $\delta\nu$ is the r.m.s. Doppler shift, and (g_2/g_1) is the ratio of statistical weights of the two states.

To give a numerical example, consider the $2S$–$2P$ line of O^{VI}, for which $\lambda_0 = 1032$ Å, $\gamma = 4 \times 10^8$, $g_2/g_1 = 2$. If we take the kinetic temperature of the O^{VI} to be 100 eV, $k = 5 \times 10^{-16}n$. Thus if $a = 10$ cm, self-absorption effects will just begin to be important for $n = 2 \times 10^{14}$ O^{VI} ions per cm^3.

From the line width and the limiting radiation values, the total radiation loss from the truncated line can be calculated to be

$$S = s_p\left(\frac{2}{\pi}\right)^{1/2} a^{5/2}\, n^{1/2} \times\left[\left(\frac{\gamma}{\nu}\right)\left(\frac{g_2}{g_1}\right)^{1/2}\left(1 + \frac{\gamma_p}{\gamma}\right)^{1/2}\right]\text{erg.cm}^{-2}.\text{sec}^{-1} \quad (55)$$

where s_p is the Planck spectral value and γ_p is the de-excitation collision frequency. $\left(\frac{\gamma}{\nu}\right) \approx \alpha^3 = (1/137)^3 = 3{\cdot}9 \times 10^{-7}$ for resonance lines. One can see that in this case, where $k(o)a \gg 1$ by virtue of high electron density, that the emission losses vary as the 5/2 power of the radius and as the 1/2 power of the density, in contrast to the case where $ka \ll 1$. In the high density limit, however, the radiation losses are already enormous.

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THE FEASIBILITY OF A SUPERCONDUCTING PROTON LINEAR ACCELERATOR

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Abstract—The use of superconducting resonators for a proton linear accelerator is shown to reduce greatly the radio-frequency power requirements and, in principle, to make a c.w. high intensity machine practicable. Rough estimates of costs do not preclude the proposal on economic grounds. Some of the technical problems involved are also discussed.

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1. INTRODUCTION

AN ideal accelerator for nuclear research would be one with the following characteristics:

(a) a duty cycle as close to unity as possible
(b) high intensity
(c) high energy resolution
(d) as large a fraction as possible of the accelerated protons available as an external beam
(e) a low background radiation level
(f) an easily adjustable energy.

The proton linear accelerator possesses all the required characteristics. Technically, however, it is a very complicated machine to build. The major difficulties are associated with the enormous amount of radio-frequency power that is required. It is for this reason that the 50 MeV proton linear accelerator (P.L.A.) at the Rutherford Laboratory operates only at a duty cycle of 1 per cent although, in principle, it could operate at a 100 per cent duty cycle. We have, therefore, been looking into the possibility of applying superconducting principles to proton linear accelerator design. A linear accelerator is very suitable for this purpose since it consists of a resonating cavity enclosed in a vacuum envelope and therefore requires little modification to permit the use of cryogenic techniques.

2. SUPERCONDUCTIVITY

In the absence of a magnetic field, the d.c. bulk resistivity of a superconductor falls abruptly to zero below the critical temperature T_c. The phenomenon is restricted to about 18 metallic elements and a similar number of compounds, T_c lying in the range 0·35°–18°K. Superconductivity is quenched by a sufficiently intense magnetic field which may be either external or due to currents flowing in the superconductor itself. The critical field for quenching depends both on the material and the temperature. It may be as high as several kilogauss but becomes zero at T_c.

At radio frequencies in the range of interest for a P.L.A. (i.e. 200–800 Mc/s), the surface resistivity R_s is small but finite. The variations of R_s with temperature and frequency are shown in Fig. 1 for lead ($T_c = 7{\cdot}2°\text{K}$). The form of variation is

$$R_s = R_n.r = R_n\, A(\omega)\, \phi(t) \qquad (1)$$

where R_n is the surface resistivity just above T_c, ω the frequency and t the reduced temperature T/T_c. Below 1000 Mc/s $A(\omega)$ is independent of material and R_s varies as ω^2. In the range $0{\cdot}55 < t < 0{\cdot}82$, $\phi(t)$ is given by (PIPPARD, 1954; STURGE, 1958)

$$\phi(t) = t^4(1 - t^2)(1 - t^4)^{-2}. \qquad (2)$$

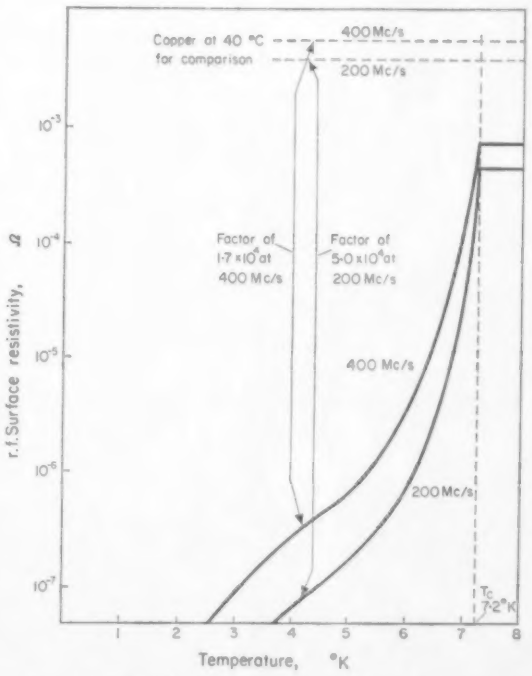

FIG. 1.—Radio-frequency surface resistivity vs. temperature for superconducting lead.

These equations are useful for interpolating from published experimental data which is fairly plentiful but cannot be used completely for a theoretical prediction of R_s since this involves knowing the effective number of conduction electrons per atom. As in the classical skin effect, the fall-off of current density with depth is exponential, the relaxation length (skin depth) involved being of the order of 10^{-5} cm.

No great advantage is gained by cooling a normal conductor such as copper to low temperature. The classical skin depth is thereby reduced, but when it becomes comparable with the collision mean free path of the conduction electrons in the atomic lattice, the classical skin effect theory breaks down and the surface resistivity approaches an asymptotic value R_∞ which (for 200–400 Mc/s) is only a factor of 10 lower than the room temperature value. This phenomenon is known as the anomalous skin effect and is illustrated by Fig. 2.

For a more detailed account of the r.f. phenomena mentioned above see PIPPARD, (1954).

3. RADIO-FREQUENCY DISSIPATION IN A PROTON LINEAR ACCELERATOR

In a conventional P.L.A. large amounts of r.f. power are required to off-set resistive dissipation in the cavity walls while building up and maintaining the electric fields needed for the acceleration process. These power requirements are so high (of order 60 kW per MeV gained by the beam) that the r.f. has to be pulsed to keep the mean power to be generated by the valves and handled by the cooling system within reasonable limits. The r.f. resistive dissipation depends on the electromagnetic field distribution in the resonator and on the surface resistivity of its walls. If the field intensities are kept constant so that the same accelerating rate and resonator geometry are maintained, the r.f. power dissipation will vary directly as the surface resistance. Such a change will also be seen as a change in the cavity Q-value defined by

$$Q = \frac{\omega \times \text{Energy stored}}{\text{Energy lost/second}} \tag{3}$$

the numerator remaining fixed while the denominator varies as the surface resistivity.

Equation (3) may also be written in the form

$$Q = K\left(\frac{377}{R}\right) \tag{4}$$

where K is a dimensionless constant, in principle calculable from the resonator geometry, R the surface resistivity of the walls and 377 the impedance of free space (both in ohms). Measurements of Q and the use of equation (4) constitute the most popular method of measuring R, both in the superconducting and the anomalous regions. Micro-wave calorimetry has also been used.

The Rutherford Laboratory P.L.A. uses copper resonators operating at 200 Mc/s and 40°C for which the classical skin effect theory gives a surface resistivity of $3{\cdot}92 \times 10^{-3}\ \Omega$. 400 Mc/s is a possible frequency for extensions beyond 50 MeV: the appropriate surface resistivity is $5{\cdot}52 \times 10^{-3}\ \Omega$.

The corresponding figures for superconducting lead are deduced below. The critical temperature of lead is 7·2°K and data on its r.f. behaviour is available. Materials with higher critical temperatures do exist, for example, niobium (9·1°K) and niobium–tin (18°K)

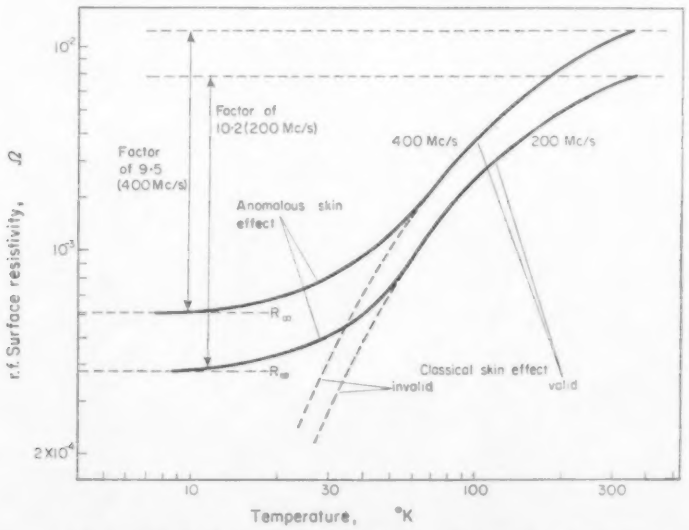

FIG. 2.—Radio-frequency surface resistivity vs. temperature for copper.

but no r.f. measurements appear to have been made on them. R_s has been measured for tin at $t = 0{\cdot}82$ and at frequencies ranging from 220 to 1700 Mc/s (STURGE, 1958). R_n for both tin and lead has also been measured and is found to be equal to within 1 part in 200 (CHAMBERS, 1952). The surface resistivity of lead at 200 and 400 Mc/s and at 4·2°K ($t = 0{\cdot}58$) can then be found by use of equation (1). It is $8{\cdot}0 \times 10^{-8}\ \Omega$ at 200 Mc/s and $3{\cdot}22 \times 10^{-7}\ \Omega$ at 400 Mc/s.

It follows from the above data and the arguments in the first paragraph of this section that the resistive dissipation can be reduced by factors of 5×10^4 at 200 Mc/s and $1{\cdot}7 \times 10^4$ at 400 Mc/s by using superconducting lead resonators at 4·2°K in place of copper resonators at 40°C.

Table 1 lists the principal parameters of various sections of a high-energy P.L.A. using copper at 40°C and lead at 4·2°K. The powers quoted are the resistive losses to the structure and would constitute the main thermal load on the refrigeration plant.

The resistive dissipation in the superconducting resonators is so small that, even allowing for the power fed into a 1 mA beam, it appears entirely feasible to supply it continuously. In fact, the Q values of the resonators are such ($\sim 4 \times 10^9$ unloaded) that the build-up time constant from a constant power source is about 3 seconds so that the r.f. could not conveniently be pulsed.

4. MAGNETIC FIELDS

It is necessary to ensure that no portion of the superconducting resonator experiences a magnetic field greater than the critical value which for lead at 4·2°K is 500 gauss. In a drift-tube loaded cylinder resonating in the mode used in the P.L.A. (loaded E_{010}) the maximum r.f. magnetic field occurs at drift-tube centres. In a 200 Mc/s Alvarez structure (WILKINS, 1955b) the field amplitude at the 50 MeV point is 180 gauss and at the 150 MeV point about 250 gauss; in a 400 Mc/s structure these figures become 250 gauss and 350 gauss. There should be an abrupt drop in magnetic field strength on passing to π-mode structure at 150 MeV since the drift-tubes are much shorter. It is also necessary to ensure that stray magnetic fields from focussing devices inside the drift tubes are not excessive but this does not appear to be difficult.

5. REFRIGERATION

A 50 MeV P.L.A. operating at 200 Mc/s and 100 feet in length is taken as an example. The r.f. dissipation using superconducting lead resonators at 4·2°K should be 68 W (see Table 1). The total surface area of the resonators would be 114 m². Assuming surface emissivities of 5 per cent, the radiative heat influx from shields at liquid nitrogen temperatures would amount to 5 W. The heat conducted to the resonators could be made as low as 13 W by using Nylon or P.T.F.E. in the mechanical supports and stainless steel in the r.f. feed lines. These materials combine good mechanical properties with low thermal conductivities at low temperatures (SCOTT 1960). The total heat load at 4·2°K would therefore be 86 W, of which 79 per cent would be due to the r.f. dissipation.

The thermal loads on the liquid nitrogen shields of area 163 m² would be about 2·5 kW which corresponds to a liquid nitrogen evaporation rate of 12 gallons per hour. Cooling such an accelerator from room temperature would involve the evaporation of 2000 gallons of liquid nitrogen, followed by a period of 7–10 hours for the helium refrigeration plant to cool the resonators from 77°K to 4·2°K.

6. FOCUSSING AND INJECTION

Superconductivity may also be exploited in the magnetic focussing devices needed in a high-intensity P.L.A. provided that the field experienced by the

TABLE 1

Final energy (MeV)	Copper 40°C				Lead 4·2°K			
	200 Mc/s throughout		400 Mc/s after 50 MeV		200 Mc/s throughout		400 Mc/s after 50 MeV	
	Length (ft)	Total peak power	Length (ft)	Total peak power	Length (ft)	Total peak power	Length (ft)	Total peak power
50	100	3·4 MW	—	—	100	68 W	—	—
148	300	19 MW	240	19 MW	300	380 W	240	970 W
600	1208	86 MW	883	86 MW	1208	1700 W	883	4900 W
1500	3030	167 MW	2170	167 MW	3030	3300 W	2170	9600 W

0–50 MeV Alvarez structure.
50–148 MeV Alvarez structure (WILKINS, 1955a).
148–600 MeV π-mode structure (THOMPSON, 1958).
600–1500 MeV Travelling wave structure (LOACH, 1961).

exciting coils does not exceed the critical value. By means of simple line integrals (AUTLER, 1960) it may be shown that in a conventional quadrupole the field experienced by the exciting coils is approximately $ga^2/21$ where g is the field gradient in the gap of radius a, and l is the length of the exciting coil. At the 4 MeV point of a 200 Mc/s P.L.A. the required values are $g = 1500$ gauss cm^{-1} and $a = 1$ cm. If $l > 0{\cdot}2$ cm the field experienced by the coils is < 4 kG which is the critical field for niobium at 4·2°K. This metal, which is easily drawn into wire, has a critical temperature of 9·1°K.

PANOFSKY (square aperture) quadrupoles (HAND and PANOFSKY 1959; VAN der MEER, 1959) may be easier to manufacture. Their normal disadvantage of high power consumption no longer applies . They may be adapted to a roughly circular form and the yoke flux is less than in a conventional quadrupole with the same g and a. The maximum field occurs at the corners of the square aperture and is given by $ga\sqrt{2}$. This region is occupied by windings, so that this field must not exceed the critical field, a criterion easily satisfied with the above parameters.

Another alternative is the use of solenoids inside the drift tubes. The fields required are high (50–100 kG) but the use of niobium–tin (KUNZLER *et al.*, 1961) may make the method feasible.

An output current of many hundred micro-amperes could be obtained with an input current of 1 mA if a buncher were used before the first cavity. A 1 mA proton beam at 4 MeV could be obtained from a d.c. injector (SEITZ *et al.*, 1960), the high injection energy easing the focussing problems in the early sections of the first cavity.

7. COSTS

An analysis of the costs of a superconducting P.L.A. has shown that both the capital and running costs are not much greater than those for a 'conventional' pulsed P.L.A. The capital cost of the accelerating structure and vacuum vessels would amount to 65 per cent of the total, and that of the helium refrigerator to 15 per cent. The r.f. system would account for only 6 per cent of the total cost since the power required is modest and, of course, there would be no modulators. These estimates were based on a 200 Mc/s machine using lead at 4·2°K. Other superconducting materials may make it feasible to work at 400 Mc/s with a reduction in the size and cost of the accelerating structure.

8. SUNDRY PROBLEMS

The aim of this paper has been to indicate the feasibility and advantages of using superconducting resonators for a P.L.A. Clearly there are many technical and scientific problems to be investigated before the design and construction of an actual accelerator could be contemplated. The foremost of these is undoubtedly whether the low surface resistivity values quoted in Section 3 which were measured on small specimens could be reproduced on surfaces many tens of square metres in area. It is hoped shortly to use resonant cavities with areas of one or two square metres to investigate this and also to assess the potentialities of niobium and other superconductors.

The proton beam will exert both a resistive and a reactive effect on the cavities. The reactive effect would result in detuning of the cavity which would be serious with an unloaded Q value of 4×10^9. A resistive load at room temperature could, however, be loop-coupled into the cavity and the Q value brought down to 5×10^6 without increasing the thermal load on the helium refrigerator. Detuning by the beam would then be much less serious. It is estimated, for example, that a 1 mA beam would produce a phase change of about 15°. Problems of frequency stability and inter-tank phasing of a long multi-cavity machine would also be eased by the reduced Q value.

Other important problems include focusing the intense proton beam so that very few protons impinge on the drift-tubes, and the effect of multipactor loading during initial build-up of the r.f. fields. This build-up will be slow due to the high Q-values (time constant 4 msec for a Q of 5×10^6), but the problem has only to be faced when the r.f. is first switched on and not at every pulse as in a modulated P.L.A.

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RESEARCH NOTES

Re-ignition of high current spark discharges

(*Received* 19 *April* 1961)

1. INTRODUCTION

SPARK discharges have been employed for various purposes by previous workers (BELLASCHI, 1934; STEKOLNIKOV, 1947; BRAUDO, 1959). With the advent of thermonuclear research the spark gap has gained prominence in many switching applications (FITCH and MCCORMICK, 1959; SMART, 1959), and the possibility of repetitive switch operation has stimulated interest in the recovery properties of spark gaps after the passage of a high current discharge (CHURCHILL, 1960). Re-ignition data are available for surge currents up to 23 kA (MCCANN and CLARK, 1943), but there appears to be no published recovery work for discharges of 235 kA as employed in the present investigation.

2. APPARATUS

Figure 1 shows the essentials of the apparatus. The high current is obtained by discharging the 40 kJ capacitor bank through the four-electrode spark gap assembly. In order to provide the well-defined, uni-directional initial current pulse required in recovery studies (EDELS and CRAWFORD, 1956), the resistor, R_D, is chosen to critically damp the generator circuit. Figure 2 depicts the current waveform.

The restriking voltage is produced by a single stage impulse generator in which R_2C_2 governs the rise-time and R_1C_1 the fall-time. These time constants are arranged so that the impulse waveform approximates a step-function over a 50 μsec period as shown in Fig. 3(a).

The main discharge current is measured by means of a coaxial tubular shunt and C.R.O. To eliminate excessive photographic recording of the restriking voltage transient, the waveform is observed with the aid of an uncalibrated distortionless potential divider and C.R.O. and the amplitude is determined by a d.c. measurement of the voltage to which the impulse capacitor, C_1, is charged.

3. EXPERIMENTAL TECHNIQUE AND RESULTS

Technique

The technique employed here for the measurement of free recovery characteristics of high current spark discharges is similar to that adopted by EDELS and CRAWFORD (1956) in their work on low current arcs. After passing the surge current through the test gap a period is allowed during which the gap freely recovers. Then at a pre-determined time after the discharge a unit-function voltage is applied to the gap and re-ignition is observed oscillographically. The re-ignition voltage is taken as the 100 per cent breakdown voltage, and for each point is determined by about 20 tests. The procedure is repeated at varying delay times in order to determine the full recovery characteristic.

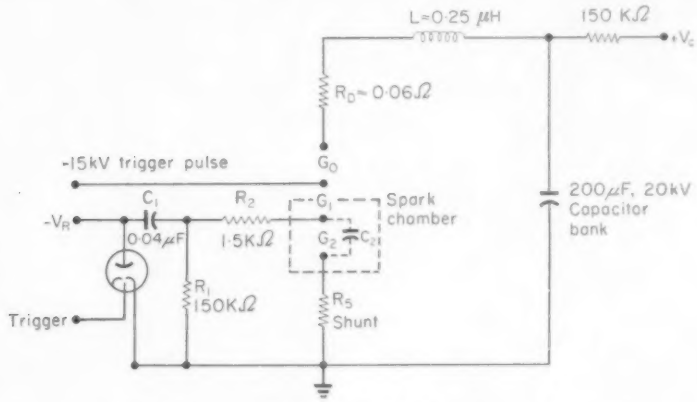

FIG. 1.—High current generator and restriking voltage generator.

Results

Figure 4 shows the recovery of spark gaps in air at atmospheric pressure for 235 kA discharges between electrodes of various sizes and materials.

A comparison of re-ignition voltage characteristics for hydrogen, air and argon can be seen in Fig. 5 for 235 kA discharges between tungsten rod electrodes.

4. DISCUSSION

4.1 *General recovery features*

Two trends are observed in the recovering spark channel: (1) a rapid initial recovery up to about 2 msec and (2) a slower rate thereafter as the re-ignition voltage approaches the sparking potential of the gap. For most characteristics the fully-recovered time is about 20 msec, and in all cases investigated is less than 100 msec.

Two forms of breakdown are observed for the re-ignition discharge. For delay times of about 750 μsec and greater, the re-ignition takes the form of a spark breakdown which can be detected by a voltage collapse across the gap (Fig. 3b). At

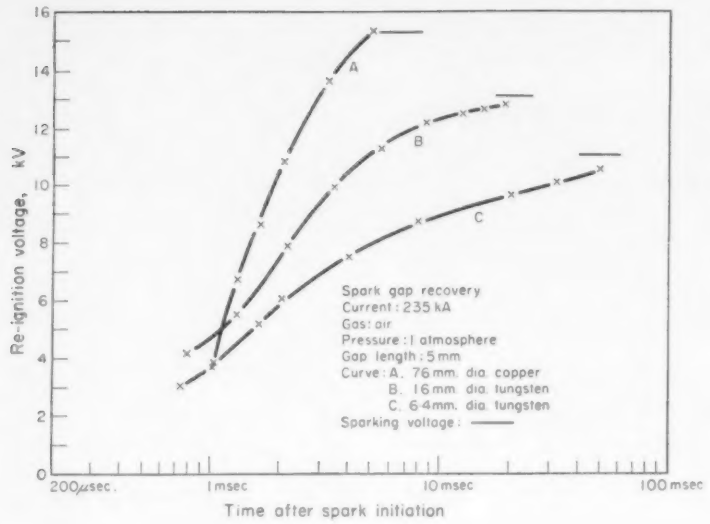

FIG. 4.—Recovery characteristics for air.

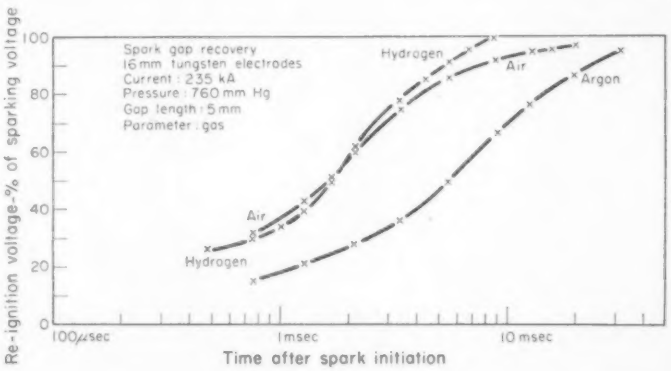

FIG. 5.—Comparison of recovery characteristics in different gases.

earlier times, a thermal breakdown occurs in which the re-ignition depends upon an energy balance between the instantaneous channel losses and power input to the gap from the probing voltage source. The time during which slow breakdown occurs depends upon the conductivity of the gap and the internal impedance of the restriking voltage generator, since upon application of the probing voltage, conduction in the gap results and the generator output decreases due to regulation (Fig. 3c). In the present investigation, re-ignition measurements are made only during the spark breakdown period.

4.2 *Effect of discharge parameters*

The re-ignition characteristics in Fig. 4 show that electrode size has a considerable influence upon the overall gap recovery. For copper electrodes the fully recovered time is about one-half that of tungsten under similar experimental conditions. Variation of the initial discharge current in the range 100 to 235 kA produced no significant change in the recovery results depicted in Fig. 4. Gas pressure (200 to 760 mmHg) and electrode separation (1 to 5 mm) also had little effect upon the recovery characteristics.

4.3 *Re-ignition in various gases*

Figure 5 shows the recovery order as: hydrogen, air and argon—with argon being much slower than the other two. These results are seen to be consistent with diffusion rates and the bulk properties of the gases concerned.

The best reproducibility of results was found for hydrogen (3%), while the scatter for argon and air was 5% and 10%, respectively.

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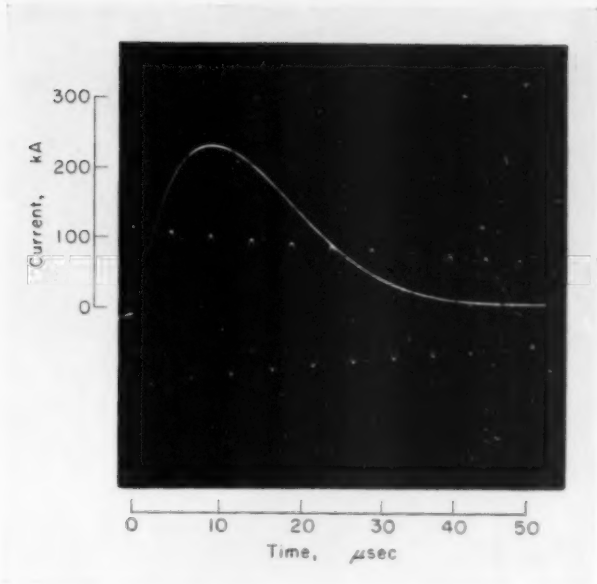

FIG. 2.—Current waveform.

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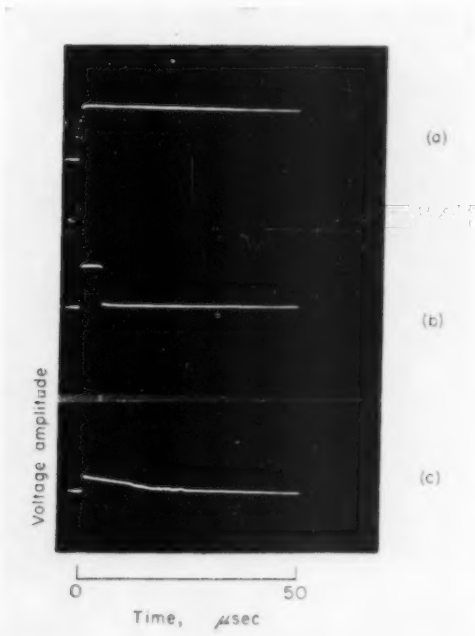

FIG. 3.—Restriking voltage waveforms.

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Magnetic configurations and the energy loss problem in the toroidal pinched discharge

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A MAJOR problem in the study of the toroidal pinched discharge is the elucidation of energy loss processes. The distribution of X-ray emission from the conducting torus walls (PAYNE and KAUFMAN, 1959) has supported the suggestion of WARE (1959) that projecting discontinuities may be important in 'anchoring' lines of force which are elsewhere drawn into the discharge column, thereby providing an easy escape mechanism for charged particles. Experiments by Miles (see WARE, 1959) have shown the existence of appreciable currents to the torus walls.

The aluminium torus Sceptre III (WARE, 1959) is appropriate for a study of the problem, since it is fitted with cylindrical copper liners which project about 1 cm. into the discharge space and shield the insulating gaps. Moreover, the vessel is made up from a large number of short cylindrical sections and is thus, imperfectly toroidal. The initial stabilizing field is closely toroidal. Wall discontinuities may thus lead to magnetic field irregularities at various points along the discharge column. This note describes magnetic probe measurements, made in a straight section between two liners, designed to investigate the effect of these irregularities.

The section had seven entry ports, shown in relation to the copper liners in Fig. 1. Ten magnetic search coils, spaced at 2·5 cm intervals, were introduced through the entry ports inside a quartz tubular envelope. The integrated signals from the probes were displayed on a multi-channel oscilloscope. Plots were obtained of average axial and azimuthal field components over eight discharges, with probes moved to cover the whole diameter. All measurements corresponded to peak current in the pulse. When the condenser bank energy supply was charged to 20 kV, peak current was 140,000 A with a primary to secondary turns ratio of 16:1 and an initial axial field of 1000 gauss.

Azimuthal ('B_θ') and axial ('B_ϕ') components of field, obtained at ports 1, 2, 3 and 4 in turn are plotted in Figs. 2 and 3. Corresponding plots were obtained at ports 5, 6 and 7 along vertical diameters. The curves were generally similar to each other. Significant differences in the B_ϕ plots occurred mainly near the outer wall; otherwise differences between curves were of the same order as experimental error. The shape of the B_θ plots implied the existence of a fairly uniformly conducting, contracted current channel, with insignificant changes (caused by liners) in the axial direction.

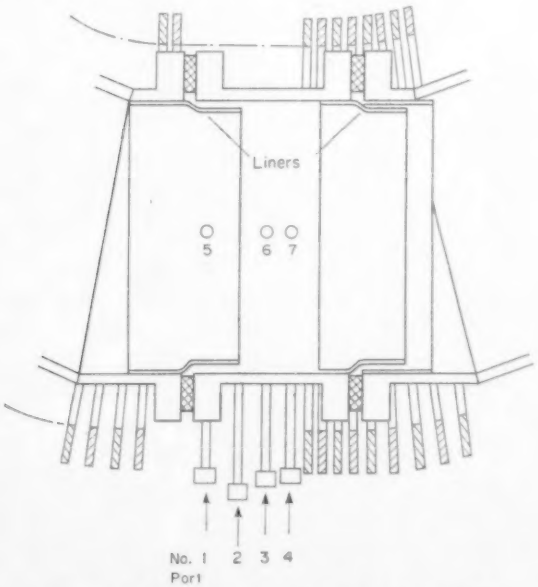

FIG. 1.—Arrangements of entry ports in Sceptre III.

The axial field, enhanced to ~4,500 gauss in the centre, was reduced to small values near the walls. Within the liner, these were always negative. Very close to the walls, the field tended to rise again towards the initial value. This was probably due partly to the ingress into the torus volume of flux initially held in the probe entry ports and partly to flux held near the metal walls by virtue of irregularities in wall geometry.

A further test of the uniformity of the magnetic configuration along the discharge column was to estimate the extent to which lines initially in the axial direction near the wall were drawn in by the contraction. Thus, in Fig. 4, lines of force initially in the area *ABCD* could be regarded as filling the area *EFGD*, where the total fluxes in the two areas are equal. The results are summarized in Fig. 5, where approximate field maps, derived from measurements along the four horizontal diameters, are shown. It is seen that within the liner region, the lines were drawn in farther than between the liners. Field gradients arising from these variations were, however, comparatively small (Fig. 3). The map ignores the B_θ component, which is thereby assumed uniform in the axial direction. A generally similar picture was obtained along the three vertical diameters.

Thus, neither the plots of azimuthal, nor of axial field components show any gross distortion by the liners of the magnetic configuration which might lead to excessive particle loss along lines of force to the walls.

The effects sought in the above investigations might also be expected if the initial axial magnetic field showed marked departures from uniformity. Lines of force would then intersect the walls in the perturbed regions. Two of the magnetic field coils were therefore short-circuited to give a 7 per cent reduction of field at the axis and a 25 per cent reduction at the wall. No significant change was detected in the discharge field distribution, measured in a plane adjacent to the disturbance, in the fluctuations on the magnetic field oscillograms or in the total current. A second non-uniformity was tried by removing two of four symmetrically placed return leads from the magnetic field windings. Each lead normally carried one-quarter of the current in the magnetic field windings and the omission of two return leads produced a transverse component of ~2 per cent of the initial field at all points round the torus. Thus, a line of force, initially paraxial at the centre, would reach the wall after two revolutions of the torus and thereby establish an easy escape path for charged particles in the early stages of the current pulse. No significant change in discharge current configuration, or of total current, was observed.

The experiments suggest that geometrical irregularities were unimportant in producing a significant current drain to the walls of a magnitude resolvable by the probe technique. Studies of X-ray emission show, however, that fast electrons impinge on projecting discontinuities, though information is lacking on whether they do so (a) by crossing lines of force or (b) by following 'anchored' lines of force from the central column as originally postulated. Present experiments suggest that process (b) is inapplicable until particles approach the walls closely enough for those lines of force which actually

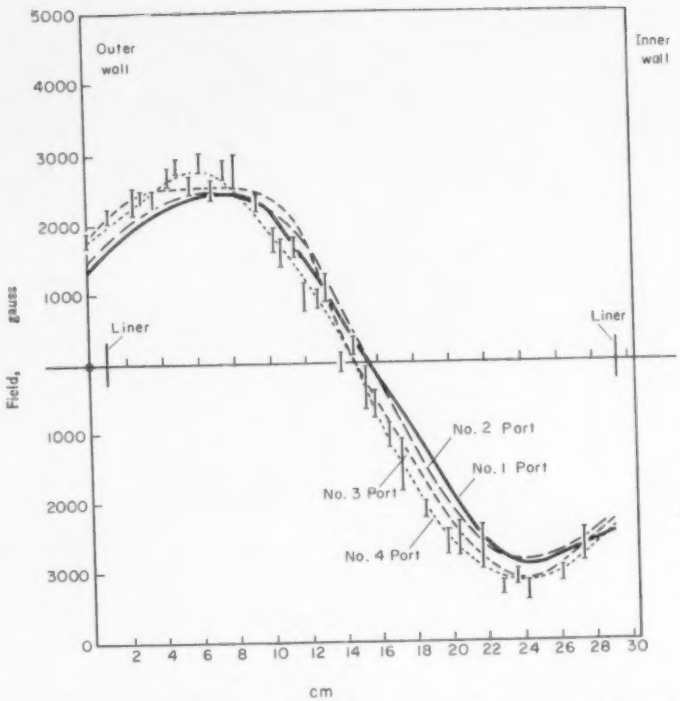

FIG. 2.—Azimuthal field plots along four horizontal diameters of Sceptre III. (Experimental points shown for Port No. 4 only to illustrate errors.)

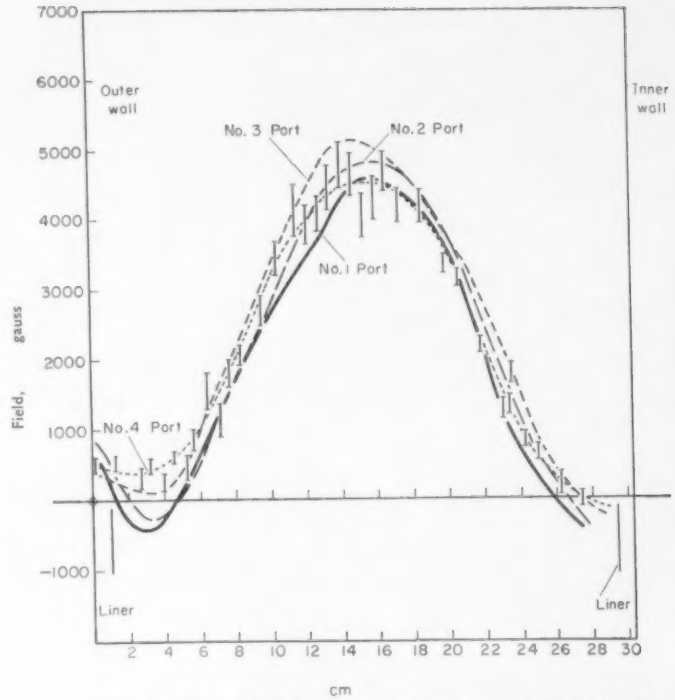

FIG. 3.—Axial field plots for the same four horizontal diameters as in Fig. 2. (Experimental points shown for Port No. 4.)

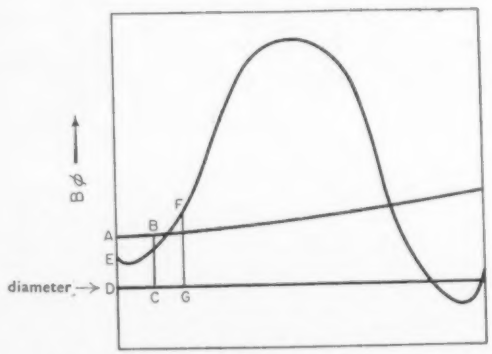

FIG. 4.—Movement of axial lines of force to peak current.

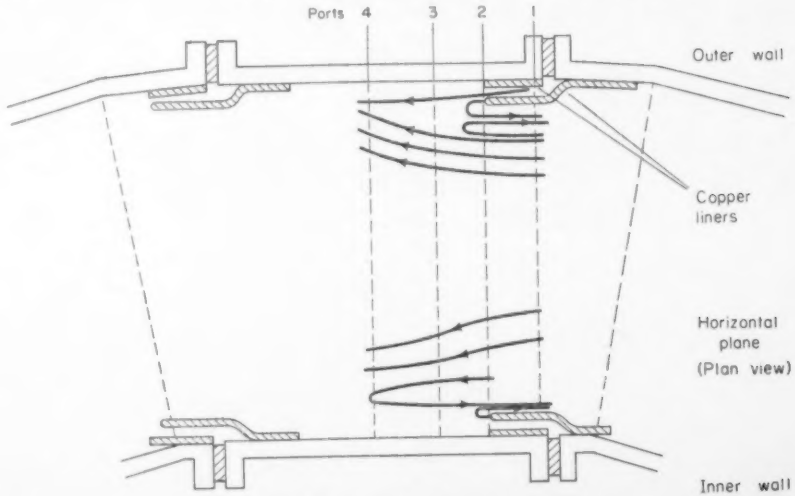

FIG. 5.—Map of axial lines of force at peak current.

pass into the liners to guide them to the copper surfaces. The experiments do not exclude the possibility of particles, particularly fast ones, making several circuits of the torus along lines which ultimately intersect the walls at points remote from the region investigated. The total current of fast electrons is believed not to exceed a few hundred amperes (PAYNE, 1961) which is beyond the resolution of the probe technique.

Acknowledgments—This work has been carried out as part of an investigation into energy loss processes from the Sceptre discharge under the general supervision of DR. A. A. WARE. It is a pleasure to acknowledge the work of MESSRS. G. A. BARNETT and H. B. MUNDY in the design of the special probe sector and of MESSRS. J. E. READ, V. C. CLOKE and J. A. MCKINNON for their operation of the torus equipment. MR. M. J. BOLTON gave valuable assistance in experimental work and data reduction.

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Observations with a small theta-pinch

(*Received* 7 *July* 1961)

WE have operated a small theta-pinch apparatus with the following parameters:

peak magnetic field on axis	80 kG
period of oscillation of the field	2·5 μsec
capacitor energy	1·3 kJ at 20 kV.

The discharge took place in a quartz tube of 2·5 cm internal diameter, and was excited by a coil consisting of two single-turn straps around the outside of the tube, 1 cm wide and separated by 1 cm. The straps were connected in parallel, giving a slight mirror effect. The entire apparatus was simple and compact. The capacitor was made up in the laboratory and consisted of six units connected to a common strip line; each unit was made of copper foil and 'melnex' sheet, folded in a zig-zag. The capacitor was discharged through the coil by means of a mechanically triggered spark gap switch, filled with a thin sheet of dielectric which was punctured by a sharp metal point, and thus had to be replaced manually after each shot. The capacitor and switch inductances were each about 1 mμH, in comparison with the coil inductance of 23 mμH. Since making the observations described here, we have developed an automatic dielectric switch of which details will be given in a letter.

In most of our work we used a weak pre-ionization device, consisting of a 30 Mc/s continuous oscillator coupled capacitively to the discharge in such a way that the power appearing at the tube wall as heat was about 10 watts. In the absence of this pre-ionizing field, breakdown of the gas still occurs but we think that high-frequency parasitic oscillations in the main current circuit then play an important part in the breakdown process. Typical phenomena in deuterium are illustrated in Fig. 1, which shows oscillograms from a stilbene scintillator placed a few inches from the discharge tube. In cases 1(a), and 1(b) no separate pre-ionization field was applied; conditions were almost the same in the two cases, except that in (b) the transmission line between the capacitor bank and the switch was a few inches shorter. In case 1(c) the pre-ionization oscillator was in use. The stilbene scintillator was not heavily screened; in cases (a) and (b) the output is due to both neutrons and X-rays, each of which were detected separately in other similar shots using a BF_3 counter and a NaI scintillator. We have many oscillograms of types (a), (b) and (c), the broad features of each type being repeatable. Particularly noticeable was the fact that discharges of type (a) always gave large scintillator outputs during the third and fifth half-cycles of coil current. In case (b) breakdown appears to occur one half-cycle earlier, which we attribute to a difference in the parasitic oscillations in the main current circuit. It is difficult to measure the amplitude of these, but with a short probe measuring the electric field outside the discharge tube and near the straps we found persistent oscillations at about 50 Mc/s, whose amplitude was comparable with that due to the voltage step across the coil just after closing the switch. Such oscillations can be accounted for crudely in terms of the circuit parameters. In case 1(c), where the gas is weakly ionized before the switch is closed, we attribute the scintillator counts during the first half-cycle to X-rays from runaway electrons accelerated in the azimuthal electric field. If the oscillating axial magnetic field is assumed uniform, a calculation of electron trajectories in such a field shows that energies of several MeV are attainable. At higher gas pressures the observed behaviour is different and hard X-rays are not produced during the first half-cycle.

In order to investigate the X-rays we set up two scintillation counters a few inches away from the discharge tube. The first was the stilbene scintillator as used above; the stilbene was in the form of a cube of side 2·4 cm, and was screened with 3·0 mm of aluminium. The second counter used a slab of NaI, 2·0 mm thick, screened with 0·1 mm of aluminium. Each counter was enclosed in a separate steel can to reduce electrical interference; the thickness of the can walls was 0·9 mm. The X-rays also had to penetrate the 1·4 mm quartz wall of the discharge tube. The oscillograms shown in Fig. 2(a) illustrate the results obtained with a helium discharge. Figure 2(b) illustrates a similar shot, but with a large sheet of lead, 3·2 mm thick, interposed between the discharge tube and the scintillators. The oscillograms were closely repeatable. It will be seen that the NaI scintillator gives an output which is confined to the first half-cycle of coil current in case 2(a), whereas it is confined to the second half-cycle in 2(b). The stilbene signal is confined to the second half-cycle in both cases, although it is much smaller in case (b) These results strongly suggest that the X-ray output occurs in two well-separated energy bands, a low-energy group emitted only during the first half-cycle and a high-energy group ($>$200 kV) emitted only during the second half-cycle. The stilbene scintillator responds preferentially to the hard component because of its extra 3 mm of aluminium screen, whereas hard X-rays tend to pass right through the thin NaI crystal. When the lead sheet is inserted the hard component produces sufficient secondary radiation to excite the NaI. The similar production of soft X-rays in Scylla has been reported and examined in detail (LITTLE *et al.*, 1961).

Figure 3 is an oscillogram from a shot with deuterium at a pressure of 65 μ. There was an internal magnetic probe (outside diameter of case 1·3 mm) present on the tube axis, about 4 mm away from the median plane. The stilbene scintillator had a close-fitting lead cap, 3·2 mm thick, which almost surrounded it; the NaI scintillator had no additional shielding. The stilbene shows a symmetrical pulse centred around the peak of the second half cycle of current, which we attribute to neutrons; we have other similar oscillograms but no independent neutron measurement in this case. The behaviour resembles that found in Scylla (BOYER *et al.*, 1960) both in the shape and position of the neutron pulse and in the way in which the X-ray output commences earlier, at approximately the time of coil-current zero.

We have made some measurements with magnetic probes. Figure 4 shows a probe oscillogram with the 1·3 mm probe on the tube axis, in the median plane. This record also closely resembles one from a probe similarly positioned in Scylla. In order to assess the effects of the probe case on the discharge we placed a small search-coil outside the discharge tube in the median plane, at a point where the circuit magnetic field was null in the absence of a discharge. Coils so positioned measure a quantity which is essentially the magnetic moment of the gas current. A check with no discharge (vacuum) showed that when displaced from the null point the integrated signal from the search coil faithfully followed the circuit current trace; at the null position the search coil output was virtually zero. Two typical oscillograms with the search coil are shown in Fig. 5(a) and (b). In case (b) the internal probe described above was positioned on the axis in the median plane; in case (a) it was completely withdrawn. This result shows that probes at the centre of the discharge have a large influence upon it. We have found similar but less pronounced effects as the internal probe is moved to other positions in the discharge.

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* On leave from University College, London.

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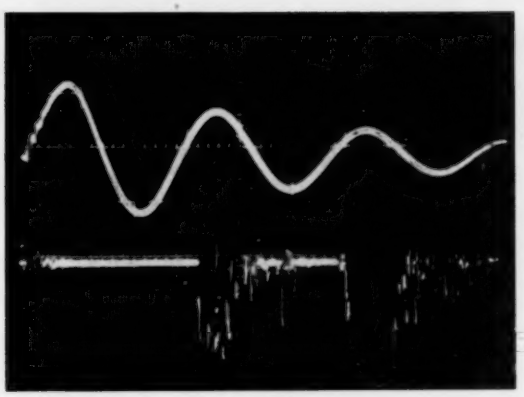

FIG. 1(a).

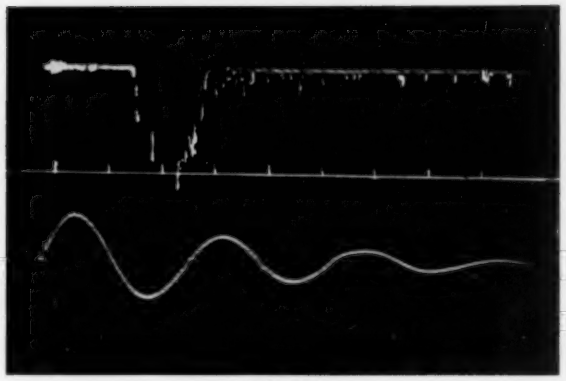

FIG. 1(b).

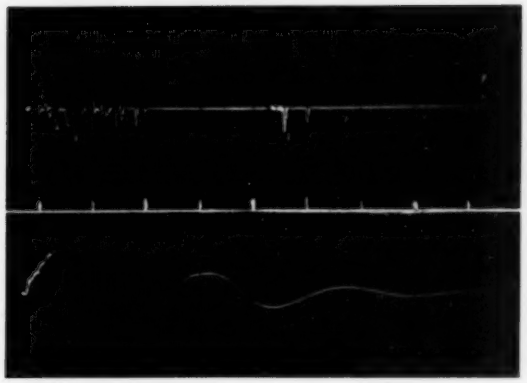

FIG. 1(c).

FIG. 1(a).—D_2 at 20 μ. Capacitor voltage 20 kV. Before shortening transmission line. No pre-ionization. Upper trace: coil current. Lower trace: stilbene scintillator without lead screen. Time scale: 1 μsec/div.

(b) D_2 at 23 μ. Capacitor voltage 18 kV. After shortening transmission line. No pre-ionization. Upper trace: stilbene scintillator without lead screen. Lower trace: coil current, vertical scale not as in (a). Time scale: 1 μsec/div.

(c) D_2 at 23 μ. Capacitor voltage 18 kV. With 30 Mc/s pre-ionization oscillator in use. Upper trace: stilbene scintillator without lead screen. Lower trace: strap current. Time scale: 1 μsec/div.

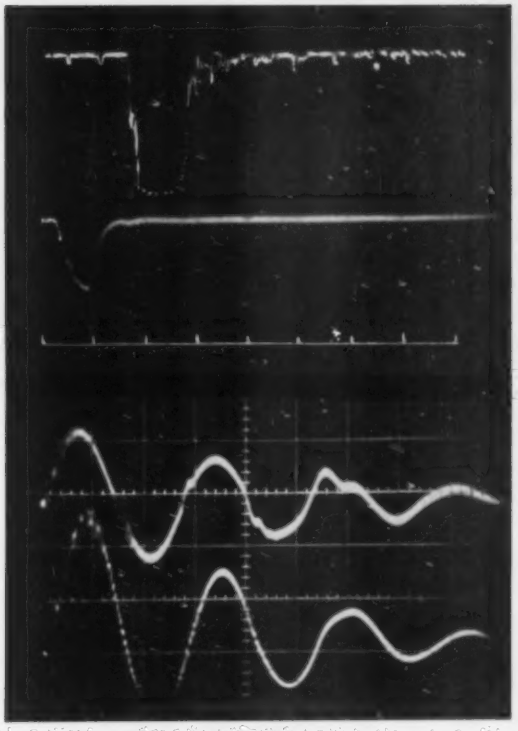

FIG. 2(a).

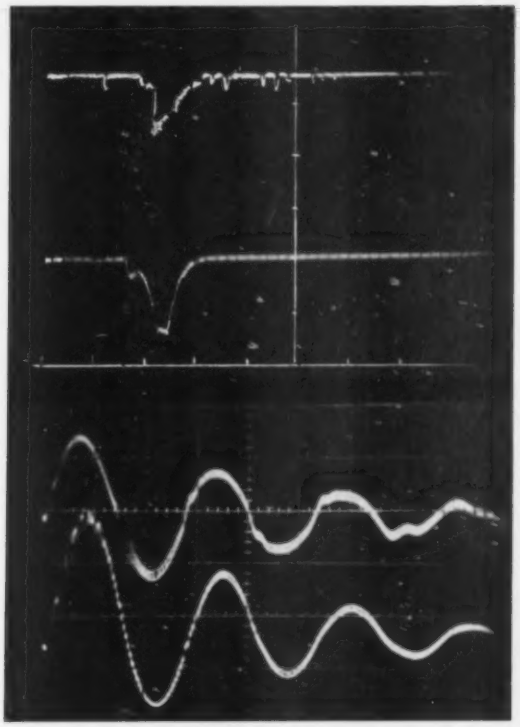

FIG. 2(b).

FIG. 2(a).—He at 60 μ. Capacitor voltage 18 kV. With pre-ionization. Top trace: stilbene scintillator without lead screen. Second trace: NaI scintillator without lead screen. Third trace: external search coil (essentially gas current). Bottom trace: coil current. Time scale: 1 μsec/div.

(b) He at 65 μ. Capacitor voltage 18 kV. With pre-ionization. Top trace: stilbene scintillator with 3·2 mm lead screen. Second trace: NaI scintillator with 3·2 mm lead screen. Other two traces as in 3(a). Time scale: 1 μsec/div.

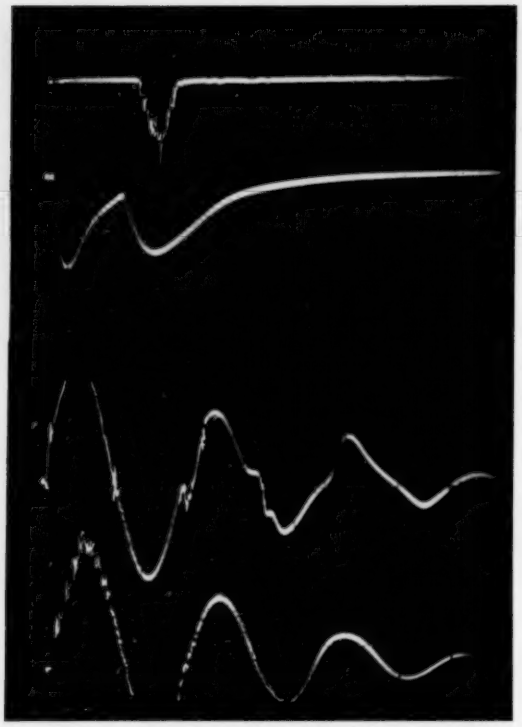

FIG. 3.—D_2 at 65 μ. Capacitor voltage 18 kV. With preionization. Top trace: stilbene scintillator with 3·2 mm lead cap. Second trace: NaI scintillator without lead cap. Third trace: internal probe (see text). Bottom trace: coil current. Time scale: 1 μsec/div.

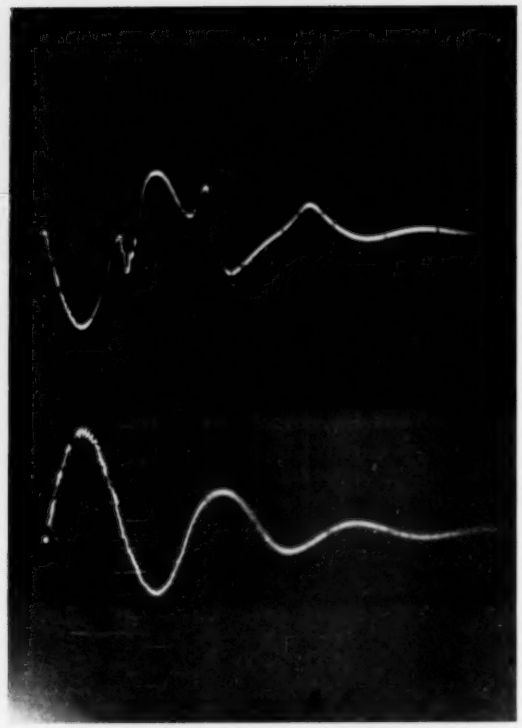

FIG. 4.—D_2 at 90 μ. Capacitor voltage 18 kV. With preionization. Top trace: internal probe (see text). Bottom trace: coil current. Time scale: 1 μsec/div.

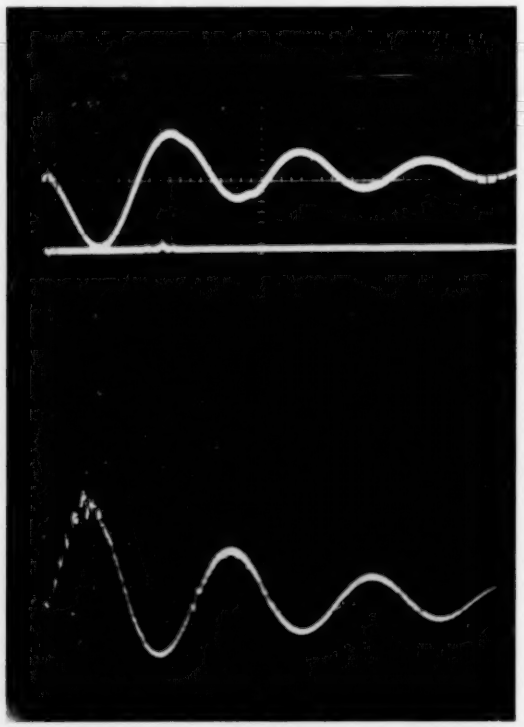

FIG. 5(a).

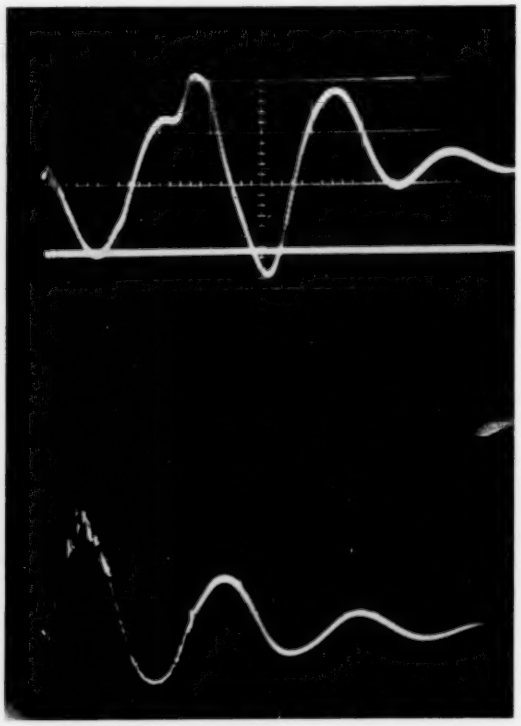

FIG. 5(b).

FIG. 5(a).—D_2 at 65 μ. Capacitor voltage 18 kV. With preionization. Internal probe withdrawn. Top trace: search coil at null point (integrated signal). Second trace: stilbene scintillator with lead shield. Bottom trace: coil current. (b) D_2 at 65 μ: Capacitor voltage 18 kV. With preionization. Internal probe at the centre. Top trace: search coil at null point (integrated signal). Second trace: stilbene scintillator with lead shield. Bottom trace: coil current.

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PRODUCTION OF AN AZIMUTHALLY VARYING MAGNETIC FIELD*

R. A. MESHCHEROV and E. S. MIRONOV

(*Received* 22 *June* 1960)

Abstract—A method is given for calculating the shape of pole surface required to produce an azimuthally varying magnetic field having a given depth and mean magnetic field intensity variation. Model experiments on a 1·5 m cyclotron having an azimuthal magnetic field variation are described.

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1. POLE-SHAPE CALCULATIONS

To calculate the pole shape required to give a magnetic field strength in the median plane having some specified form $H_z = F(r,\phi)$ one can use a method which assumes that the pole surfaces are magnetic equipotential surfaces. For axially symmetric poles, the scalar magnetic potential in cylindrical co-ordinates can be given in the form

$$P(r,\phi,z) = \sum_{k=1}^{\infty} v_{2k-1}(r,\phi)z^{2k-1}. \qquad (1)$$

The coefficients of this series can be obtained from the following recurrence relation, derived from Laplace's equation:

$$v_{2k+1} = -\frac{1}{2k(2k+1)} \times \left(\frac{\partial^2 v_{2k-1}}{\partial r^2} + \frac{1}{r}\frac{\partial v_{2k-1}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_{2k-1}}{\partial \phi^2}\right). \qquad (2)$$

Clearly,

$$v_1 = \left.\frac{\partial P}{\partial z}\right|_{z=0} = -H_z(r,\phi). \qquad (3)$$

Using equations (1)–(3), it is possible to calculate the shape of the pole surfaces for any arbitrary distribution $H_z(r,\phi)$, which may be defined either analytically or graphically. In particular, this method can be used to calculate the pole shapes needed to produce the field $H_z = H_0(1 + Ar^2 + Br\cos n\phi)$ which was first proposed by THOMAS[2]. However, the pole profile required for such a field is very complex and would be very difficult to machine.

This particular magnetic field variation may be produced much more easily by using flat sector plates[3]. Since there is no accurate method of calculating the magnetic field for such poles the problem was solved by assuming that the intensity of the magnetic field between the middle of the sector and the middle

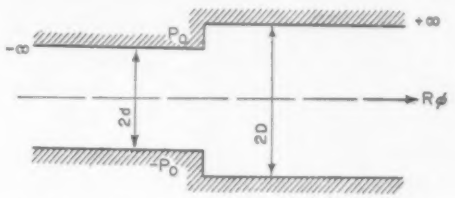

FIG. 1.—Shape of the plane parallel poles assumed for the purpose of calculating the magnetic field strength distribution.

of the recess has the same ϕ-dependence as that between the plane parallel pole pieces represented in Fig. 1. It is obvious that this assumption can be valid only when

$$R\alpha \gg d \text{ and } R\left(\frac{2\pi}{n} - \alpha\right) \gg d$$

where R is the radius, α is the sector angle and n is the number of sectors.

Figure 2 shows the regions on the complex planes

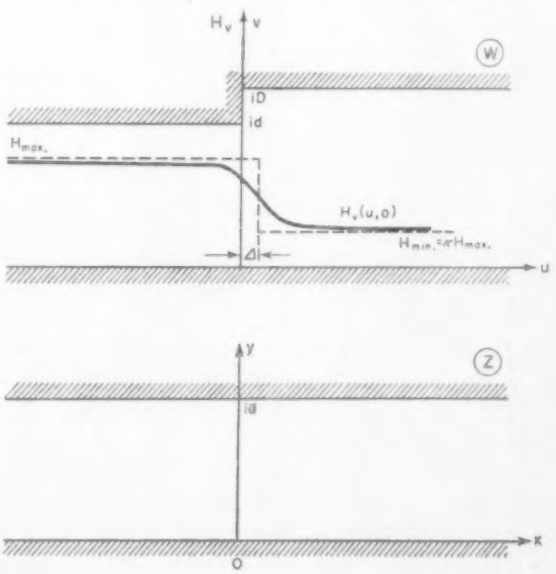

FIG. 2.—Regions on the w and z complex planes.

* Translated by D. L. ALLAN from *Atomnaya Energiya* **10**, 127 (1961).

which transform one into the other with the aid of the formula:

$$w = \frac{d}{\pi} \ln \frac{\sqrt{e^{\pi z/d} + \kappa^2} - \kappa\sqrt{e^{\pi z/d} + 1}}{\sqrt{e^{\pi z/d} + \kappa^2} + \kappa\sqrt{e^{\pi z/d} + 1}} - \frac{D}{\pi} \ln \frac{\sqrt{e^{\pi z/d} + 1} - \sqrt{e^{\pi z/d} + \kappa^2}}{\sqrt{e^{\pi z/d} + 1} + \sqrt{e^{\pi z/d} + \kappa^2}} \quad (4)$$

where $\kappa = d/D$.

From complex potential theory it follows that the magnetic field between the stepped pole pieces shown in Fig. 1 is equal to

$$H(w) = H_u + iH_v = -iH_{\max}\kappa \frac{\sqrt{e^{\pi z/d} + 1}}{\sqrt{e^{\pi z/d} + \kappa^2}}, \quad (5)$$

where $H_{\max}$ is the magnetic field intensity at an infinite distance to the left.

The intensity distribution $H_v = f(u)$ in the median plane between the stepped poles was calculated from equations (4) and (5) using an infinite expansion. The results of this calculation are shown in Fig. 3.

In many cases, particularly when studying the motion of ions in an idealized magnetic field of the sector type[4], it is desired to represent $H_v = f(u)$ in terms of an equivalent stepped field (see Fig. 2). Such a field is characterized by the parameter Δ – the difference between the co-ordinates of the individual steps of the pole surface and the equivalent magnetic field

$$\Delta = \frac{1}{H_{\max}(1-\kappa)} \times \left\{ \int_{-\infty}^{\infty} H_v(u)\,du - \int_{-\infty}^{0} H_{\max}\,du - \int_{0}^{\infty} \kappa H_{\max}\,du \right\} = \frac{d}{\pi(1-\kappa)} \left[\frac{1+\kappa^2}{\kappa} \ln \frac{1+\kappa}{1-\kappa} - \ln \frac{16\kappa^2}{(1-\kappa^2)^2} \right]. \quad (6)$$

2. PRELIMINARY EXPERIMENTS

The experiments were carried out on an electromagnet with 370 mm diameter pole pieces separated by 90 mm. Two 370 mm diameter circular steel plates (thickness 16 mm) were placed between the cylindrical poles and to these were attached 52·5° sectors (thickness 5 mm). The gap between the sectors was 40 mm and consequently $d = 20$ mm, $D = 25$ mm and $\kappa = 0{\cdot}8$. The measurements were made with a central magnetic field intensity $H_0 = 6000$ oersteds. The experimentally determined values of the quantity $H_z(R,\phi)/H_{z\max}$ are shown in Fig. 3, where $H_{z\max}$ is the maximum value of the magnetic field strength beneath the middle region of a sector. As one should expect, the best agreement with calculation was found

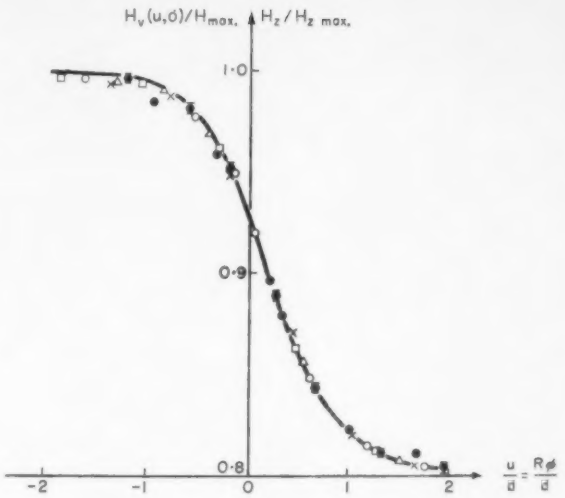

FIG. 3.—Comparison of calculated and experimental results. The continuous curve represents the calculated results; the points denote results of measurements taken at various radii (in mm):
●—50; ×—70; □—90; △—110; ○—130; ⊤—145.

at large distances from the centre; at $R = 50$ mm the experimental values deviate considerably from the calculated values.

Within the range of radii $50 < R < 150$ mm one can assume that

$$\kappa = \frac{d}{D} \simeq \frac{H_{z\,\mathrm{rec}}}{H_{z\,\mathrm{sect}}}$$

where $H_{z\,\mathrm{sect}}$ and $H_{z\,\mathrm{rec}}$ are the values of the magnetic field intensity under the middle of a sector and under the middle of a recess, respectively, for $R = \mathrm{const}$. Consequently, within this range, the boundaries of the equivalent magnetic field produced by the sector plates will be straight lines which do not pass through the origin and the magnetic field intensity, averaged over ϕ, decreases smoothly as R increases from 50 to 145 mm (Fig. 4, curve 1). The equivalent magnetic field boundaries are indicated in Fig. 5 by the dotted lines and the sector plate boundaries are represented by the thicker continuous lines.

Equation (6) can be used to calculate the shape of the sector plates required to produce any dependence $\bar{H}_z(R)$ within the radius limits $50 < R < 145$ mm. For example, to produce the field $\bar{H}_z(R) = \mathrm{const}$. 4 mm was cut off each side of the sector plates parallel to the edges, since for $\kappa = 0{\cdot}8$ and $d = 20$ mm, $\Delta \simeq 4$ mm. Magnetic field measurements with sector plates having this shape showed that $\bar{H}_z(R) \simeq \mathrm{const}$. over the radius range $50 < R < 145$ mm (see Fig. 4, curve 2) but for small values of R, when this method of calculation is inapplicable, the average field strength as

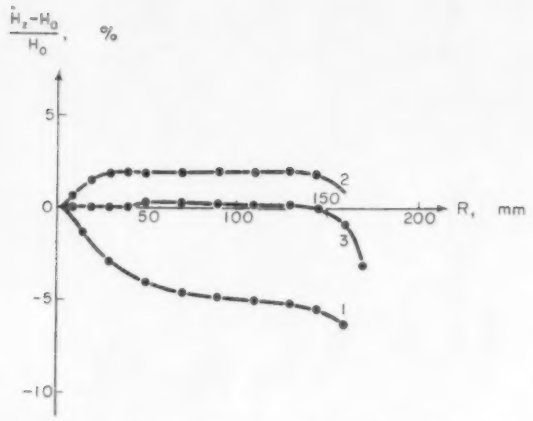

FIG. 4.—Radial dependence of the magnetic field intensity averaged over φ.

before differs considerably from the value of $\bar{H}_z$ at large R. The field was adjusted by cutting off the apex of each sector and placing 40 mm diameter disks in the centre. The thickness of the disks, calculated from the condition $H_0 = \bar{H}_z$, was equal to

$$\delta_0 = D - \frac{d}{\frac{n\alpha}{2\pi} + \frac{d}{D}\left(1 - \frac{n\alpha}{2\pi}\right)} = 2{\cdot}5 \text{ mm}. \qquad (7)$$

In order to obtain a smoothly varying field, the transition from the central disk to the sectors was accomplished in two steps (represented by the continuous heavy lines in Fig. 6). The mean magnetic field intensity distribution obtained with this central disk is shown in Fig. 4 (curve 3), from which it will be seen that $\bar{H}_z$ is constant to quite good accuracy.

The shape of the sectors was then changed (Fig. 6, dashed curve) so that the mean value of the magnetic field intensity increased with R. The incremental increase in the mean magnetic field intensity $\Delta\bar{H}_z = f(R)$ due to this change in shape is shown in Fig. 7. The ~0·1 per cent difference between the experimental

FIG. 5.—Shape of the sector pole pieces and of the boundaries of the equivalent magnetic field (dotted lines).

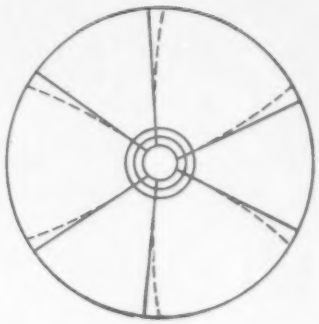

FIG. 6.—Shape of pole pieces required for a given field dependence $H_z(R)$ in the range $0 < R < 145$ mm. For $H_z(R) = $ const. the sector boundaries are as indicated by the continuous heavy lines: for $H_z(R)$ equal to a monotonically increasing function the sector boundaries are as indicated by the dashed lines.

and calculated values is caused partly by machining errors and partly by errors of measurement.

3. MAGNETIC FIELD EXPERIMENTS ON A 1·5 m CYCLOTRON MODEL

Model experiments with a 1·5 m cyclotron were carried out using the electromagnet referred to above. The cylindrical pole pieces were replaced by conical pole pieces having a diameter of 300 mm and circular steel plates (diameter 2 0 mm) were placed between the magnet poles to simulate the lids of the cyclotron accelerating tank. In order to produce a magnetic field having a maximum depth of variation of about $\pm$15 per cent (this figure includes the effect of the chamber) it was necessary to choose $d = 16$ mm and $D = 22$ mm. Three 60° sectors having a thickness $D - d$ equal to 6 mm were attached to each circular plate and, in accordance with the value of Δ for $\kappa = 0{\cdot}73$, 4·4 mm was removed from both sides of each sector parallel to the edge. In addition, the apex of each sector was trimmed off to make room for a central disk (diameter 42 mm and thickness 3·5 mm). Measurements were made with $H_0 = 14500$ oersteds and, as the measurements proceeded, the thickness of the central disk was made more and more accurate ($\delta_0 = 3{\cdot}2$ cm).

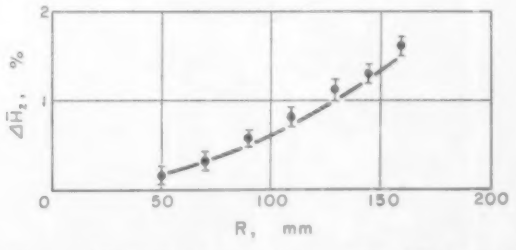

FIG. 7.—Incremental increase in the mean field intensity $\Delta\bar{H}_z = f(R)$ due to changing the shape of the sectors.

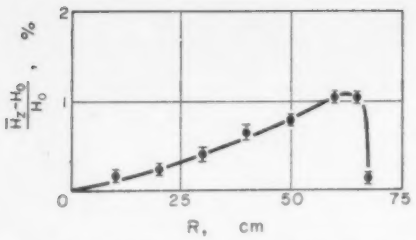

FIG. 9.—$\bar{H}_z(R)$ for the 1·5 m cyclotron ($H_0 = 14000$ oersteds).

For $R > 110$ mm there is a sharp drop in $\bar{H}_z(R)$ caused by edge effects and, to maintain the required field shape, steel correcting shims were inserted over the range $110 < R < 140$ mm.

Since the mean field intensity distribution derived from the model measurements agreed well with the calculated distribution, we proceeded with the manufacture of the lids of the vacuum tank, sector pole pieces and the correcting shims for the 1·5 m cyclotron using the data obtained from the model experiments.

Figure 8 gives a general view of the vacuum tank lids for a 1·5 m cyclotron with an azimuthally varying magnetic field. After making a final adjustment of the magnetic field using large-headed screws we obtained a magnetic field $\bar{H}_z(R)$ having the required shape (Fig. 9). The agreement between the magnetic field strength measurements made on the model and on the final cyclotron magnet was so good that no modifications to the sectors, central disks or peripheral correcting shims were necessary.

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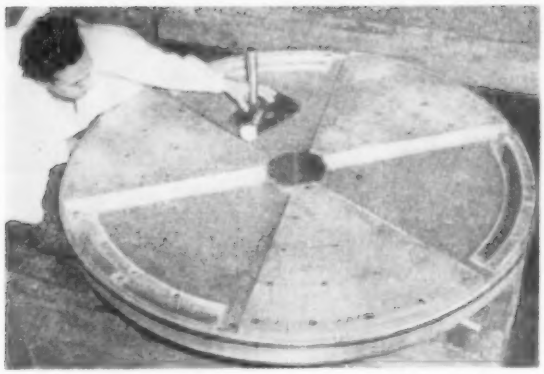

FIG. 8.—General view of the 1·5 m cyclotron vacuum chamber lid.

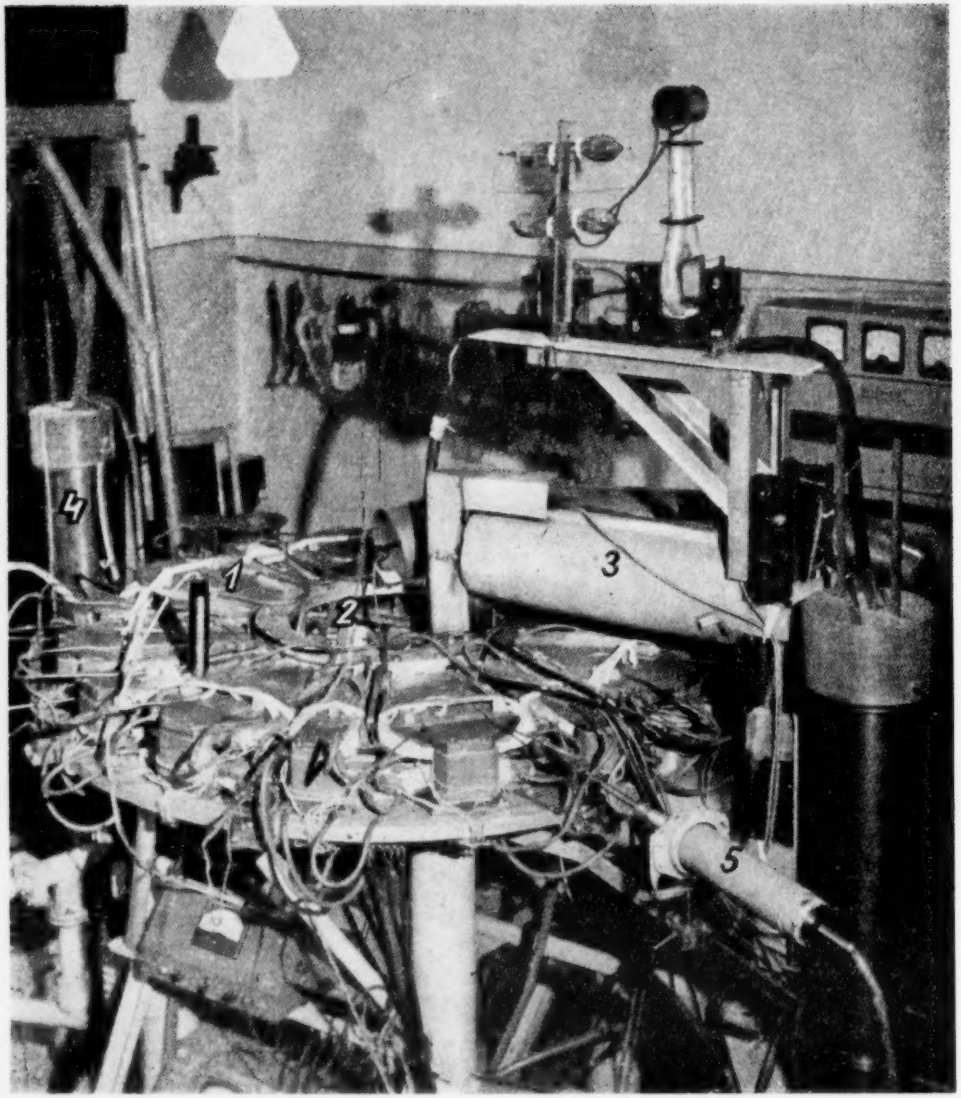

FIG. 1.—General view of the ring phasotron model. (1) Electromagnets; (2) electron injector; (3) induced acceleration core; (4) high-frequency generator; (5) photomultiplier.

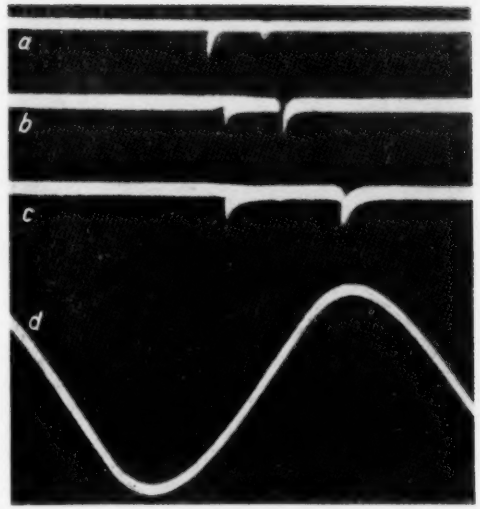

FIG. 2.—Accelerated beam pulses. (a) Without pulsed core; (b) with pulsed core; (c) beam accelerated to final energy; (d) magnetic flux in the main core.

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LETTERS TO THE EDITORS

Model of a ring phasotron*

(*Received* 28 *May* 1960)

THE idea of a ring phasotron,—i.e. a strong focusing accelerator with a magnetic field which is constant with respect to time—was suggested by KOLOMENSKII, PETUKHOV and RABINOVICH in 1953[1] and independently by SYMON[2] in 1955. The construction of such an accelerator would represent a notable advance in the direction of obtaining higher beam intensities and towards the realization of the idea of a colliding beam accelerator.[3,4,5]

A model of a ring phasotron with radial sectors has been built at the Joint Institute of Nuclear Studies. In deciding upon the parameters and general design of this model the authors have allowed for the possibility of changing several of its characteristics over quite a broad range so that it will be possible to make a close study of the behaviour of the particle beam and to try out various ways of stacking it.

A general view of the accelerator is shown in Fig. 1. The model has an electromagnet period of eight, each element having positive and negative sectors and also two straight sections. The azimuthal extent of the positive sectors, which provide the radial focussing, is 22° 30′ while the negative sectors, which provide the vertical focusing, and the straight sections have an azimuthal extent of 7° 30′. The inclusion of negative sectors makes the outermost orbit of a ring phasotron rather larger than that of a conventional strong focusing accelerator. This particular feature of a ring phasotron is characterized by the circumference factor of the machine; this quantity is defined as the ratio of the maximum radius of the orbit to the minimum radius of curvature. In our case this is approximately 3.

The magnetic field in the sectors is produced by current coils wound on the pole-piece surface following the curvature of the instantaneous orbit. The coils are arranged so that the absolute magnitude of the magnetic field increases with the orbital radius R according to the relation $H = H_0\left(\frac{R}{R_0}\right)^4$, from 42 oersteds at $R = 35$ cm to 340 oersteds at $R = 59$ cm. The electromagnet in this model has the special feature that simultaneously with the rise in magnetic field, the gap between the poles increases in proportion to the radius. Thus, the working region in the gap is 2 cm high (in the vertical direction) at the initial radius and 4 cm high at the final radius. Increasing all the dimensions of the electromagnet sectors in proportion to the radius results in geometrically identical instantaneous orbits and this, together with the requirement of a constant mean value for the magnetic field exponent k (in our model, $k = 4$), leads to dynamically equivalent orbits.[6,7] In other words, the frequency of the free oscillations remains constant during the acceleration process.

By changing the ratio of the fields in the positive and negative sectors it is possible to change the number of betatron oscillations per revolution from 1 to 3 in the vertical direction and from 2·5 to 3·5 in the radial direction. The fact that it is possible to vary the free oscillation frequency over such a wide range makes it possible to study the effect of the different resonances on the free oscillations.

The model is intended for the acceleration of electrons. Electrons with energies from 20 to 40 keV can be injected either continuously or in pulses. The electrons are accelerated by a circumferential electric field with an e.m.f. from 10 to 20 V per revolution and a frequency of 450–500 c/s. To obtain better particle capture into the acceleration regime a pulsed betatron core is used which provides an e.m.f. of up to 600 V per turn after approximately 1 μsec.

Experiments have shown that electrons are captured into the acceleration regime when the core is not inserted but the beam intensity is much less in this case.

Figure 2 presents photographs of the accelerated beam (a) without pulsed core and (b) with pulsed core. Figure 2 shows also (c) a beam accelerated to 450 keV, i.e. to the energy which the circumferential electric field is able to impart to the electrons. By combining the circumferential field with a high frequency field the model is capable of producing an electron beam with an energy of 2 MeV.

Results obtained when first starting up this model showed that the initial adjustment was highly critical and so also were the main magnet characteristics. In addition, as expected from the theory, many different sorts of resonance were observed. These had a substantial effect upon the intensity of the accelerated beam.

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Effectiveness of a waveguide as the accelerating system of an electron synchrotron†

(*Received* 31 *August* 1960)

IN an earlier paper[1] the possibility of using a waveguide as the accelerating system of an electron synchrotron was discussed but no estimate was given of the efficiency of the proposed system. The question of the efficiency of the system is discussed below.

Let a LE_{11} wave be propagated in a closed bent-diaphragm type waveguide along the azimuthal direction φ. We are interested in its phase and group velocity, the damping α and in the coupling resistance R_{coupl}. These parameters, together with the Q, determine the total shunt resistance R_{sh}; this quantity being a measure of the efficiency of the high frequency system.

* Translated by D. L. ALLAN from *Atomnaya Energiya* **9**, 491 (1960).

† Translated by D. L. ALLAN from *Atomnaya Energiya* **10**, 69 (1961).

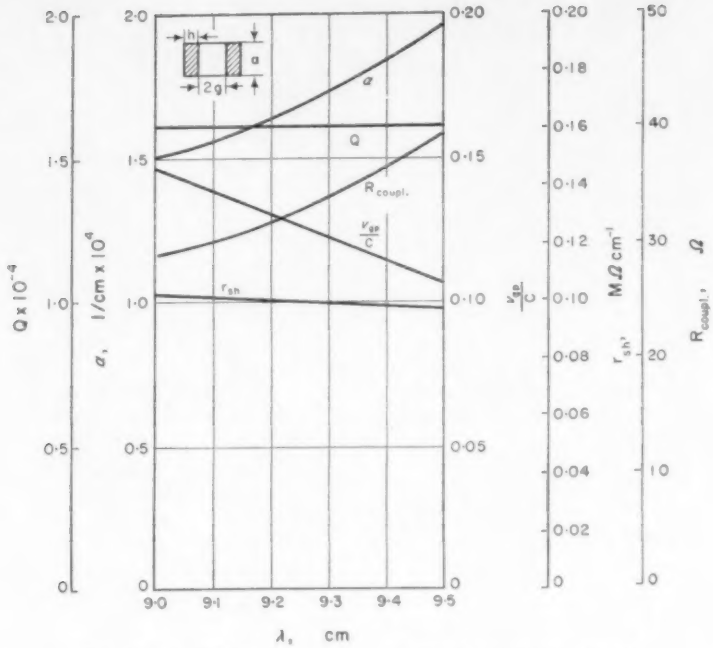

FIG. 1.—Dependence of α, Q, R_{coupl} v_{gp} and r_{sh} on wavelength.

Precise calculations are tedious but a considerable simplification can be introduced by neglecting the spatial harmonics of the wave and ignoring the effect of the curvature on the field structure across the waveguide. The first of these assumptions is quite usual in this kind of calculation[2] and the validity of the second assumption derives from the results of the earlier work[1]. Obviously, the curvature is unimportant in large synchrotrons. The calculation of the damping coefficient α gives

$$\alpha = \alpha_1 + \alpha_2. \tag{1}$$

Here
$$\alpha_1 = \frac{2W\beta_0\pi^2}{kk_0^2a^3}\left(1 + \frac{k_y^2}{\beta_0^2}\frac{\operatorname{sh} 2k_yg + 2k_yg}{\operatorname{sh} 2k_yg + 2k_yg}\right);$$

$$\alpha_2 = \frac{2Wk_y^3}{k\beta_0}\frac{\dfrac{\operatorname{ch}^2 k_yg}{\sin^2 k_0h}}{\operatorname{sh} 2k_yg - 2k_yg}\left[1 + \frac{h}{D} + \frac{2\pi^3h}{k_0^2a^3} + \frac{\pi^2h}{k_0^2a^2D} - \sin 2k_0h\left(1 - \frac{k_0^2 - \dfrac{\pi^2}{a^2}}{2k_0^3D}\right)\right],$$

where
$$W = \frac{1}{2}\sqrt{\frac{c}{\sigma\lambda}};$$

$$k = \frac{\omega}{c};\ \beta_0 = \frac{\omega}{v_\Phi};\ k_0 = \sqrt{k^2 - \frac{\pi^2}{a^2}};$$

$$k_y = \sqrt{\beta_0^2 + \frac{\pi^2}{a^2} - k^2};$$

σ is the conductivity of the walls; ω is the angular frequency; λ is the wavelength, v_φ is the phase velocity of the wave and D is the period of the system (the other dimensions are given in Fig. 1).

Similarly, for the group velocity v_{gp} and the coupling resistance R_{coupl} we find the expressions

$$v_{gp} = \frac{2k_0^2k\beta_0c}{\left(k_0^4 + \dfrac{\beta_0^2\pi^2}{a^2} + k^2\beta_0^2\right) + \left(k^2 + \dfrac{\pi^2}{a^2}\right)k_y^2\dfrac{\operatorname{sh} 2k_yg + 2k_yg}{\operatorname{sh} 2k_yg - 2k_yg} + A\dfrac{\operatorname{ch} 2k_yg}{\operatorname{sh} 2k_yg - 2k_yg}}, \tag{2}$$

where

$$A = \frac{2k_y^2k^2}{k_0\sin^2 k_0h}\left(2k_0h - \frac{\lambda^2}{4a^2}\sin 2k_0h\right),$$

and

$$R_{coupl} = 480\frac{k_y^3}{\beta_0^3}\frac{k\pi}{k_0^2a}\frac{1}{\operatorname{sh} 2k_yg - 2k_yg}. \tag{3}$$

Making use of expressions (1) and (3) it is easy to find Q and r_{sh}—the shunt resistance per unit length of the accelerator:

$$Q = \frac{\omega}{2\alpha v_{gp}};\quad r_{sh} = \frac{4\pi}{\lambda}R_{coupl}Q\frac{v_{gp}}{c}. \tag{4}$$

Figure 1 shows the dependence of α, Q, R_{coupl}, v_{gp} and r_{sh} on the wavelength λ for $v_\varphi = c$, $2g = 6$ cm, $a = 5{\cdot}85$ cm and $D = \lambda/4$. In Fig. 2 the quantities R_{coupl}, v_{gp}/c and r_{sh} are given as functions of $2g$ for $\lambda = 9{\cdot}3$ cm. It follows from Figs. 1 and 2 that r_{sh}, the quantity characterizing the efficiency of the high frequency system, varies weakly with λ but rather strongly with g: if $2g$ is increased from 2 to 6 cm, r_{sh} falls from 0·5 to 0·1 MΩ cm^{-1}. It is interesting to compare these theoretical values of r_{sh} with the shunt resistances of actual accelerators. For the 600 MeV Stanford accelerator, the interaction space of which is 2 cm, $r_{sh} = 0{\cdot}47$ MΩ cm^{-1}. This confirms that the theoretical values of r_{sh} which we have derived are not beyond the bounds of technical possibility and that high shunt resistance values are a general property of high frequency systems based on a waveguide.

From equation (4) it follows that the expression for r_{sh} has the form

$$R_{sh} = 4\pi\ \mathrm{m}\ R_{coupl}\ Q_{eff}\frac{v_{gp}}{c} \tag{5}$$

where Q_{eff} is the effective Q-value of the waveguide which takes all losses into account.

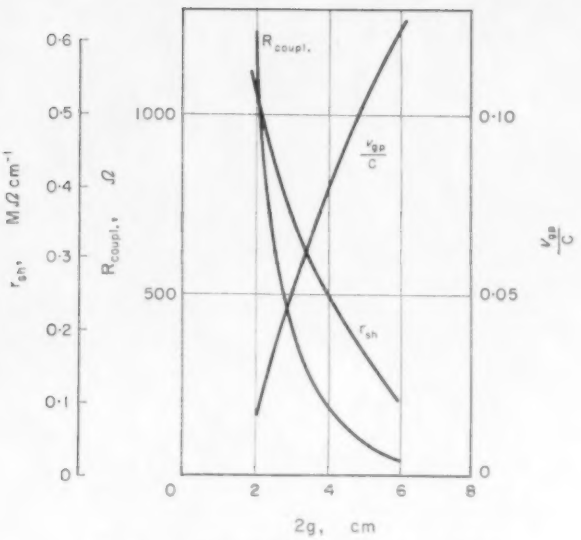

FIG. 2.—Dependence of R_{coupl} v_{gp} and r_{sh} on $2g$.

We will now compare the efficiency indices of a high frequency system of the closed diaphragm waveguide type with the efficiency indices of the best of the most recent accelerating systems employed in large synchrotrons. By using sufficiently good manufacturing techniques it is possible to produce a waveguide having the parameters: $R_{coupl} = 50\,\Omega$, $Q_{eff} = 10^4$ and $v_{gp} = 0{\cdot}1c$. With these parameters a waveguide employed in such an accelerator as the German 6 BeV DESY synchrotron with $\lambda = 10$ cm should have $R_{sh} = 2000$ MΩ. In order to obtain such a shunt resistance using a resonator system it is necessary for each of the 80 resonators in the DESY accelerator to have $R_{sh} = 25$ MΩ. This takes us far beyond the bounds of technical possibility (in fact, the resonators in the DESY accelerator have $R_{sh} = 2$ MΩ). Thus, from the point of view of practicable shunt resistances, waveguide accelerating systems have a more promising future than resonators.

It follows from equation (5) that, other conditions being equal, an increase in the multiplicity m will lead to an increase in the efficiency of the high frequency system. Consequently, from the high frequency power point of view, increasing m is an advantage. However, the relative merits of waveguide accelerating systems and the more conventional resonator systems are invariably bound up with the q-fold increase in the multiplicity. In this case, because of quantal fluctuations in the radiation, the mean square spread in phase increases by a factor of $\sqrt{q}$. To avoid this, if we want the loss of particles to remain the same as before, we must increase the amplitude of the high frequency voltage correspondingly. Using the formula given by ROBINSON[3], it can be shown that the loss of particles in the two accelerators will be equal if

$$\frac{1}{m_1}(\tan\psi_{s1} - \psi_{s1}) = \frac{1}{m_2}(\tan\psi_{s2} - \psi_{s2}). \qquad (6)$$

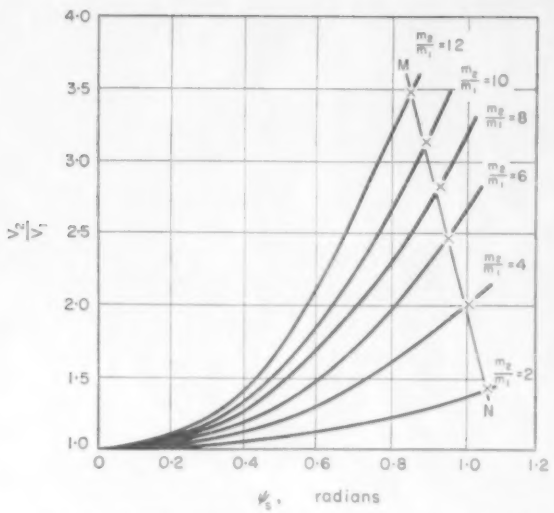

FIG. 3.—Results of the numerical solution of equation (6).

From equation (6) it follows that with the increase in multiplicity, the phase ψ_s must be increased also so that the equality of equation (6) is not upset. Since the radiation losses are identical in the two cases, this leads to an increase of voltage in accordance with the relation

$$\frac{V_2}{V_1} = \frac{\cos\psi_{s1}}{\cos\psi_{s2}}. \qquad (7)$$

However, if the multiplicity is increased R_{sh} increases also[4]. Therefore, when we allow for the fluctuations in the radiation, it is not at all certain that increasing the multiplicity is undesirable. In Fig. 3 we present the results of a numerical solution of equation (6) for various multiplicity ratios m_2/m_1. The region to the left of the line MN corresponds to phases for which a transition to higher harmonics will lead to an energy gain. It will be seen that this region contains all those values which are of the greatest practical interest; it includes the values of the equilibrium phases of the larger accelerators.

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